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THE  
UNIVERSAL ACCOUNTANT  
AND  
COMPLETE MERCHANT.

IN TWO VOLUMES.

By WILLIAM GORDON,  
of the ACADEMY, GLASGOW.

VOLUME I.

*Quid munus reipublicæ majus meliusve afferre possumus, quam  
si juventutem bene erudiamus?* CICERO.

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ТИАТИКОСА ДАРЕНИИ

БИБЛІО

ТИАНОМ АТИКІМ



At th

TO THE HONOURABLE  
THE MERCHANTS OF GREAT BRITAIN,  
UPON  
WHOSE APPLICATION, SKILL, AND ADDRESS,  
THE NATIONAL CREDIT, PROSPERITY, AND GREATNESS;  
THE MAINTENANCE AND IMPROVEMENT OF PEACE,  
AND EXTENSIVE EMPIRE,  
WITH  
THE ENCOURAGEMENT OF INDUSTRY AND INGENUITY,  
AND WHATEVER ELSE TENDS TO PROMOTE  
THE INTEREST OR DIGNITY OF THE STATE,  
CHIEFLY DEPEND;  
THE UNIVERSAL ACCOUNTANT  
IS,  
WITH GREAT SUBMISSION AND RESPECT,  
DEDICATED BY  
THEIR MOST OBEDIENT, AND  
MOST HUMBLE SERVANT,  
WILLIAM GORDON.

At the Academy, Glasgow,  
April 20. 1763.

СКАЗАНИЕ ОН ЗИТ ОГ

СКАЗАНИЕ ТАКИХ О СТИЛЮ РЯЗИ ЗИТ

УПОКИ

СКАЗАНИЕ СКА ЗИМЫ АРГАДАСИИ  
СКАЗАНИЕ СКА ЧУРЧЕНОВА: ТИДЬЮ ЛАЮГИИ ЗИТ  
СКАЗАНИЕ ГИЛМАНОВИДИ СКА ПОНАДГИИА СИИ  
СКАЗАНИЕ ДИЛДАСИИА СКА

ИТИВ

СКАЗАНИЕ СКА ТАРГИИ ЧО ГИЛМАНОВИДИ  
СКАЗАНИЕ СКА ВОЛГИ СКА КИЛДАСИИ СКА  
СКАЗАНИЕ ЗО ЧУРЧЕНОВО СКА СКАЗАНИЕ

ДИЛДАСИИА

БИЛДИССА ДАЛДАКИИ СИИ.

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СКАЗАНИЕ СКА МИЛДАСИИ СКА ИТИВ

ЧУРЧЕНОВИДИ

СКА ГИЛДАСИО ТРОИ ДИТИ

СКАЗАНИЕ КИЛДАСИИ СКА

МОЛДОВИДИ

СКАЗАНИЕ  
ДИЛДАСИИА

To THE P U B L I C.

Whoever considers the numberless transactions that diversify the business of a merchant, must easily see, that a perfect knowledge in figures and accounts, a facility and conciseness of operation, and some idea of trade in general, are necessary prerequisites to fit a gentleman with propriety for the counting-house, without which he can profit little by seeing the most important and extensive business conducted with all the skill and address it is possible to conceive.

The plan of education usually followed in schools, hath been found, by almost constant experience, to be far from answering the design; nor hath any thing hitherto published, contributed so much as could have been wished to supply that defect.

To remove this difficulty, which is one of the greatest obstacles in the pursuit of business; to render mercantile computations intelligible, easy, and concise; to illustrate the method of arranging and adjusting accounts of business, according to the practice of the most ingenious and experienced; and, in short, so to blend the merchant with the accountant, as the application may easily appear,—were the prevailing motives of this undertaking: which, if the execution

execution is adequate to the design, will, it is hoped, be of some importance to trade in general; enable many to pursue with pleasure, what otherwise, perhaps, they might have but little relished; and consequently have some title to claim the countenance and encouragement of the public.

This first volume, which is now offered to the public, contains the art of computation, as the first and leading step to perfect accountanship; the principles whereof, and application to business, are laid down in that intelligible, demonstrative, and practical method, which bids fairest for accuracy, facility, and dispatch.

The observations interspersed throughout the whole of this volume, are intended not only to show the *rationale* of every arithmetical process, but the use to be made of every rule, in real practice. Those which more immediately relate to particular branches of business, are collected from the best authors, or founded on the best intelligence. In a word, the author hath added to a particular study, and the experience of many years teaching, the conversation of the most sagacious and intelligent traders, to whom he could, by any means, have access, to render this performance, in every instance, worthy the title of **THE UNIVERSAL ACCOUNTANT.**

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ESSAY

# E S S A Y

## O N

### The EDUCATION of a YOUNG GENTLE- MAN intended for the COUNTING-HOUSE.

**I**T is a truth, which the ingenious writers of all ages have acknowledged, and constant experience has confirmed, that commerce contributes to the prosperity of states, communities, and individuals, in proportion to the wisdom of the laws and regulations upon which it is established, the privileges by which it is encouraged, and the judgment and address wherewith it is conducted. Wise institutions, and well-concerted encouragements for promoting the interest of trade, are the happy effects of good government; and such is the peculiar importance of an extensive and well-regulated commerce to these kingdoms, that it is hoped it will ever be the object of our public care. But the best regulations and the greatest privileges will signify little, unless they be rendered practical, operative, and useful, by the skill and address of the judicious and industrious merchant. It is he who employs the poor, rewards the ingenious, encourages the industrious, interchanges the produce and manufactures of one country for those of another; binds, and links together in one chain of interest, the universality of the human species, and thus becomes a blessing to mankind, a credit to his country, a source of affluence to all around him, his family, and himself. What extent of knowledge, what abilities must it require, to fit a man for so

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great

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great and valuable purposes? And yet it is certain, that there is not another class of men, in the British community, who labour under greater disadvantages, in point of education, than that of the commercial profession.

A few years are spent at the grammar-school, and perhaps a few more at the university; but so little time is allotted for the grammar-school studies, that few, very few can carry from thence the knowledge or the judgment prerequisite to university-studies; by which means a number of years is spent, and a considerable expense laid out, to very little purpose. Add to this, the low opinion that is generally entertained of the use of those studies among men of business; which, when it happens to be discovered by their children, destroys that emulation and ambition to excel, that ought to support them in the elements of learning; and, in fine, induces them to consider the whole as a formal drudgery imposed on them by custom, which continues only for four years.

At a certain age, not after certain acquisitions, a teacher of figures and accounts is applied to; and, in this case, the cheapest market is often reckoned the best. When the round of this teacher's form is once finished, the student is then turned over to the counting-house; where, if he is found qualified for nothing higher, which is too often the case, he will be employed, during the time of his apprenticeship, in copying letters, going messages, and waiting on the post-office.

The business of the counting-house is of such importance, and every moment so precious to the master, that had he talents for communicating, he hath no time for attending to the instruction of an apprentice;

prentice; who, on the other hand, hath been so little accustomed to think, that his improvement by self-application will be very inconsiderable. Besides, his time of life, and constant habit of indulgence, render him more susceptible of pleasurable impressions, than of improvement in business; the more especially when he was not previously prepared to understand it. Wherefore it is not at all surprising, if many, who, having no foundation in knowledge to qualify them for the purposes of the counting-house, profit little from the expense and the time of an apprenticeship, and from seeing the most extensive business conducted with all the skill and address of the most accomplished merchant. The consequence must, no doubt, be fatal to numbers; and the public interest, as well as private, must suffer greatly by every instance of this nature. It must indeed be acknowledged, that there have been, and still are gentlemen, who destitute of all previous mercantile instruction, without money and without friends, by the uncommon strength of natural abilities, supported only by their own indefatigable industry and application, and perhaps favoured with an extraordinary series of fortunate events, have acquired great estates. But such instances are rare, and rather to be admired than imitated. For we have likewise seen many go through all the forms mentioned above, set out with large capitals, though perhaps without any other mercantile accomplishment but an adventurous spirit, who have shone in the commercial world, while their capitals lasted, as meteors do in the natural; but, like them, soon destroyed themselves, and involved in their ruin all such who were unhappy enough to lie within the sphere

of their influence \*. Commerce is not a game of chance, but a science ; in which he who is most skilled, bids fairest for success ; whereas the man who shoots at random, and leaves the direction to fortune, may go miserably wide of the mark. Parents ought by no means to trust the future prospects of their children in the world to a foundation so weak or uncertain ; and, indeed, it is not reasonable to expect that the most substantial character in the British community, can be formed from an education which is common even to the meanest citizen.

There never was a time when the necessity of a reformation in this particular was greater, or promised more ample rewards. — When Britain, by the force of her arms, hath opened, in all quarters of the world, a passage for an unlimited commerce, which the wisdom of her councils hath established and secured by a glorious peace ; — when unanimity formerly unknown, freedom, peace, and prosperity will give new vigour to the polite arts ; — when our neighbours, the French, who have long been our rivals in commerce, will strain every nerve to recover by their trade what they have lost by the war ; — when the low interest of money, and the extent of our dominions abroad, will induce many people of fortune to strike into trade, by which means the stores abroad will be multiplied, and many more hands employed ; — it is hoped a few thoughts on the education of a merchant, will neither be unseasonable nor unacceptable.

To be able to read the English language with some

\* Novimus novitios quosdam, qui cum se mercaturæ vix dede-  
runt, in magnis mercimonii se implicantes, rem suam male ges-  
sisse ; et profecto imperitos mercatores, multis captionibus suppo-  
sitost, multorumque infidiis expositos, experientia videmus. *St.  
de mercat.*

## ESSAY ON EDUCATION. 5

ease and accuracy, is certainly prerequisite to every other study ; and it is with pleasure that we see daily improvements made in this particular. Men of education have not been ashamed of late to take upon them the direction of children in reading English, which, but a few years ago, was committed to people of very little knowledge. This is a reformation, which, as it was very much wanted, ought to be particularly encouraged and promoted ; although at the same time the purposes of it should by no means be extended, especially by those of rank and fortune, beyond its real bounds. It is imagined by some who have reaped little benefit from three or four years attendance at a grammar-school, that the new method of teaching English, will answer all the purposes intended by the study of dead languages to a man of business. But this opinion is ill founded. The study of the English language is not yet carried to a proper extent ; and if it was, it would still fall short of the purposes of a liberal education. There is no business whatever that requires a greater correspondence, or a diction more pointed and concise, than that of the merchant ; and it would require a singular strength of genius to write even correctly in the English language, unless a foundation in the Greek and Latin languages had been previously laid. The arts and sciences, by these means, are laid open to us, the most ingenious of all ages become our companions and acquaintances, whom we may upon all occasions with freedom consult.

The mind must be prepared and opened by degrees ; and before we know the grammar which respects the genius of our own language, we must go back to the source for the principles of which it is composed.

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composed. The Roman language never arrived at its greatest perfection till it called in the assistance of the Greek; and ours would have been void of force and harmony without the aid of both. Besides, no period of life is so apt for proper impressions, as the years allotted for the grammar-school, and no lessons furnish more excellent examples of correct writing and regular living than what are contained in the classics, if they are properly attended to, and judiciously improved. It is here, where youth are furnished with the first opportunity of passing a proper judgment on what they read, with regard to language, thoughts, reflections, principles, and facts, without which the knowledge of words would be very insignificant. How apt are young people, unless the knowledge of true criticism be properly laid, to admire and imitate the bright more than the solid, the marvellous more than the true, and what is external and adventitious more than personal merit and good sense? And is it not of some importance, that youth should be set to rights in particulars so essential? It is here where the taste for writing and living may be in some measure formed, the judgment rectified, the first principles of honour and equity instilled, the love of virtue and abhorrence of vice excited in the mind, provided the grammar-school studies be properly directed, and carefully pursued. *Quare ergo liberalibus studiis filios erudimus? non quia virtutem dare possunt, sed quia animum ad accipiendam virtutem præparant. Quemadmodum prima illa, ut antiqui vocabant, literatura, per quam pueris elementa traduntur, non docet liberales artes, sed mox percipiendis lacum parat; sic liberales artes non perducunt animum ad virtutem, sed expediunt.*

The study of rhetoric and composition ought by no means

means to be neglected by a young gentleman intended for the counting-room. This will give him an opportunity of reducing to practice, what formerly he had been only taught to relish. It will not only teach, but accustom him to range his thoughts, arguments, and proofs in a proper order, and to clothe them in that dress which circumstances render most natural. By this means he will not only be able to read the works of the best authors with taste and propriety, but be taught to observe the elegance, justness, force, and delicacy of the turns and expressions, and still more, the truth and solidity of the thoughts. Hereby will the connection, disposition, force, and gradation of the different proofs of a discourse be obvious and familiar to him, while at the same time he is led by degrees to speak and write with that freedom and elegance, which in any other way will be found very difficult to attain.

But to speak or write well, however necessary it may be, is not the only object of mercantile instruction. It will be of little consequence to have the understanding improved, if the heart be totally neglected. Man was made by nature for society, but the merchant both by nature and practice ; who, if he is not qualified or not disposed to act his part well, like a bad performer in a concert of music, will destroy the harmony, and render the whole disagreeable. Therefore to tune his mind to virtue and morality, to teach him to blend self-love with benevolence, to moderate his passions, and to subject all his actions to the test of reason, he must have recourse to philosophy.

The principles of law and government ought likewise to constitute a part of the mercantile plan of instruction ; by which we are taught to whom obedience

ence is due, for what it is paid, and in what degree it may be justly required: more particularly in Britain, where we profess to obey the prince according to the laws; and indeed we ourselves are secondary legislators, since we give consent, by representatives, to all the laws by which we are bound, and have a right to petition the great council of the nation, when we find they are deliberating upon any act, which we think will be detrimental to the interest of the community, with respect to commerce, or any other privilege whatever.

When a young man hath been thus accustomed to application, reason, and reflection, when his taste hath been formed and his judgment confirmed; the study of those sciences which more immediately respect the counting house, will become easy and agreeable: but it is necessary his teachers should keep up the same spirit and dignity in their instructions with which his earlier studies were animated, otherwise the design of the whole may be in danger of being frustrated.

The first care of a scholar who is put under the tuition of a new master, is to observe, to study, and to sound him; and it generally holds, that the proficiency of the one, and the authority of the other, are both in proportion to the judgment which the scholar forms of his master's prudence and abilities; for which reason, parents cannot be too strict in their inquiries concerning the temper, qualifications, and character of a master before they trust him with so important a charge, as the happiness and prosperity of their children during the whole course of their lives must depend upon it.

Writing, the elements of arithmetic, and the French

French language, should, I think, be the first objects of instruction, when a young man is sent to an academy, to be prepared for the counting-house; and these ought to be taught at particular hours on the same day. It is necessary that a young man commence the study of the French language early, that he may be able not only to translate, but speak and write the language with ease before he enters the counting-house.

Writing is a prerequisite to every other step; and therefore no time should be lost in making him as soon and as much master of the pen as possible. To teach arithmetic well, which is another leading step, requires more skill and knowledge than perhaps is attended to. It is, of all other sciences, the most necessary to the mercantile profession; and it is not a little surprising that it should by so many be so shamefully neglected. Before arithmetic is applied to computations in business, the powers, properties, and relations of numbers should be particularly taught and explained. Every rule should be demonstrated, exemplified, and illustrated in an easy and intelligible manner; and the examples so multiplied and diversified, that the learner may be thoroughly grounded, and have a reason always ready for what he doth; all the various compendiums, which serve to abbreviate operations, should be distinctly shown and demonstrated, that facility and dispatch may be equally familiar. When he hath thus become master of the capital rules in vulgar and decimal arithmetic, involution and evolution, he ought then to be introduced to geometry and algebra, which of all other studies contribute most to invigorate the mind, to free it from prejudice, credulity, and superstition, and to accustom it

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to attention, and to close and demonstrative reasoning. In the course of these studies, he should be taught a new demonstration of all his arithmetical rules; and the whole theory ought to be reduced to practice, in the mensuration of surfaces and solids, heights and distances, and in constructing the instruments he hath occasion to use.—When practice is thus joined to demonstration, the study of the sciences becomes easy, entertaining, and instructive: whereas, was a young man to hear nothing else but demonstration, he would soon be wearied of that kind of study, and consider it as very dry and insipid: but when he sees the use of mathematics, in laying down plans and maps of countries, selling land by measure, ascertaining the price of labour, and determining the quantity of liquors for a regulation of their price and duty, he must be convinced of their influence, and admire their excellency. To complete his mathematical course, he should be made acquainted with navigation and geography. The first, after such a general acquaintance with the mathematics, will require no great study: but to the last more time and reading will be absolutely necessary.

The solution of a few problems on the globe, and three or four studied harangues, will come far short of answering the design. A teacher who considers the extent of geography necessary to a merchant, must see that the knowledge of the globes is no more than the elements of what he should be instructed in. He must be made acquainted with the use of maps, the situation, extent, produce, manufactures, commerce, ports, politics, and regulations, with respect to trade, of all the nations in the world, not only by public lectures, but by private reading and conversation.

This

This will not be the work of a few days or a month; and those who allot no more time for geography, know very little of the subject. Half an hour every day for six months together spent in private instruction and examination, will perhaps be found little enough for a study so extensive and important.

When the foundation is thus properly laid by such a mathematical course as I have been describing, communicated in that demonstrative and practical manner, which will join science with judgment, and conviction with experience; the counting house must begin to open, and the *arcana mercatorum* be exposed to view. Arithmetic must again be resumed, and the former theory reduced to practice, in all the cases which can occur to the merchant, the banker, the customhouse, and insurance-office; to which every observation ought to be joined, which will serve to illustrate the use of the different examples in that particular branch of business to which they may be applicable. A proper course of reading at this period, which might be wonderfully improved by the conversation of a good master, upon the subjects of insurance, factorage, exchange, and such other branches of business, will be of singular use, not only to form the mind to business, but, when he comes to act for himself, to prevent many tedious and expensive pleas, which an ignorance in the practical arts of negotiating them is frequently apt to create.

To this course of reading, an epistolary correspondence among the students themselves might, with great propriety, be added; as it would give them the practice of folding letters in a quick and dexterous manner, accustom them to digest well whatever they read, and improve their diction, under the correction

of an accurate master, to that clear, pointed, and concise manner of writing which ought peculiarly to distinguish a merchant. Fictitious differences among merchants might likewise be submitted to their judgment, sometimes to two in the way of arbitration, and again to a jury of fifteen; whilst one would assume the character of the plaintiff, and another that of the defendant, and each give in such memorials or representations, according to the nature of the facts considered on, as he thinks most proper to support the cause, the patronage of which was assigned him. Thus will youth be accustomed to think, write, and act like men before they come upon the real stage of action; and their appearance in real life, will have nothing of that awkward and stupid manner which is generally observed in young men for some time after they enter the counting-house.

When a young man hath thus attained to a proper accuracy and dispatch in figuring, and some idea of the different branches of business with which every kind of computation is connected; it is time then to introduce the young merchant to book-keeping, which is the last; but not the least important branch of education previous to the counting-house. It is become a proverb in Holland, that the man who fails did not understand accounts. And indeed, however much a merchant, who is concerned in an extensive trade, may be employed in matters of a higher nature, and upon that account be necessitated to make use of the assistance of others in keeping his books, he ought certainly to be capable of keeping them himself; otherwise he never can be a judge, whether justice is done him in that essential particular or not; neither can he have that idea of his own business,

fulness, which is indispensably necessary to the prosperity of his trade.

This happy method of arranging and adjusting a merchant's transactions, must, like other sciences, be communicated in a rational and demonstrative manner, and not mechanically by rules depending on the memory only. The principles upon which the science is founded, must likewise be reduced to practice by proper examples in foreign and domestic transactions ; such as, buying, selling, importing and exporting for proper, company, and commission account; drawing on, remitting to ; freighting and hiring out vessels for different parts of the world ; making insurance and underwriting ; and the various other articles that may be supposed to diversify the business of the practical counting-house. The nature of all these transactions, and the manner of negotiating them, ought to be particularly explained as they occur ; the forms of invoices and bills of sales, together with the nature of all intermediate accounts, which may be made use of to answer particular purposes, ought to be laid open ; and the forms of all such writs as may be supposed to have been connected with the transactions in the waste-book, should be rendered so familiar, that the young merchant may be able to make them out at once without the assistance of copies.

As the following work is intended to be a complete course of mercantile computations and accountantship, to say more on the method of communicating them would be unnecessary. Only I would beg leave to hint, that there are many things, the knowledge of which is better inculcated by public lectures, private reading and conversation, than in the ordinary method

method of teaching, when, perhaps, there may be two or more classes to direct. The national commerce in general; the trade of the place where we live; the laws, customs, and usages relative to the business of a merchant, the penalties to which he is liable, and the privileges to which he is intitled; the duties, imposts, and other charges laid upon the British produce in other countries, with all the known maxims that relate to the prosperity of trade; will open a wide field for improvement in matters of real use to the master as well as the student.

When the education of a young gentleman is thus conducted, from his earliest years, in a manner calculated to engage his mind in the love of useful knowledge; to improve his understanding; to form his taste, and ripen his judgment; to fix him in the habit of thinking, steadiness, and attention; to promote his address and penetration, and raise his ambition to excel in his particular province; will not the transition to the counting-house be extremely easy and agreeable? His knowledge will be so particular, and his morals so secured, that he will be proof against the arts of the deceitful, the snares of the disingenuous, and the temptations of the wicked. He will, in a short time, be so expert in every part of the business of the practical counting-house, and be able to form such a judgment of every thing he sees transacted, that when he comes to act for himself, every advantage in trade will lie open to him; his knowledge, skill, and address will carry him through all obstacles to his advancement; his talents will supply the place of a large capital; and when the beaten track of business becomes less advantageous, by being in too many hands, he will strike out new paths for himself,

self, and thus bring a balance of wealth, not only to himself, but to the community with which he is connected, by branches of trade unknown before.

How few are there, even among parents, who perhaps have felt the loss of a proper education in their own practice, that consider the extent of knowledge requisite to make a young gentleman appear with dignity in the commercial life ? and how few are there among those who profess to qualify young gentlemen for the counting-house, that have knowledge in any degree proportionable to their credit ? The reason is obvious : In every other article of expense, considered as communities or individuals, we are generally profuse : but in that which relates to education, we are shamefully narrow. This false parsimony, this mistaken frugality, prevents men of genius and education from appearing as teachers, because their talents will turn out to much more account, in almost any other profession whatever ; and if circumstances should have rendered it necessary for a man of some abilities to turn his mind this way, he is obliged to divide his studies among so many different sciences, and his time among so many different classes, to secure to himself a bare subsistence, that he hath neither the leisure, the means, nor the opportunity of that reading or conversation, which is absolutely necessary to his practice, in instructing youth in the most difficult and important branch of British literature. And if this is the case with the ablest teachers, what can be expected of those who became teachers, because they were really qualified for nothing else ? For the instruction of youth in every other science, we have not only excellent institutions, but eminent masters, whose abilities are inquired into and approved.

## 16 ESSAY ON EDUCATION.

ved of, before they are admitted to the important trust : but in this case, great pretensions, which are generally taken upon the teacher's word, and low prices for the articles of education in his scheme, are credentials sufficient to procure him business, though neither the teacher nor the students reap much advantage from it.

The art of managing and forming the mind is perhaps of all others the most intricate and extraordinary, and certainly the most important ; which, that it may be sufficiently studied, ought to be properly rewarded. It is no doubt the business of magistrates, to interest themselves in the education of youth, since they are the nursery of the state, by whom it is renewed and perpetuated, and upon whom the national prosperity, as well as the national existence depends. If part of the public revenues were employed in erecting academies for training up youth to business, especially in trading cities, where every master should have a salary proportioned to the difficulty of his department ; if the most intelligent merchants were appointed as superintendants of these academies, who would take care that none should be admitted as students, whose proficiency in the languages, rhetoric, and philosophy was not previously inquired into, nor any suffered to prosecute the studies prerequisite to the counting-house, whose genius was not in some measure turned to act with dignity in the mercantile profession ; if these gentlemen would inquire often into the morals and proficiency of the students, converse frequently with the masters on the subject of trade, and admit the students according to their seniority in letters to such conversations, and, in short, take every other method of encouraging both masters and students

dents to industry and attention, that they might go through the tedious, the difficult task with alacrity and spirit; if parents, at the same time, would set that value upon education which they sometimes do upon trifles, and be but as careful in having the minds of their children adorned with virtue and good sense, as they are in setting off every thing which relates to their bodies, we would then see a reformation indeed. Was this to be the case, our youth would be long acquainted with the arts of gaining before they would learn how to spend money, and they would not be grown old in debauchery and riot, before they were initiated into business. Was this to be the case, we would soon see a spirit of industry, knowledge, humanity, and good sense diffuse itself among all ranks and denominations, whilst idleness and folly, with all their mischievous train, would be banished the streets. In one word, our teachers would be men of understanding, our young men would be senators, and our "merchants would be princes."



T H E

# UNIVERSAL ACCOUNTANT.



## P A R T I.

### THE ELEMENTS OF ARITHMETIC.

#### I N T R O D U C T I O N.

**A**RITHMETIC is the art of reckoning by numbers, which, from the various combinations of these ten Arabian characters, ۰, ۱, ۲, ۳, ۴, ۵, ۶, ۷, ۸, ۹, teacheth to calculate with expedition, exactness, and ease.

To render operations more short and expressive, with great propriety have been introduced the following

#### C H A R A C T E R S.

The sign	+	- - -	Addition,	- - -	More.
	-	- - -	Subtraction,	- - -	Lefs.
	×	- - -	Multiplication,	- - -	Into.
	÷	- - -	Division,	- - -	By.
	=	- - -	Equality,	- - -	Equal to.
	:	signifies	{ Proportion,	read	{ As to.
	::			- - -	{ So is.
	—	- - -	Majority,	- - -	Greater than.
	—	- - -	Minority,	- - -	Lefs than.
	✓ <sup>2</sup>	- - -	Extraction of,	- - -	Square-root.
	✓ <sup>3</sup>	- - -	Extraction of,	- - -	Cube-root.

It is not my present purpose, nor would it be material, to trace this useful art back to its original, or carry it through all the different steps of its improvement. The Lombards, no doubt, imported it into Britain; and, for its after improvements, we have been equally obliged to the productions and the practice of the ingenious.

VOL. I.

A

C H A P.

## CHAP. I. NUMERATION.

**N**UMERATION teacheth to read or write any number known, or proposed.

## RULE I.

To read any number, divide it by commas into classes of three characters each, beginning at the right hand; over the right-hand figure of the third class, make a point; over that of the fifth, two points; over that of the seventh, three points, &c. The number to the left hand of the first comma, express by thousands; that which hath over it the first point, by millions; that which hath over it the second, by billions; that which hath over it the third, by trillions, &c. The left-hand character of each class is expressed by hundreds, the middle one by tens, and the right-hand one by units.

## RULE II.

To write down any proposed number, reverse the former rule; and, if any intermediate place is wanting, fill it up with the cipher (0). Upon these principles is effected the following table.

## NUMERATION-TABLE.

0 0 0 . 0 0 0 1 . 0 0 0 . 0 0 1 . 0 0 0 . 0 0 1 . 0 0 0 . 0 0 1 .	Units.
1 2 3 . 4 5 6 . 7 8 9 . 1 2 3 . 4 5 6 . 7 8 9 .	Tens.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Hundreds.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Thousands.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Tens of Thousands.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Hundreds of Thousands.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Millions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Tens of Millions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Hundreds of Millions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Thousands of Millions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Tens of Thouſ. of Millions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Hundreds of Thouſ. of Mil.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Billions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Tens of Billions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Hundreds of Billions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Thousands of Billions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Tens of Thouſ. of Billions.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Hundreds of Thouſ. of Bill.
1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 :	Billions.

OB-

## O B S E R V A T I O N .

It is obvious from the table, that all numbers increase in a decuple proportion; and, consequently, that, in a series of numbers, every figure hath a local, as well as simple value: hence ciphers, though they have no simple value, when annexed to significant figures, remove those figures so many steps from the units place, and increase their value accordingly.

## C H A P . II . ADDITION of INTEGERS .

**A** DDITION teacheth to find a sum which shall be equal to several homogeneous ones given.

## R U L E .

Place the numbers, homogeneous under homogeneous successively; then, beginning with the lowest, or units place, find the sum thereof by collecting them all together, and, of that sum, or total, write down under the column of units what belongs to that name, and carry the number of tens to be added with the next column, being the denomination to which, in the very nature of numbers, it belongs: of the sum or total of the column of tens, write likewise down, under that column, the units place, whose value will be tens, and carry the tens to be added with their homogeneous column, which is hundreds. Proceed thus through the whole, and take down the sum of the last column all together, there being no other to which it can be carried.

## E X A M P L E S .

(1.)	(2.)	(3.)	(4.)
75468	74367	478561	59374
37546	43243	37456	38
18764	67548	5937	473
34654	32764	5478	89
21687	13487	5684	9
85463	54728	—	45
273582	286137	—	—

4 OF ARITHMETIC. Part I.

*Illustration of the first example.*

$3+7=10+4=14+4=18+6=24+8=32$ ; of which total, 2 belongs to the units column, and accordingly taken down there; the 3 in its local value is 30, or 3 tens; therefore,  $3+6+8+5+6+4+6=38$  tens, or 380; of which total, 8 belongs to the place of tens, where it is taken down, and the 3, or 300, carried to be added with the column of hundreds; thus,  $3+4+6+6+7+5+4=35$ , or 3500; therefore 5 falls to be noted down in the place of hundreds, whilst the 3 remains to be added with the column of thousands, to which it belongs; thus,  $3+5+1+4+8+7+5=33$ , or, according to its local value, 33,000; of which total, the 3 to the right hand is taken down in the place of thousands, and the other 3 carried to the column of tens of thousands; thus,  $3+8+2+3+1+3+7=27$ , or, in its local value, 270,000; of which sum the 7 is taken down in its homogeneous column, and the 2 in the place of hundreds of thousands, to which it naturally belongs. The whole taken together becomes 273,582.

O B S E R V A T I O N S.

1. Had the units place of any of the sums of the columns been 0, it is plain that 0 would have been taken down in that place.

2. Had any of the sums consisted but of one place only, there would have been nothing to carry to the next column.

3. The reason of this manner of operation is sufficiently demonstrated in the illustration, being a plain deduction from the nature of numbers; and, as all the parts of any thing whatever must be equal to the whole, the sum or total, thus found with sufficient accuracy, must be equal to the several given numbers taken together.

4. Operations in this rule may be proved by dividing the numbers into two or three different classes, finding the sums of these severally, and collecting their totals again into one; which, if the operations were right, will agree with that total which was taken at once. Men of business prove their summations by adding first upwards and then downwards.

E X A M P L E S.

## Chap. III. S U B T R A C T I O N. 5

## E X A M P L E S.

7458	54856
5978	34785
4567	67857
—	18003
8452	34158
3456	51213
3978	242869 upwards.
—	242869 downwards.
33889 = 33889	

5. Expedition, as well as exactness in calculating, depends much upon improving the memory; and, as addition occurs more frequently in business than almost any other rule in arithmetic, both dispatch and accuracy are absolutely necessary: Wherefore, when one is sufficiently accustomed to add the figures one by one, he should be gradually led, to take them two by two, three by three, by either or more, as appears convenient; which will not only promote dispatch, but be less liable to error. Thus, for instance, the first four figures in the units place, of the first example, to one who hath been ever so little accustomed to addition, will at once present the sum of 18. and the other two figures 14=32, &c.

## CHAP. III. SUBTRACTION of INTEGERS.

SUBTRACTION finds the difference, called the *remainder*, betwixt a lesser number, called the *subtrahend*, and a greater, called the *minuend*, and is the converse of addition.

## R U L E.

Place the numbers homogeneous under homogeneous, the subtrahend in a line directly under the minuend. Then, beginning at the units place, if the figure in the subtrahend be equal to that corresponding in the minuend, write down a cipher for the difference; if less, take down the figure which represents the difference; but, if greater, increase it by 10, and then take down the difference; remembering at the same time, that however oft the minuend

## 6 OF ARITHMETIC. Part I.

end must be so increased, the next place in the subtrahend must be likewise increased by unity. Proceed thus till the whole remainder is taken down and completed.

### EXAMPLES in INTEGERS.

	(1.)	(2.)	(3.)	(4.)
Minuend,	8467548	546317	47856478	5741563
Subtrahend,	3254138	257168	39764789	3194585
Remainder,	5213410	289149		

#### *Illustration of the second example.*

Beginning with the units place, because 8 is more than 7, increase 7 with 10, and 17 becomes the minuend; therefore,  $17 - 8 = 9$ , or  $10 - 8 + 7 = 9$ ; because the minuend was thus increased by 10, the next figure in the subtrahend must be increased by 1, which in effect is 10, and then it will be  $6 + 1 = 7$ ; but still the corresponding figure in the minuend is less, and therefore the same increase must be repeated, and then it will be  $10 + 1 - 7 = 4$ . In the same manner, and for the same reason, the next figure in the subtrahend must be increased by 1, and it will become 2; the correspondent figure to which in the minuend is 3, and their difference, without any increase, is 1; which is noted down, and nothing carried to the next figure in the subtrahend, &c.

### OBSERVATIONS.

1. When all the figures in the minuend are greater than, or some of them equal to their correspondents in the subtrahend, it will be obvious, that the difference of the figures put down as correspondents, must, when taken as one sum, be the difference or remainder required; for as all the parts of any number taken together are equal to the whole, so the difference of all the parts of any two numbers make together the difference of the wholes.

2. When any figure in the subtrahend is greater than its correspondent one in the minuend, the latter, before subtraction, is increased by 10; and, for that reason, the next subtrahend figure is increased by 1: because, from the nature of numbers, 10 in any place is equal to 1, in the next place to the left; therefore an equal number increases both factors, and

## Chap. IV. MULTIPLICATION. 7

and the difference must accordingly be equal: for the same difference will always exist betwixt 9 and 17, as betwixt 19 and 27, or betwixt 29 and 37, *ad infinitum*.

3. If one number is to be subtracted from several, several from one, or several from several, it is plain that they must be reduced to two factors before subtraction, by addition.

4. The accuracy of operations in this rule may always be proved by adding the remainder to the subtrahend, whose sum, when the operation is right, will be equal to the minuend; because the subtrahend and remainder are the parts of the minuend which is considered as the whole.

## CHAP. IV. MULTIPLICATION of INTEGERS.

**M**ULTIPLICATION serveth instead of many additions; and, from two numbers given, called the *multiplier* and *multiplicand*, findeth a third, called the *product*, which shall repeat the multiplicand so oft as the multiplier contains unity. For the more expeditious management of this rule, it will be necessary to commit to memory the following

TABLE OF MULTIPLICATION.

$2 \times 2 = 4$	$3 \times 3 = 9$	$4 \times 5 = 20$	$5 \times 8 = 40$	$6 \times 12 = 72$	$8 \times 12 = 96$
$2 \times 3 = 6$	$3 \times 4 = 12$	$4 \times 6 = 24$	$5 \times 9 = 45$	$7 \times 7 = 49$	$9 \times 9 = 81$
$2 \times 4 = 8$	$3 \times 5 = 15$	$4 \times 7 = 28$	$5 \times 10 = 50$	$7 \times 8 = 56$	$9 \times 10 = 90$
$2 \times 5 = 10$	$3 \times 6 = 18$	$4 \times 8 = 32$	$5 \times 11 = 55$	$7 \times 9 = 63$	$9 \times 11 = 99$
$2 \times 6 = 12$	$3 \times 7 = 21$	$4 \times 9 = 36$	$5 \times 12 = 60$	$7 \times 10 = 70$	$9 \times 12 = 108$
$2 \times 7 = 14$	$3 \times 8 = 24$	$4 \times 10 = 40$	$6 \times 6 = 36$	$7 \times 11 = 77$	$10 \times 10 = 100$
$2 \times 8 = 16$	$3 \times 9 = 27$	$4 \times 11 = 44$	$6 \times 7 = 42$	$7 \times 12 = 84$	$10 \times 11 = 110$
$2 \times 9 = 18$	$3 \times 10 = 30$	$4 \times 12 = 48$	$6 \times 8 = 48$	$8 \times 8 = 64$	$10 \times 12 = 120$
$2 \times 10 = 20$	$3 \times 11 = 33$	$5 \times 5 = 25$	$6 \times 9 = 54$	$8 \times 9 = 72$	$11 \times 11 = 121$
$2 \times 11 = 22$	$3 \times 12 = 36$	$5 \times 6 = 30$	$6 \times 10 = 60$	$8 \times 10 = 80$	$11 \times 12 = 132$
$2 \times 12 = 24$	$4 \times 4 = 16$	$5 \times 7 = 35$	$6 \times 11 = 66$	$8 \times 11 = 88$	$12 \times 12 = 144$

### R U L E.

When the multiplier consists of any number within the bounds of the table, the product is found at once, by multiplying every figure, or place of the multiplicand into the multiplier, one after another, beginning with the units place; and the several products

8 OF ARITHMETIC. Part I.

ducts are wrote down as the several sums in addition; but, when the multiplier exceeds the bounds of the table, the product of every particular digit must be taken by itself, the first figure of every particular product placed directly below its respective multiplier, to answer the local value thereof, and the sum of these several products will be the product required.

E X A M P L E S.

(1.)	(2.)	(3.)
875467543 Multiplicand.	437546754	47845678978
8 Multiplier.	12	11
7003749344 Product.	5250561048	

*Illustration of the first example.*

Beginning with the units place, by the table,  $8 \times 3 = 24$ , of which product 4 is taken down in its own place; then  $8 \times 4 = 32 + 2$ , in the second place of the last product, = 34, in its local value = 340, whereof 4 falls to be noted down in its own place, viz. the place of tens; again,  $8 \times 5 = 40 + 3 = 43$ ; 3 is taken down in its own place, and 4 is reserved to be carried to the product of the succeeding figure; then  $8 \times 7 + 4 = 60$ , here 0 is noted;  $8 \times 6 + 6 = 54$ , 4 is noted;  $8 \times 4 + 5 = 37$ , note 7;  $8 \times 5 + 3 = 43$ , note 3;  $8 \times 7 + 4 = 60$ , note 0;  $8 \times 8 + 6 = 70$ , which is taken down all together, because there is no new product to which the figure in the highest place could be carried.

Example (4.)	(5.)	(6.)
745678	549356	67540573
345	678	30405
—	—	—
3728390	3	3
2982712	X	X
2237034	*	1
—	3	3
257258910		

(5.)	(6.)
549356	67540573
678	30405
—	—
337702865	3
270162292	X
202621719	1
—	3
20535571122065	3

O B S E R V A T I O N S.

- When the multiplier consists of any number within the bounds of the table, the reason of the operation will be plain

## Chap. IV. MULTIPLICATION. 9

plain from what hath been said in addition, since multiplication is only a repetition of that rule, so oft as the multiplier contains unity; to be satisfied of which any one may make the experiment at pleasure, by finding the sum of any multiplicand repeated as oft as the multiplier contains unity.

2. When the multiplier exceeds the bounds of the table, the products are taken partially, and the sum of these products must as certainly be the whole product, as it is true that the whole is equal to all its parts taken together. The reason of placing the first figure of every particular multiplier's product below its respective multiplying figure, will appear from this consideration, that as each figure hath a simple and local value, both these values must be retained in the product: for instance, in the multiplier of the 4th example, 4, from the place in which it stands, is really 40, and consequently the first figure of its product is not 2, but 20; for which reason it must stand in the place of tens. For the same reason, 4 in the multiplier of the 6th example is 400; and therefore 2, the first figure of its product, is 200, and for that reason stands in the place of hundreds.

3. Operations in this rule may be proved by shifting the factors, the reason of which is obvious; but more expeditiously by casting out the 9s. For an example, take an illustration of the proof of example 4.

\*  $7+4=11$ , exceeding 9 by 2, and  $2+5+6=13$ , excess  $4+7=11$ , excess  $2+8=10$ , excess 1, which is noted on the right side of the crois. The same is done by the multiplier thus  $3+4+5=12$ , excess 3, noted on the left of the cross; then  $3\times 1=3$ , noted, as it does not exceed 9, on the top of the cross. If it had been 9, 0 would have been noted; if more than 9 or 9s, the excess. After the same manner are the 9s cast out of the sum of the products, and the last excess is found to be 3, which is set at the bottom of the cross, and proves the operation to be right, being equal to the figure at the top. For because in whatever place any figure stands, taken in its simple value, according to the place in which it stands, it will be equal to what remains, after all the 9s contained in its value are taken away; it follows, that the sum of all the figures of which any number consists, considered simply as so many units, is equal to the remainder, after all the 9s are taken out of that number, which can be found in the real value of each figure of which it consists. Hence, if this sum be less than 9, it is equal to what remains when all the 9s possible are taken out of that number. But if this

10 OF ARITHMETIC. Part I.

sum is equal to, or exceeds 9, the remainder, when the 9s are taken out, will be equal to what remains when the 9s are taken out of the given number; because the number of 9s in any number must be equal to the number of 9s which is contained in the several products, and in the sum of the excesses of 9s in those parts.

**CONTRACTIONS IN MULTIPLICATION.**

1. When there are ciphers on the right of either or both factors, they may be neglected in the operation, but annexed to the sum of the products.

**E X A M P L E S.**

(1.)

$$\begin{array}{r} 7415678000 \\ 123000 \\ \hline 22247034 \\ 88988136 \\ \hline 912128394000000 \end{array}$$

(2.)

$$\begin{array}{r} 534000 \\ 560 \\ \hline 3204 \\ 2670 \\ \hline 299040000 \end{array}$$

$$\begin{array}{r} 2 \\ \times \\ 3 \\ \hline 6 \\ 3 \\ \hline 6 \end{array}$$

Hence, to multiply by 1 and any number of ciphers annexed to it, is only to annex those ciphers to the multiplicand.

2. When unity is in the place of tens of the multiplier, the product may be found in one line, by adding the product of that place in the multiplication; and the same method may be extended by practice, to 2 or 3 in the place of tens.

**E X A M P L E S.**

(1.)

$$\begin{array}{r} 874675432 \\ 15 \\ \hline 13120131480 \end{array}$$

(2.)

$$\begin{array}{r} 84670000 \\ 23 \\ \hline 1947410000 \end{array}$$

$$\begin{array}{r} 5 \\ \times \\ 8 \\ \hline 40 \\ 40 \\ \hline 40 \end{array}$$

3. It will be found convenient, in applicate questions, to work by the component parts of the multiplier, which, for any small number, will be found in the table; but if the multiplier be such a number, for which no component parts can

## Chap. IV. MULTIPLICATION. 11

can be exactly found, the nearest component parts must be taken, and the multiplicand being added so often to the last product, as the product of the component parts comes short of the given multiplier, or so often subtracted from it, as the product of the parts exceeds the given multiplier, the sum in the one case, and remainder in the other, will give the true product.

### E X A M P L E S.

(1.)

$$54537543 \text{ by } 56$$

$$\begin{array}{r} \\ 7 \\ \hline 381762801 \\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} ^o \\ 2 \times ^o \\ ^o \end{array}$$

$$3054102408$$

(2.)

$$3741500 \text{ by } 74$$

$$\begin{array}{r} 8 \\ \hline 299320 \\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ 2 \times 2 \\ 4 \end{array}$$

$$\begin{array}{r} 269388000 \text{ Product of } 72 \\ 7483000 \text{ Product of } 2 \\ \hline \end{array}$$

$$276871000 \quad 74$$

The proof is taken by casting the nines out of the given multiplier, and not the artificial ones.

4. If the given multiplier is a number exceeding the bounds of the table, multiply by as many tens as the multiplier consists of places save one, the last product by the first figure on the left hand, the next in order by the succeeding place, &c. the sum of the products of these places gives that required.

### E X A M P L E S.

(1.)

$$8 \times 436843246 \text{ by } 578$$

$$10$$

$$\begin{array}{r} \\ 10 \\ \hline 7 \times 4368432460 \\ 10 \\ \hline \end{array}$$

$$\begin{array}{r} \\ 5 \\ \hline 43684324600 \end{array}$$

(2.)

$$74563000 \text{ by } 6002$$

$$10$$

$$\begin{array}{r} \\ 10 \\ \hline 745630 \end{array}$$

$$\begin{array}{r} \\ 10 \\ \hline 7456300 \end{array}$$

$218421623000$  = the product of 500.

$30579027220$  = the product of 70.

$3494745968$  = the product of 8.

$752495396188$  = the product of 578.

$$74563000$$

$$6$$

$$447378000000$$

$$149726000$$

$$447527126000$$

## 12 OF ARITHMETIC, Part I.

5. If the multiplier be any number near 100, 1000, 10000, &c., increase the multiplicand by as many ciphers as there are figures in the multiplier, and subtract the multiplicand from itself thus increased as often as the multiplier wants units of that by which the multiplicand was increased.

## EXAMPLES.

$$(1.) \text{ Multiply } 8754687 \text{ by } 999 \quad (2.) \text{ 4378 into } 9998$$

999 is 1 short of 1000  
 Therefore  $8754687000$

8754687	43780000
	$8756 = 2 \times 4378$
	$43771244 = 9998 \times 4378$

$\frac{8754687}{8745932313}$

6. When the multiplier can be parted into periods which are multiples of one another, the operation may be contracted in the following manner.

## EXAMPLES.

$$\begin{array}{r} 5697487 \\ 96488 \\ \hline \end{array} \quad (1.)$$

$$\begin{array}{r} 45579896 = 8 \times 5697487 \\ 273479376 = 6 \times 45579896, \text{ because } 60 \times 8 = 480 \\ 546958752 = 2 \times 273479376, \text{ because } 200 \times 480 = 96000 \\ \hline 549739125656 = 5697487 \times 96488 \end{array}$$

*Note.* One number is said to be the multiple of another, when it contains it a certain number of times without any remainder.

Or in a reversed order, thus

$$(2.) \text{ 5742135 into } 52575$$

52575	
28710675	= $50000 \times 5742135$ for 50000
143553375	= $500 \times 28710675$ for 2500
430660125	= $3 \times 143553375$ for 75
301892747625	$= 5742135 \times 52575$

7. If the multiplier be a repetend of the same figure, multiply by one of the repeating figures ; and the figures of that product

duct added, as if they had been wrote down in as many products as the multiplier repeated the same figure, give the product required.

## EXAMPLES.

(1.)  $\begin{array}{r} 547856789 \\ \times 22222 \\ \hline 1095713578 \end{array}$ (2.)  $\begin{array}{r} 54018 \\ \times 3333 \\ \hline 162054 \end{array}$ 

$$\begin{array}{r} 12174473565158 \\ \times 180041994 \\ \hline \end{array}$$

8. When the repeating figure is a high digit, collect the product of as many ones as there are digits in the multiplier, from the multiplicand, according to the rule in the last contraction; which product being multiplied into the repetend, will give the true product.

Example.  $\begin{array}{r} 784325634 \\ \times 7777777 \\ \hline \end{array}$  $\begin{array}{r} 861462839519374 \\ \times 7 \\ \hline \end{array}$  Products collected for 111111. $\begin{array}{r} 6030239876635618 \\ \times 7777777 \\ \hline \end{array}$  Product of 7777777.

There is another contraction for finding the product of any series of repeating figures, more elegant than any of the preceding, but it will come in more properly in the next chapter.

## CHAP. V. DIVISION of whole Numbers.

**D**IVISION findeth how oft one number is contained in another, and is a compendious method of subtraction, in the same sense that multiplication is a compendious method of addition.

## R U L E.

Place the dividing number, called the *divisor*, on the left of the dividend, or number to be divided, and on

## 14 OF ARITHMETIC, Part I.

on the right of the dividend place the quotient, as in the examples following. The factors being thus placed, point off so many places from the right of the dividend, as are equal to, or not exceeding the product of the divisor, into any of the 9 digits, and this is called a *dividual*: in which, having considered how often the divisor is contained, note the number of times in the quotient, then subtract the product of that quotient-figure, after it is multiplied into the divisor, from the dividend, and to the remainder affix the next place in the dividend for a new dividual, with which proceed as before; and if the divisor is not once contained in any dividual, increase the quotient with a cipher, before any new place is taken down to the right of the dividual; if any thing remains after all the places are taken down from the dividend, it is called the *remainder*, which, with the divisor, expresseth some parts of unity, the number whereof is ascertained by the remainder, and the quality by the divisor.

## EXAMPLES.

(1.)	(2.)
Divisor. Dividend. Quotient.	
12 ) 475678 ( 3963975	15)7846735(523115
36	75
<hr/>	<hr/>
115	34
108	30
<hr/>	<hr/>
76	46
72	45
<hr/>	<hr/>
47	17
36	15
<hr/>	<hr/>
118	23
108	15
<hr/>	<hr/>
(10) Remainder.	85
	75
	<hr/>
	(10)

Illustration

*Illustration of Example first.*

The divisor 12 is found in the first dividend 47, three times, which is noted in the quotient, and being multiplied into the divisor, presents a product of 36, which is brought down below its own dividend 47, and subtracted therefrom, by which means we discover a remainder of 11: To this remainder, the next figure in the dividend being affixed, we are presented with a new dividend of 115, which contains the divisor 9 times; consequently 9 is noted in the quotient, and multiplied into the divisor, the product 108, being subtracted from its dividend 115, leaves a remainder of 7 to be increased by the next figure in the dividend, *viz.* 6; with which, and the remaining part of the dividend, we proceed as before, and at last there remains 10, which being taken up with the divisor, and noted after the integral part of the quotient, expresseth ten twelfth parts of one.

## O B S E R V A T I O N S .

1. Since, by the above method of division, the dividend is taken into as many dividuals as possible, and the quotient taken out of the first dividual as near as possible, the defect made the foundation of the succeeding dividual; and this operation being repeated so oft as there were places in the dividend to bring down, or quotient-figures to note, it will be plain, if there hath been no error in the operation, that all the parts of the dividend have been added, and the number of times the divisor is contained in those parts hath been separately found: And since all the parts taken together are equal to the whole, it must follow, that however often the divisor is contained in those parts which constitute the dividend, so often must the divisor be contained in the whole dividend.

2. The best proof of operations in this rule is made by multiplying the quotient into the divisor, whose product added to the remainder, if any, must be exactly equal to the dividend.

A proof less certain, but much more expeditious, may be made by casting out the nines, as in multiplication, considering the integral part of the quotient as a multiplicand, the divisor as a multiplier, and the dividend — the remainder as a product. The proof by the nines, in either case, can only be applied to integers or decimals; so that, upon the whole, the best and most general proof of multiplication is division, and, *vice versa*, of division, multiplication.

Example

## 14 OF ARITHMETIC, Part I.

on the right of the dividend place the quotient, as in the examples following. The factors being thus placed, point off so many places from the right of the dividend, as are equal to, or not exceeding the product of the divisor, into any of the 9 digits, and this is called a *dividual*: in which, having considered how often the divisor is contained, note the number of times in the quotient, then subtract the product of that quotient-figure, after it is multiplied into the divisor, from the dividend, and to the remainder affix the next place in the dividend for a new dividual, with which proceed as before; and if the divisor is not once contained in any dividual, increase the quotient with a cipher, before any new place is taken down to the right of the dividual; if any thing remains after all the places are taken down from the dividend, it is called the *remainder*, which, with the divisor, expresseth some parts of unity, the number whereof is ascertained by the remainder, and the quality by the divisor.

## EXAMPLES.

(1.)	(2.)
Divisor.	Dividend.
12 ) 475678	( 3963975 ) 7846735( 523115
<u>36</u>	<u>75</u>
<u>115</u>	<u>34</u>
<u>108</u>	<u>30</u>
<u>76</u>	<u>46</u>
<u>72</u>	<u>45</u>
<u>47</u>	<u>17</u>
<u>36</u>	<u>15</u>
<u>118</u>	<u>23</u>
<u>108</u>	<u>15</u>
(10) Remainder.	<u>85</u>
	<u>75</u>
	(10)

Illustration

*Illustration of Example first.*

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Example

## 16 OF ARITHMETIC. Part I.

Example (3.)

$$\begin{array}{r}
 375)5478989(14610\overset{2}{3}\overset{1}{4}\overset{2}{9} \\
 375 \quad 375 \\
 \hline
 1728 \quad 73289 \\
 1500 \quad 102270 \\
 \hline
 43830 \\
 2289 \quad \hline \\
 2250 \quad 5478989 \text{ Proof by multiplication.} \\
 \hline
 398 \\
 375 \\
 \hline
 239
 \end{array}$$

(4.)

$$25476)88350768(3468$$

$$\begin{array}{r}
 76428 \\
 \hline
 119227 \\
 101904 \ 3 \ X \ 6 \\
 \hline
 173236 \\
 152856 \\
 \hline
 \end{array}$$

173236  
152856  
 $\hline$

203808 Proof by  
203808 casting out  
 $\hline$  the nines.  
o

### CONTRACTIONS IN DIVISION.

1. In dividing by unity, the quotient will be found just equal to the dividend; therefore in dividing by 1, and any number of ciphers, if as many places are cut off from the right of the dividend, as there are ciphers to the right of 1 in the divisor, the number to the left of the separating point in the dividend will be the integral part of the quotient, and that to the right will be the remainder, or fractional part.

### EXAMPLES.

Quot. Rem.	Quot. Rem.	Quot. Rem.
1000)4578. 567	1000)545. 78	10000)597. 8451

2. For the same reason, when there are ciphers to the right of any divisor, an equal number of ciphers or figures may be cut off from the right of the dividend, and the remaining figures to the left being divided by the significant figures in the divisor, will quote the integral part, and the figures on the right of the point, annexed to the last remainder, will give the fractional part.

### EXAMPLES.

## E X A M P L E S.

$$\begin{array}{r}
 \text{(1.)} \\
 25|000)35784675|000(1431387 \quad 35|00)37645|67(1075\overset{2}{3}\overset{6}{3}\overset{6}{6}\overset{7}{7} \\
 \underline{25} \\
 \underline{\underline{107}} \\
 \underline{100} \\
 \underline{\underline{78}} \\
 \underline{75} \\
 \underline{\underline{34}} \\
 \underline{25} \\
 \underline{\underline{96}} \\
 \underline{75} \\
 \underline{\underline{217}} \\
 \underline{200} \\
 \underline{\underline{175}} \\
 \underline{175} \\
 \underline{\underline{0}}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{(2.)} \\
 35|67(1075\overset{2}{3}\overset{6}{3}\overset{6}{6}\overset{7}{7} \\
 \underline{35} \\
 \underline{\underline{264}} \\
 \underline{245} \\
 \underline{\underline{195}} \\
 \underline{175} \\
 \underline{\underline{(2067)}} \\
 \end{array}$$

*Note.* As an equal number of ciphers was cut off from both factors, there was nothing to constitute a remainder, and therefore there is no fractional part.

3. When the divisor consists but of one or two figures, the operation may be performed by a mental multiplication and subtraction; in which case, no part of the work needs to be noted but the quotient, and it may stand as in any of the subjoined

## E X A M P L E S.

$$\begin{array}{r}
 \text{(1.)} \quad \text{(2.)} \quad \text{(3.)} \\
 5|47856743 \quad 8007456 \quad 75645 \div 25 = 3025\overset{2}{3}\overset{6}{6} \\
 \underline{5} \\
 \underline{\underline{47}} \\
 \underline{45} \\
 \underline{\underline{2}} \\
 \underline{1348\frac{3}{5}} \\
 \quad \quad \quad \underline{70} \\
 \quad \quad \quad \underline{\underline{0}}
 \end{array}$$

4. When the divisor is a composite number, divide by its component parts continually, and the last quotient gives the integral part of the answer. For the fractional part, multiply the last remainder by the last divisor but one, and to the product add the remainder belonging to that divisor; multiply this sum by the next preceding divisor, to which add its cor-

## 18 OF ARITHMETIC. Part I.

respondent remainder; and thus proceed, till you have multiplied by the first divisor, and added in the first remainder.

Example. Divide 7456 $\frac{7}{85}$  by 75

$$75 = 3 \times 5 \times 5 \quad ) \quad 7456785$$

$$5) \overline{1491357}$$

$$3) \overline{298271} : 2$$

$$99423 : 2$$

The remainder is found  
by saying

$$2 \times 5 + 2 \times 5 = \frac{6}{75}$$

Or the remainders may be valued as in the following

## EXAMPLE.

Divide 37841 by 48

$$48 = 8 \times 6 \quad ) \quad 37841$$

$$8) \overline{6306\frac{5}{6}}$$

For 2 when reduced to 6ths =  $\frac{1}{3}$   
and  $\frac{1}{3} + \frac{5}{6} = \frac{7}{6}$ , but  $1 \times 17 = 17$ .  
 $8 \times 6 = 48$ .

$$788 \text{ Rem. } 2\frac{5}{6} = \frac{17}{48}$$

The last method will be found to be most convenient for practice, but it will be best understood when we are come to reduction of vulgar fractions.

5. When both factors are commensurable, it will shorten the division considerably, to abridge both factors according to the following method.

$$\begin{array}{r} 57456 \text{ by } 56 \\ 4) \overline{56} \quad ) \quad 57456 \\ 2) \overline{14} \quad ) \quad 14364 \\ \hline 7 \end{array}$$

1026 Quotient.

$$\begin{array}{r} 11) \overline{88} \quad ) \quad 47415665 \\ 8 \quad ) \quad 4310515 \\ \hline 537564\frac{3}{8} \end{array}$$

This and the preceding contraction are founded on the same principles, *viz.* That if equal quantities be divided by equal quantities, the quotients will be equal; and though they may at first view seem to be one and the same, they will be differently applied, as may be observed in the sequel.

Supplement

*Supplement to contractions in multiplication.*

1. To multiply by a whole number and a fraction, find the product of the integral part as before, and take parts of the multiplicand for the fraction.

Multiply 3475 by $5\frac{1}{2}$	54789	6758	Note $\frac{1}{4}$ of 3 = $\frac{3}{4}$ of 1.
$\frac{5\frac{1}{2}}{—}$	$\frac{4\frac{1}{3}}{—}$	$\frac{3\frac{3}{4}}{—}$	
17375	219156	4) 20274	
1737 $\frac{1}{2}$	18293 $\frac{1}{3}$	5068 $\frac{1}{2}$	
19112 $\frac{1}{2}$	—	25342 $\frac{1}{2}$	
	255742		

2. The digit 9 hath a property peculiar to itself, that whatever other digit, with any number of ciphers annexed, is divided by it, the quotient will consist wholly of such digits, and so many 9ths of an unit over; hence the following method of multiplying by repetends.

E X A M P L E S.

(1.)	(2.)	(3.)
575 by 666	4745 by 777	3987 by 5555
$\frac{6000}{—}$	$\frac{7000}{—}$	$\frac{50000}{—}$
9) 3450000	9) 33215000	9) 189350000
$\underline{383333}$	$\underline{3690555}$	$\underline{21038888}$
383 Subtract.	3690 Subtract.	2103
382950 Product.	3686865 Product.	21036785

This last contraction will be demonstrated immediately after division of decimals, where it will be better understood.

## C H A P. VI.

## ESSAY on MONEY, WEIGHTS, and MEASURES.

**I**N the preceding chapters we have endeavoured, with as much perspicuity and conciseness as possible, to show the properties and combinations of abstract or pure numbers, as far as concerned the fundamental rules of arithmetic, in a method that leads gradually to ease, dispatch, and certainty in calculation. But, before we can show the use of these rules in matters of business, it will be proper to give some account of the monies, weights, and measures, which the British merchant hath occasion to be acquainted with; as a proper knowledge of these is not only intimately connected with, but a *fine qua non* in the mercantile business.

## I. OF MONEY.

In the first ages of commerce, there was little occasion for computation, as one commodity was bartered for another by the bulk; a custom which, even at this day, prevails among the savage unpolished nations of Chili on the South sea, in the land of Jesso on the Pacific ocean, and other barbarous countries: but by degrees, as improvements were made in the world, something new was added daily to the conveniences of life; and as such a method of bartering commodities was found to be difficult and inconvenient, it was agreed among mankind to make choice of one commodity, which being in general and constant esteem, an equivalent quantity of it might always remove the difficulty of bartering in kind. To determine therefore this substance that should be in universal and constant esteem, they made choice of gold and silver, not only because they were divisible and portable, but because they were more valuable than other metals. Since there was a considerable difference in the nature of these two metals, and gold was more precious than silver, both on account of its rarity and intrinsic worth; besides the expense in working gold far exceeds the charge which attends the working silver, as appears by the tax paid upon each to the sovereign lords of mints, that upon gold being only 5 of the hundred, and that on silver 20: it was therefore just to ascribe a greater value to gold. And because the baser metal ought to be given in greater measure, that what was wanting in quality might be made up in quantity;

it

it was likewise found necessary to fix a proportion between them by some certain and determinate rule; whence it is, that, in the practice of commerce, though formerly the proportion of gold to silver was settled as ten to one, yet the matter is so settled at present, throughout the greatest part of Europe, that 1 ounce of gold is worth about 15 ounces of silver. When this substance was agreed upon at first to be a common equivalent for any of the conveniences of life, the particular quantity of it to be given as the value of any thing else, was determined by the bulk and weight only; but afterwards, to save the trouble of proving this weight upon every occasion, it was coined into certain forms by public authority, and impressed with a mark of distinction, expressing the quantity each piece contained, so that it should always have the same determined value, and be every where the same both for matter and weight.

In Britain, as in all other trading places, the current money or specie is either gold, silver, or copper. The standard of gold coin is 22 carats of fine gold, mixed with 2 carats of alloy, in the pound weight Troy; and the standard of silver coin is 11 ounces and 2 pennyweights of fine silver, mixed with 18 pennyweights of copper. These masses being thus proportioned, and respectively divided into pieces of a certain weight, upon which the current stamp, authorised by the prince, is impressed, constitute the several coins we meet with in Britain, the value whereof is determined by an imaginary piece, called the *pound Sterling*, by which we buy and sell, and keep all accounts.

The division of the pound Sterling is as follows.

Farthings, marked - qrs.  
4 = 1 penny, - - d.  
48 = 12 = 1 shilling, s.  
960 = 240 = 20 = 1 pound, L.

When shillings and pence are wrote together, they are often in figures distinguished thus, 4|5, i.e. 4 s. and 5 d.  
10, i.e. 10 s. and |10, i.e.  
10 d.

The coins used in Britain, with their value.

GOLD COINS. L. s. d.  
The Guinea, = 1 10  
Half-guinea, = 0 10 6  
Quarter guinea, = 0 5 3

SILVER COINS.

The Crown, = 0 5 0  
Half-crown, = 0 2 6  
Shilling, = 0 1 0  
Sixpence, = 0 0 6

COPPER COINS.

The Halfpenny, = 0 0 0 5  
Farthing, = 0 0 0 4

There

There are some gold pieces bearing the stamp of other countries which are likewise current in Britain, namely, the Moidore, = L. 1, 7 s. and the Joannes, = L. 3, 12 s. and the half and quarter ditto.

The pound of gold Troy, including the alloy, is divided into  $44\frac{1}{2}$  parts, which are stamped into guineas, and into 89 parts, when stamped into half-guineas.

### OBSERVATION.

If an exact proportion between gold and silver is not maintained, and fixed unalterably, according to some universal rule adopted by the generality of the European nations, the consequence may be dangerous to a kingdom in the affair of money.

Suppose that in some particular kingdom a money-system prevails, that shall raise the gold above its real value, and that in this regulation, instead of the common proportion of 1 to 15, that now obtains, an ounce of gold is allowed to be equivalent to 16 ounces of silver, since such an alteration would raise the gold  $6\frac{2}{3}$  per cent. above its value, and reduce the silver to just so much below its worth; it is evident that this increase of the current price of gold would naturally cause the silver to be exported out of the kingdom; and as gold would be imported in its stead, and increase greatly, the nation must unavoidably lose  $6\frac{2}{3}$  per cent. of all the silver that would be thus exported.

On the other hand, should the silver money be raised above its value, so that 14 ounces should be deemed equivalent to one ounce of gold, while the proportion should stand thus, the silver money would not only continue in the kingdom, but also increase greatly, and the gold coin would be exported in the same proportion; by which means the nation would sustain a loss of  $7\frac{1}{7}$  per cent. Moreover, from these variations two absurdities would follow; the one is, that both the prince and the people would lose of that part of their monied property  $6\frac{2}{3}$  per cent. should the above disproportion fall upon the gold coin; and  $7\frac{1}{7}$ , should it fall upon the silver. The other inconvenience would be, that there would be no specie to circulate in the kingdom, but either gold only, or silver only, according as the one or the other of these metals should happen to be estimated above its true proportion. To maintain an exact proportion therefore between gold and silver, are essential points of good conduct, with regard to the preservation

servation of money, that are by no means to be neglected. There is another consideration, however, that hath a surprising effect on the money-matters of a kingdom, and that is the balance of trade; which when it is against a nation, its money must be carried to foreign nations, to pay for the excess of goods imported above those which have been exported; but if it is in its favour, the money will not only continue in it, but also increase and multiply.

*The Laws of ENGLAND relating to MONEY.*

By 20th *Edw. I.* merchants are prohibited from trafficking with money, and importing clipped coin, under the pain of forfeiture.

9th *Edw. III. c. 1.* Gold or silver plate, or coin, not to be exported without licence, under the pain of forfeiture. Search to be made for false coin imported.

Money not to be impaired in weight or alloy, 25th *Ed. III. c. 13.*

No coin to be current but the King's own, and any person may refuse foreign coin, 27th *Edw. III. c. 14.*

Foreign coin not to be current in England, but to be melted down, 17th *Rich. II. c. 1.*

Coin or plate found in the custody of persons ready to pass the seas, or in any ship, to be forfeited to the King, 2d *Hen. IV. c. 5.*

Treason to clip or file money, 3d *Hen. V. c. 6.*

Gold to be received in payment by the King's weight, 9th *Hen. V. c. 11.*

The mint-master to keep to his alloy, and receive silver at the true value, on pain of double damages, 2d *Hen. VI. c. 12.*

Coins of gold and silver to continue current, notwithstanding they may be cracked or worn, but not, if they are clipped; monies clipped to be exchanged at the mint. Coin transported to Ireland above 6 s. 8 d. or Irish coin imported above 3 s. 4 d. to be forfeited. A circle to be made round the outside of money, 14th and 15th *Hen. VIII. c. 12.*

Counterfeiting, impairing, &c. of coin, or foreign coin made current, is made high treason by 14th *Eliz. c. 3.* and 4. and by 18th *Eliz. c. 1.* and 7.

Silver coin melted down to be forfeited, and double value, 13th and 14th *Ch. II. c. 31.*

Gold and silver delivered into the mint to be assayed, coined, and delivered out, according to the order and time of bringing in, 18th *Ch. II. c. 5.*

Buying or selling clippings or filings, L. 500 penalty. Persons melting coin to be imprisoned six months, besides forfeiture, &c. Persons apprehending money-coiners, &c. to have L. 40 reward; and guilty persons discovering two others, to be pardoned, 6th and 7th Will. III. c. 17.

Persons bringing plate to the mint to be coined, not to pay for coinage, but to have the same weight of money delivered out. Persons keeping public houses to have no manufactured plate but spoons. Molten silver or bullion not to be shipped off, without a certificate from the lord mayor that oath hath been made that it is foreign bullion, under the penalty of L. 200; and officers may seize the bullion as forfeited. Gold or silver, not exceeding L. 200,000, may be exported with a licence. Guineas not to go for more than 22 s. 7th and 8th Will. III. c. 19.

Hammered silver coin brought to the mint, to be received at 5 s. 4 d. per ounce; receivers of taxes, &c. to receive money at 5 s. 8 d. per ounce, to be delivered back to the bringers in; and receivers, &c. to be paid into the exchequer, with an allowance of the deficiency in recoining. Silver-plate, &c. to contain 11 ounces and 10 pennyweights of fine silver in every pound, and to be marked with the two initial letters of the worker's name, on pain of forfeiture. Plate received at 5 s. 4 d. per ounce to be melted down, 8th and 9th Will. III. c. 7. and 8.

It is made high treason, to make any stamp, die, mold, &c. for coining, excepting by persons employed in the mint, &c. Conveying such out of the mint, the same. Colouring metal resembling coin like gold or silver, or marking it on the edges, is likewise high treason: and mixing blanched copper with silver, to make it heavier, and look like gold, or receiving or paying counterfeit milled money, is felony, 8th and 9th Will. III. c. 26.

Hammered silver coin may be refused in payment, as not being the lawful coin of this kingdom, 9th Will. III. c. 2.

Any person may cut, break, or deface pieces of silver money, suspected to be counterfeit, or diminished otherwise than by wearing; but if they should, upon trial, appear to be lawful money, &c. to stand to the loss, 9th and 10th Will. III. c. 21.

No person to make or coin any farthings or halfpence, or pieces to go for such, of copper, under the penalty of L. 5 for every pound weight, 9th and 10th Will. III. c. 33.

On a scarcity of silver coin, for remedy guineas were sunk to 21 s. by proclamation 3d Geo. I.

Persons

Persons counterfeiting broad pieces of gold, or uttering them knowingly, to be guilty of treason, 6th *Geo. II.* c. 26.

Washing, gilding, or altering the impression of any real or counterfeit shilling, or sixpence, or brass-money, to make the one pass for a guinea, or half-guinea, or the other for a shilling or sixpence, is high treason. Knowingly uttering false money, for the first offence six months imprisonment, for the second two years imprisonment, and for the third felony without benefit of clergy. If any person, knowingly uttering false money, shall have about him any other false money, he shall suffer one year's imprisonment; and coiners of halfpence or farthings, two years imprisonment, &c. 15th *Geo. II.* c. 28.

Quarter-guineas were ordered to be coined in the 1st *Geo. III.* of which some had been struck in the reign of *Geo. I.* but were become so rare, that they were scarcely to be met with.

## II. OF WEIGHTS.

As the security of commerce depends much on the justness of weights, most nations have taken care to prevent their being falsified. The standard of weights in Britain is kept in the exchequer, by a particular officer, called the *clerk or comptroller of the market*. By the 27th chapter of *Magna Charta*, the weights are to be the same all over England; but as commerce flourished, and introduced a greater variety of commodities, it was found convenient to vary the original weight, and likewise invent others better calculated for dispatch in business, which hath introduced a diversity of weights, in almost every different county or province. The first of all the weights used in Britain was a grain of wheat, picked out of the middle of the ear, which, being well dried, became the least denomination of Troy weight, now used for gold, silver, jewels, seeds, liquors, bread, and medicines.

### TABLE OF TROY WEIGHT.

#### Grains.

24 = 1 pennyweight, dwt.
480 = 20 = 1 ounce, ʒ.
5760 = 240 = 12 = 1 pound, lb.

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### TABLE OF TROY WEIGHT.

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24 = 1 pennyweight, dwt.
480 = 20 = 1 ounce, ʒ.
5760 = 240 = 12 = 1 pound, lb.

## APOTHECARIES WEIGHT

Is deduced from Troy; but convenience taught them to vary the division, for compounding their medicines, according to the following

## T A B L E.

## Grains.

$$20 = 1 \text{ scruple}, \frac{3}{2}.$$

$$60 = 3 = 1 \text{ dram}, \frac{3}{2}.$$

$$480 = 24 = 8 = 1 \text{ ounce}, \frac{3}{2}.$$

$$5760 = 288 = 96 = 12 = 1 \text{ pound, lb.}$$

## AVOIRDUPOISE WEIGHT

Was rather introduced by chance, and confirmed by custom, than fixed by any law. The Troy weight was in practice found to be too small for coarse and heavy goods, such as grocery wares, pitch, tar, rosin, wax, tallow, flax, hemp, &c. copper, tin, iron, lead, steel, fish, flesh, butter, cheese, salt, &c.; for which, and other such goods, it was thought proper to allow a greater weight than the law had provided, which in this weight exceeds the Troy by  $\frac{1}{3}$ , one pound Avoirdupoise being equal to 1 lb., 2  $\frac{1}{2}$ , 11 dwt.  $15\frac{1}{2}$  grains Troy. In lead, they give only  $19\frac{1}{2}$  cwt. to the tun or foder.

TABLE of AVOIRDUPOISE  
GREATER WEIGHT.

## Ditto LESSER WEIGHT.

lb.

Drops.

$$28 = 1 \text{ quarter, qt.} \quad 16 = 1 \text{ ounce.}$$

$$112 = 4 = 1 \text{ hundred weight, cwt.} \quad 256 = 16 = 1 \text{ pound.}$$

$$2240 = 80 = 20 = 1 \text{ tun.}$$

$$3584 = 224 = 14 = 1 \text{ stone.}$$

After the union, when the weights in Scotland were attempted to be reduced to English standard, it was found that the Scotch Troy pound was equal to 7600 grains, and the English Avoirdupois to 7000: hence, the Scotch, Paris, or Amsterdam pound will be to the pound Avoirdupoise as 38 to 35. Besides the Scotch Troy weight, commonly known by the name of *Dutch weight*, whereof a table is subjoined, there is another weight derived from it, called *Tron weight*, which, in different places, consists of a heavier or lighter pound,

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## Chap. VI. ESSAY ON WEIGHTS. 27

pound, according as custom hath established it. The pound Tron weight runs from 20 to 24 ounces generally, and in some places, but rarely, falls as low as 19 ounces.

### TABLE OF SCOTCH TROY, OR DUTCH WEIGHT.

Grains.	<i>Note</i> , The Tron weight is divided in the same manner as the Troy in the adjacent table, excepting the pound into ounces, of which there is no certain regulation.
36 = 1 drop.	
576 = 16 = 1 $\frac{1}{3}$ .	
9216 = 256 = 16 = 1 lb.	
147456 = 4096 = 256 = 16 = 1 stone.	

Wool weight is founded on, and derived from Avoirdupoise, the pound in both being the same, but the greater weights different.

### TABLE OF WOOL WEIGHT.

#### Pounds.

7 =	1 clove.
14 =	2 = 1 stone.
28 =	4 = 2 = 1 todd.
182 =	26 = 13 = $6\frac{1}{2}$ = 1 wey.
364 =	52 = 26 = 13 = 2 = 1 sack.
4368 =	624 = 312 = 156 = 24 = 12 = 1 last.

- Note* 1. Some few authors make 8 pounds in the clove.  
 2. Woolstaplers generally purchase their wool by the todd, but sell it again, when sorted and stapled, by the pack, consisting of 240 pounds.

### THE REFINERS WEIGHTS.

#### Blanks.

24 =	1 perrot.	<i>Note</i> , What they denominate carats are the $\frac{1}{24}$ of a lb, an $\frac{1}{3}$ ,
480 =	20 = 1 mite.	or any other weight.

### THE WEIGHTS FOR MERCHANTISES USED IN HOLLAND.

24 grains = 1 drachm; 3 drachms, or 72 grains, = 1 gros; 30 grains = 1 engel; 10 engels, or 4 gros and 2 grains, =

D 2 1 loot;

28      OF ARITHMETIC. Part I.

1 loot; 16 loots, or 8 ounces, = 1 mark; 2 marks = 1 pound; 8 pounds = 1 stone; 165 pounds = 1 waggon, or wage; 400 pounds = 1 load; 15 pounds = 1 lispond; 20 lisponds = 1 schippond.

DUTCH WEIGHTS for GOLD and SILVER.

32 aces = 1 engel, 20 engels = 1 ounce, 8 ounces = 1 mark, for gross gold.

24 parts = 1 grain, 12 grains = 1 carat, 24 carats = 1 mark, for fine gold.

*Note.* The mark weights are about 1 per cent. lighter than the Troy weight of London.

WEIGHTS for MERCHANDISE used in HAMBURG.

2 loots = 1 ounce, 16 ounces = 1 pound, 10 pounds = 1 stone of wool or feathers, 14 pounds = 1 dispound, 20 pounds = 1 stone of flax, 8 dispound = 1 center = 120 pounds Avoirdupoise of London. 16 pounds of feathers or wool is a dispound, and 20 dispounds = 1 scippound of the same. 16 dispounds = 1 tun of butter or tallow.

100 pounds of Hamburg = 98 of Amsterdam =  $103\frac{1}{2}$  of Antwerp =  $107\frac{1}{4}$  of London. See the table in the comparison of weights and measures.

III. MEASURES.

The same necessity that introduced money and weights, may justly be supposed to have introduced measures; as the most certain method of ascertaining quantities bought and sold. The common measures used in Britain are those which follow.

I. CLOTH MEASURE, which is of four kinds.

1. The yard = 4 quarters = 16 nails = 36 inches; by which are measured and sold all kinds of English woollen cloths, linen, wrought silks, tape, &c.

2. The English ell =  $1\frac{1}{4}$  yard = 20 nails = 45 inches, by which is measured chiefly a species of fine linen called Holland.

3. The Flemish ell =  $\frac{3}{4}$  yard = 12 nails = 27 inches, chiefly applied to tapestry.

4. The

4. The Scotch ell =  $1\frac{1}{3}\frac{1}{8}$  yard = 37 inches, by which green linen, and most of the private or house manufactures in the country, are bought and sold.

## II. CORN MEASURES, which are of two kinds.

1. English, in which the pint is found to weigh 1 pound Troy, and the several denominations are as follow.

### TABLE OF ENGLISH DRY MEASURE.

#### Pints.

8=	1 gallon.
16=	2= 1 peck.
64=	8= 4= 1 bushel.
256=	32= 16= 4= 1 comb.
512=	64= 32= 8= 2= 1 quarter.
2560=320=160=40=10=5=	1 wey.
5120=640=320=80=20=10=2=	1 last.

Corn, salt, coals, lead-ore; and other dry goods, are measured according to this table. Corn is generally sold in England by the quarter, 5 whereof are reckoned to the ton of freight. The ton of wheat weighs between 2200 and 2500 pounds Avoirdupoise; of rye, between 2100 and 2240 pounds ditto; and of barley, between 1700 and 1800.

By the standard in his Majesty's exchequer, every round bushel with a plain and even bottom,  $18\frac{1}{2}$  inches wide throughout, and 8 inches deep, is esteemed a legal corn bushel, and will contain  $2150\frac{2}{5}$  cubic inches; consequently the corn-gallon contains  $268\frac{4}{7}$  cubic inches.

2. Scotch, in which the boll of meal weighs 8, and amongst the farmers in the north, frequently 9 stone Dutch, or Scotch Troy weight; is divided as follows.

#### Lippies.

4=	1 peck.
16=	4= 1 firlot.
64=	16= 4= 1 boll.
1024=256=64=16=	1 chalder.

*Note*, The wheat firlot in Scotland contains  $21\frac{1}{4}$  pints Scotch measure, and the bear or barley firlot 31; hence the Scotch wheat firlot is to the English corn-bushel as 100 to  $99\frac{2}{5}\frac{1}{5}$ .

#### 3. Liquid

3. Liquid measure was originally raised from Troy weight, as is evident from several statutes, enacting, that 8 pounds Troy of wheat, properly prepared, should weigh one gallon of wine measure, the divisions and multiples whereof should form the other measures, and be the common standard throughout the whole kingdom; yet the invention of a new weight introduced likewise a new gallon adjusted thereto, exceeding the former in the proportion of Avoirdupoise weight to Troy, which serves to proportion the several denominations of ale and beer measure. The sealed gallon at Guildhall, which is the standard for wines, spirits, mead, perry, cyder, vinegar, honey, oil, &c. is supposed to contain 231 cubic inches, on which supposition, the other measures raised therefrom will contain proportionally; yet, by actual experiment, made in 1688, in presence of the Lord Mayor and commissioners of excise, this gallon was only found to contain 224 cubic inches; notwithstanding it was agreed to continue the computation, upon the supposition of 231 cubic inches to the gallon, as before.

## TABLE of WINE MEASURE.

Solid inches.

$$231 = 1 \text{ gallon.}$$

$$4158 = 18 = 1 \text{ runlet.}$$

$$7276\frac{1}{2} = 31\frac{1}{2} = 1\frac{3}{4} = 1 \text{ barrel.}$$

$$9702 = 42 = 2\frac{1}{3} = 1\frac{1}{2} = 1 \text{ terce.}$$

$$14553 = 63 = 3\frac{1}{3} = 2 = 1\frac{1}{3} = 1 \text{ hogshead, (bhd.)}$$

$$19279 = 84 = 4\frac{2}{3} = 2\frac{2}{3} = 2 = 1\frac{1}{3} = 1 \text{ puncheon.}$$

$$29106 = 126 = 7 = 4 = 3 = 2\frac{1}{4} = 1\frac{1}{2} = 1 \text{ butt.}$$

$$582:2 = 252 = 14 = 8 = 6 = 4\frac{1}{2} = 3 = 2 = 1 \text{ tun.}$$

Ale and beer measure, as was formerly observed, is deduced from the Avoirdupoise weight, and therefore the gallon must be much larger than the gallon in wine measure. The standard ale-quart, kept in the exchequer, hath been found by experiment to contain just  $70\frac{1}{2}$  cubic inches; consequently the ale gallon must contain 282 cubic inches. Hence

## ALE-MEASURE TABLE.

Cubic inches.

282	= 1	gallon.
2256	= 8	= 1 firkin.
4512	= 16	= 2 = 1 kilderkin.
9024	= 32	= 4 = 2 = 1 barrel.
13536	= 48	= 6 = 3 = 1 $\frac{1}{2}$ = 1 hhd.

## TABLE OF BEER MEASURE.

Solid inches.

282	= 1	gallon.
2538	= 9	= 1 firkin.
5076	= 18	= 2 = 1 kilderkin.
10152	= 36	= 4 = 2 = 1 barrel.
15228	= 54	= 6 = 3 = 1 $\frac{1}{2}$ = 1 hogshead.
30456	= 108	= 12 = 6 = 3 = 2 = 1 butt.

*Note,* This distinction, or difference betwixt ale and beer measure, is only used in London; for, in all other places, the following table of beer or ale, whether strong or small, is to be observed according to a statute of excise, made in the year 1689.

*Note;* In measuring soap and herrings, 8 gallons is considered as a firkin.

Cubic inches.

35 $\frac{1}{4}$	= 1	pint.
282	= 8	= 1 gallon.
2397	= 68	= 8 $\frac{1}{2}$ = 1 firkin.
4794	= 136	= 17 = 2 = 1 kilderkin.
9588	= 272	= 34 = 4 = 2 = 1 barrel.
14382	= 408	= 51 = 6 = 3 = 1 $\frac{1}{2}$ = 1 hogshead.

In Scotland, the excise and breweries use the English measure; but retailers and victuallers in the country use the Scotch pint, of  $103\frac{3}{4}$  solid inches, whose divisions and multiples are as follows.

4 gills = 1 mutchkin, 2 mutchkins = 1 chopin, 2 chopins = 1 pint, 2 pints = 1 quart, and 4 quarts = 1 gallon.

4. Long

4. Long measure, among other improvements, took its rise from wheat, three grains of which, properly prepared, were in length made the measure of an inch, as in the

## T A B L E.

## Grains.

3=	1 inch.
36=	12= 1 foot.
108=	36= 3 = 1 yard.
594=	198= $16\frac{1}{2}$ = $5\frac{1}{2}$ = 1 pole.
23760=	7920= 660 = 220 = 40=1 furlong.
190080=	63360=5280 =1760 =320=8=1 mile.

Note, 4 poles, or 22 yards, is the length of Gunter's chain, consisting of 100 links, each link =  $73\frac{2}{5}$  inches. But the chain for surveying in Scotland should be 74 feet.

Note, 3 miles = 1 league, and 20 leagues = 1 degree, by common reckoning; but a degree of a great circle, measured upon the surface of the earth, has been found, by the best geographers, to be equal to 69 English miles.

5. Square measure was founded upon long measure, and is differently divided in England and in Scotland.

In English square measure, 144 square inches = 1 foot square; 9 feet square = 1 square yard;  $30\frac{1}{2}$  square yards = 1 pole; 40 poles = 1 rood; and 4 roods = 1 acre.

Though the statute pole be  $16\frac{1}{2}$  feet, in measuring fens and woodlands, they use a pole of 18 feet, and for forests 21 feet. 40 poles in length and 4 in breadth, or 220 yards in length, and 22 in breadth, make a statute acre.

Masons measure their hewn work by the English foot, painters and plasterers by the yard. Glaziers reckon only 8 inches to their lineal foot, and 64 to their square foot.

The square measure used in Scotland is thus divided.

36 square ells=1 fall, 40 falls=1 rood, and 4 roods=1 acre.

The Scotch acre by statute is to the English as 100,000 to 78694.

In Scotland, slaters, masons, and pavers use the square ell and the fall in measuring their work, and the land-surveyors, the fall, the rood, and the acre.

There are some commodities sold by the dozen, of which we reckon 12 = 1 dozen, 12 dozen = 1 small gross, and 12 small gross = 1 great gross.

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## Chap. VI. ESSAY ON MEASURES. 33

Paper is sold by the following denominations, in which 24 sheets = 1 quire, 20 quires = 1 ream, and 10 reams = 1 bale.

Parchment thus: 12 skins = 1 dozen, 15 dozen = 1 roll. Yarn thus: 120 threads = 1 cut, 2 cuts = 1 heer, 6 heers = 1 hank, and 4 hanks = 1 spindle.

In glass, 5 pounds = 1 stone, and 24 stone = 1 seam. The freight of bale-goods is often determined by the tunnage, in which 40 solid feet are reckoned = 1 tun.

Having thus given a brief and succinct account of the measures used in Great Britain, for the benefit of the dealer in foreign spirits, &c. I shall add Mr Postlethwayt's account of the different measures and vessels used in most parts of the world.

### OF MEASURES FOR WINE AND VINEGAR.

" The vessels for containing wine and brandy have different names, according to the quantities they contain, and the countries where they are made use of.

The vessel called in Germany *woeder*, made use of for keeping the wines that grow upon the Rhine and the Moselle, do ordinarily contain 14 aams of Amsterdam, but sometimes they contain more and sometimes less.

The aam of Amsterdam is a measure of 4 anckers, reckoning the ancker of 2 steckans.

The steckan contains 16 mingles, each of which makes two pints.

The verge, or verteel, of the wines upon the Rhine and the Moselle, &c. is reckoned but 6 mingles, that of brandy is counted 6½ mingles, as we shall see hereafter.

The hogshead of Bourdeaux, according to the just measure, should contain 12½ steckans, or 200 mingles of wine and lee; and 12 steckans, or 192 mingles clear wine; so that the tun of Bourdeaux, consisting of 4 hogsheads, contains 50 steckans, or 800 mingles, wine and lee, and 48 steckans, or 768 mingles clear wine.

The tun of Bayonne, and other places thereabout, is reckoned 240 steckans, measure of Amsterdam, there being likewise 4 hogsheads to a tun.

In England, and especially at London, they reckon the hogshead 63, and the tun 252 gallons. The said gallon weighs 7½ pounds weight of London; so that the 63 gallons, or the hogshead, should weigh 472½ pounds, and the tun

1890 pounds weight of London. The said gallon is said to contain 4 Paris pints.

The hogshead of Bourdeaux should contain 110 pots with the lee, and 100 pots clear wine, measure of the said place; so that the said pot of Bourdeaux contains about 2 mingles of Amsterdam.

The Bourdeaux tun of wine should weigh, with the hogshead, 2000 pounds weight; and, in marine terms in freighting of ships, by a tun is meant 2000 pounds weight; so that, when it is said any ship is of so many tuns, it is to be understood that the ship can carry so many times 2000 pounds weight; though in Holland, Flanders, and other northern countries, they only talk of last, containing 2 tuns each, or 400 pounds weight.

The Rhenish and Moselle wines are ordinarily sold at Amsterdam, the former at so many florins of about 20 each, current money, and the latter so many rixdollars, of 50 stivers each, current money, for the aam of 20 verges, or verteels, the verge being, at that rate, 6 mingles, as already said.

French, Spanish, and Portugal wines are sold at so many pound gross the tun of 4 hogsheads, and there is ordinarily 1 *per cent.* rebate for payment in ready money, both buyer and seller paying brokerage, each 6 stivers *per tun.*

The muid of Paris contains 150 quarts, or 300 pints with the lee, and 280 pints clear wine, measure of Paris.

There are all over France a great many vessels for keeping of wine, different from one another, according to the custom of the several provinces where they are made use of; of which, though there be scarce any possibility to give an exact account, we shall here set down the regular fractions of the muid of Paris, 3 of which make the tun of France; and, as we have occasion to speak of the measures of the other provinces of France, we shall give as distinct an account as we can of their contents.

The pint of Paris is a measure pretty well known all over the world; 2 of those pints make 1 quart, 4 quarts 1 sextier, and 36 sextiers 1 muid of Paris; 3 of which (as is already said) make 1 tun of France.

The measure they make use of in Provence is called a *mille-role*; that of Thoulouse should weigh 130 pounds, and ought to contain 66 Paris pints, which is about 100 pints of Amsterdam.

At

At Montpelier, and several other places of Languedoc, their muid contains 18 sextiers, and the sextier 32 pots; so that the muid, which makes but 35 steckans, or 560 mingles of Amsterdam, makes 756 pots of Montpelier; by which it appears that the pot of Montpelier is  $\frac{1}{8}$  less than the mingle of Amsterdam. However, you must here take notice that the casks of Montpelier are not all of an equal measure, some being bigger than others; and, in several places of Provence, as well as High and Low Languedoc, they frequently transport wines, oils, and other such goods, in vessels made of goat-skins.

The butts and pipes of Seville, Malaga, Alicant, Lisbon, Port a Port, Canaries, and isles of Fagel, &c. are likewise of different sizes; for the tun of Malaga, consisting of 2 butts or pipes, (which they call *persemyn* at Amsterdam), is reckoned only 36 or 37 steckans; and those of other places are reckoned at 25 or 26 steckans the butt or pipe.

As for the wine at Hamburg and Lubeck, it is sold at so many rixdollars, of 48 stivers, or 3 marks lubs per tun.

#### Of BRANDY.

French, Spanish, and Portuguese brandy, is ordinarily put into big casks, which some call *pipes*, others *butts*, others *pieces*, viz. according to the custom of the places, there being no positive measure regulated for that liquor.

In France it is ordinarily put into great casks, which they call at Bourdeaux *pieces*, at Rochelle, Nantes, Cognac, Montguion, the isle of Rhé, &c. *pipes*, which (as we have already said) contain some more than others, there being some which hold at Amsterdam from 60 to 90 verges, or verteels; and they reduce those measures into hogsheads, by reckoning as under, for

hoghead makes	27 verges of Cognac, Montguion, Rochelle, and the isle of Rhé.
	29 of Nantes, and other places in Britany and Anjou.
	32 of Bourdeaux, and other places in Guienne.
	32 of Bayonne, and places thereabouts.
	30 of Amsterdam, and other places of Holland.
	30 of Hamburg and Lubeck.
	27 of Embden.

In Provence and Languedoc, they sell it at so much the quintal, or 100 weight with the cask.

At Bruges they call the verges *sefters*, of 16 stoups to a scster, which they sell at so much a-stoup.

At London, and generally through all England, they count only by gallons, as we have said already.

The mingle of brandy weighs, at Amsterdam, 2 pounds 4 ounces; and the verge, or verteel, about 14 pounds; at which rate, the 30 verges must weigh about 320 pounds.

At Bourdeaux, though pieces of brandy contain from 50 to 90 verges, they reckon but 32 to the hogshead; the verge is something less than  $3\frac{1}{2}$  pots.

You must know that whatever there is at Bourdeaux in a piece of brandy more than 50 verges, is called by the farmers of the King's duty *excès*, or an excess, and pays so much *per verge*, besides the duties of sortie, or exportation (as they call it) for the 50 verges.

Those that make brandy seldom or never put it in small barrels, or tierces, except it be designed for some particular places in America, or elsewhere, where those small measures are advantageously sold to people, who, perhaps, would not be able to buy a pipe at a time; for a piece of brandy that contains perhaps  $1\frac{1}{2}$  of an ordinary piece, costs but very little more of freight and carriage than one that contains  $\frac{1}{2}$  or  $\frac{1}{3}$  less.

At Hamburg it is likewise sold at so many pounds gross, of  $7\frac{1}{2}$  marks lubs *per pound* gross, or at so many rixdollars in banco; but at Lubeck it is paid in current money, there being no bank.

At Bremen, Copenhagen, and Embden, it is also sold at so many rixdollars; and in this last place the hogshead is counted but 27 verges.

At London it is sold by the tun of 252 gallons; and, in short, in every country according to the custom of the country, which must always be strictly inquired into by the dealers for their government.

#### Of MEASURES for OIL of OLIVES.

The oil of olives is ordinarily kept in butts or pipes, containing from 20 to 25 steckans, at 16 mingles a-steckan; and there go 717 mingles, or 1434 pints of Amsterdam, to the tun of oil. They reckon at Genoa that the barrel of oil of olives weighs  $187\frac{1}{2}$  pounds nett, of their weights, which make 125 pounds of Amsterdam; and 14 barrels make 717 mingles of the said place, or thereabout.

At

At Leghorn, the barrel of oil of olives weighs 85 pounds of their weight, which is a little more than 59 pounds of Amsterdam.

In Provence they sell it by the measure of that country, called *millerole*, containing 66 Paris pints, which make about 100 pints of Amsterdam; and, in some places of that country and of Lower Languedoc, they put it in certain vessels made of goat-skins, as they do the wine.

In Spain and Portugal it is put in butts and pipes, to be carried over seas, and sometimes in great earthen vessels called jars.

#### Of MEASURES for FISH OIL.

Coarse fish oil is ordinarily kept in barrels, containing from 15 to 20 steckans each.

#### Of MEASURES for HONEY.

Honey is kept in many different sorts of vessels of wood and earth, and sold in some places by measure, and in other places by weight.

At Amsterdam they sell it at so many pounds gross *per ton*, consisting of 6 tierces or aams, or by so many florins the barrel, or the 100 weight. The Bourdeaux and Bayonne honey is sold at Amsterdam from 30 l. to 40 l. gros the tun.

#### Of ROUND MEASURES for GRAIN, &c.

As the great diversity of measures of capacity renders it very troublesome for merchants to calculate the quantities thereof, it will be very necessary to give an account of those that are used in the principal places of Europe for trade.

The last is of several sorts, but all comprehended in these two, *viz.* the sea-last, and that used by land.

A last is reckoned at sea both with regard to measure and weight, according to the nature of the goods.

In measure, there are allowed to a last of goods 2 tuns, or 8 hogsheads of wine, 5 pieces of brandy or prunes, 12 barrels of herrings or pease, 13 barrels of pitch, 4 pipes or butts of oil of olives, and seven quarters or barrels of fish oil.

By weight, there is generally allowed to the last 4000 lb. but, as wool is bulky, they reckon only 2000 lb. to the last thereof,

thereof, and 3600 lb. of almonds; and so likewise they make some abatements of several other sorts of goods, in proportion to their bulk.

The last is not the same in all places, there being some difference introduced by custom in the several countries of Europe.

#### Of the MEASURES of CAPACITY of AMSTERDAM and HOLLAND, &c.

The last of Amsterdam contains 27 muds, and each mud 4 scheppels.

Or, otherwise, the last of Amsterdam contains 36 sacks, and the sack 3 scheppels.

So that the mud is  $\frac{4}{3}$  of the scheppel, and the scheppel is only  $\frac{1}{3}$  of the mud.

A last of wheat commonly weighs between 4200 and 4800 lb. rye between 4000 and 4200 lb. and barley between 3200 and 3400 lb.

But those commodities are so much subject to alteration, by their humidity, &c. that there is but little certainty in their weight.

The last of Amsterdam makes 19 sextiers of Paris, or 38 bushels of Bourdeaux; and three lasts make 4 muds of Rouen.

The last of Munickendam, Edam, Purmeran, and several other places of North Holland, is reckoned equal to that of Amsterdam.

But that of Hoorn and Enchuyzen, being likewise towns in North Holland, is of 22 muds, or 44 sacks, of 2 scheppels each; and so is that of Muyden, Naerden, and Weesop, small towns in the neighbourhood of Amsterdam.

At Haerlem they reckon 38 sacks to the last, their sacks consisting of 3 scheppels, 4 of which make 1 hoedt of Rotterdam, and 14 of those sacks make 1 hoedt of Delft.

The last of Alckmaer, in North Holland, contains 26 sacks.

They reckon 44 sacks to the last of Leyden, and 8 scheppels to the sack.

The last of Rotterdam, Delft, and Schiedam, is composed of 29 sacks, and the sack of 3 scheppels, of which  $10\frac{2}{3}$  make one hoedt; where it is to be observed, that the last of those places is 2 per cent. more than that of Amsterdam.

At Tergow they reckon 28 sacks to the last, 3 scheppels to the sack, and 32 scheppels to the hoedt.

Of the LAST of UTRECHT.

At Utrecht they reckon 25 muids, or sacks, to the last,  $10\frac{1}{2}$  of which sacks make 1 hoedt of Rotterdam.

The last of Amersfort is composed of 64 scheppels.

That of Montfoort, Ysselstein, Vianen, &c. is greater than that of Rotterdam; it is composed of 18 muids, and the mud of 2 sacks.

Of the LAST of FRIESLAND.

The last of Leeuwarden, Haerlingen, and other towns of West Friesland, is composed of 33 muids.

And that of Groningen in East Friesland is of the same measure.

Of the LAST of GUELDERLAND, and country of CLEVES.

The last of Nimeguen, Arnhem, and Dresburg, is composed of 22 mouvers, and the mover of 4 scheppels, 8 of which mouvers make 1 hoedt of Rotterdam.

At Thiel they reckon 33 scheppels to the last.

At Burenande 68 scheppels.

At Haerderwick they reckon 11 muids to 10 of Amsterdam.

Of the LAST of OVER-YSEL.

The last of Campen is of 25 muids for corn, 9 of which make 1 hoedt of Rotterdam.

And 9 muids of Zwoll make likewise 1 hoedt of Rotterdam.

The last of Deventer contains 36 muids of 4 scheppels each.

Of the LAST of ZELAND.

The last of Middleburg is composed of  $4\frac{1}{2}$  sacks of 2 scheppels each, or a little more; and that of Flessing, Zirickzee, the Brill, and some other places, is somewhat different from it, the sack being there reckoned  $2\frac{1}{2}$  scheppels.

Of the LAST of BRABANT.

The last of Antwerp is composed of 38 verteels, of which  $37\frac{1}{2}$  make 1 last of Amsterdam.

Their verteel is composed of 4 mukens, and 32 verteels make the sack for oats.

At

## 40 OF ARITHMETIC. Part I.

At Brussels they reckon 25 sacks equal to the last of Amsterdam.

At Malines they reckon 28 verteels equal to the last of Amsterdam.

The last of Louvain is composed of 37 muds, and each mud of 8 halsters.

At Breda and Steenbergue they reckon  $33\frac{1}{2}$  verteels to the corn last, and 29 for oats; and 13 verteels make 8 sacks, or 1 hoedt of Rotterdam.

At Bergen-op-zoom, they allow 34 verteels to the last of corn, and  $28\frac{1}{2}$  for oats.

That of Bois-le-duc is composed of  $20\frac{1}{2}$  mouvers, 8 of which make 1 hoedt of Amsterdam.

### Of the LAST of several towns in FLANDERS.

The last of Ghent is composed of 56 halsters for corn, and of 38 for oats. Their mud is composed of 6 sacks, each sack of 2 halsters.

At Bruges, the last is composed of  $17\frac{1}{2}$  hoedts for corn, and  $14\frac{1}{2}$  for oats, equal to the last of Amsterdam.

At St Omers, the last is reckoned  $22\frac{1}{2}$  raziers, the razier consisting of 2 scheppels.

At Dixmude, they reckon  $30\frac{1}{2}$  raziers to the last of wheat, and 24 for oats.

At L'Isle, they reckon 41 raziers to the last of wheat, and 30 for oats.

At Gravelin, they reckon  $22\frac{1}{2}$  raziers to the last of corn, and  $18\frac{1}{2}$  for oats.

Eighteen raziers of Dunkirk are equal to one hoedt of Rotterdam.

### Of the LAST of LIEGE.

The last of Liege is composed of 96 sextiers, of 8 muds each: they reckon the corn last of Tongres 15 muds, and that for oats but 14.

### Of the LAST of GREAT BRITAIN and IRELAND.

The last of London consists of  $10\frac{1}{2}$  quarters, or barrels, composed of 8 bushels each, and the bushel of 4 gallons.

The bushel weighs between 56 and 60 lb. and 10 bushels of England make about 1 last of Amsterdam.

In Scotland they reckon 38 bushels to the last, and 18 gallons to the bushel; and in Ireland the same thing.

Of

I.

Chap. VI. ESSAY ON MEASURES. 41

Of the LAST of DANTZICK.

At Dantzick they reckon 36 scheppels to the last, which is equal to 58 scheppels of Amsterdam.

They likewise reckon 16 schipponts to the last, and 340 lb. to the schippont, which makes 5440 lb. to the last; but they give only 15 schipponts, or 5100 lb. weight, the last of oats.

They likewise divide their last at Dantzick into 16 sextiers, measure of Paris, or 20 bushels of Bourdeaux.

They buy and sell their corn at Dantzick, as every other thing, by Polish florins and gros.

Of the LAST of RIGA.

At Riga they reckon 46 loopen to be equal to the last of Amsterdam; and they buy and sell it by rixdollars of 3 florins, or 90 Polish gros.

Of the LAST of KONINGSBERG.

Six lasts of that place are equal to 7 of Amsterdam.

Of the LAST of COPENHAGEN.

They have there several lasts, which differ from one another considerably, according to the different sorts of grain, or other commodities that are measured by them. Ricard makes mention of three several sorts of lasts usual in Copenhagen, viz. of 42 barrels, of 80 scheppels, and of 96 scheppels.

Of the LAST of STOCKHOLM.

At Stockholm they reckon 23 barrels to the LAST.

Of the LAST of HAMBURG, BREMEN, and EMBDEN.

The last of Hamburg consists of 90 scheppels.

At Bremen they reckon 40 scheppels to the last; and 8 lasts of Bremen have held out to 7 lasts, 18 muds, and 1 scheppel at Amsterdam.

At Embden they reckon  $15\frac{1}{2}$  barrels to the last.

Of the MUID, &c. of FRANCE.

The principal measure made use of for grain, &c. at Paris, and most other places of the kingdom, is called *muid*.

The muid contains 12 sextiers, and the sextier 12 bushels. The sextier of good wheat weighs between 244 and 248 lb. marc weight.

They divide the sextier of oats into 24 bushels, which again are subdivided into several smaller measures.

Nineteen sextiers of Paris are reckoned equal to 1 last of Amsterdam.

The muid of Rouen contains 12 sextiers, which are equal to 14 of Paris: it ought to weigh about 3360 lb. marc weight, and makes 28 bushels of Bourdeaux.

Four muds of Rouen are reckoned equal to 3 lasts of Amsterdam.

The sextier of corn weighs 210 lb. weight of Rouen, and is divided into 2 mines, and the mine into 4 bushels.

The muid of Orleans ought to weigh 600 lb. and is composed of 12 mines, equal to  $2\frac{1}{2}$  sextiers of Paris, or 5 bushels of Bourdeaux.

The measure made use of at Lyons, called *asnée*, is divided into 6 bushels, equal to  $1\frac{1}{3}$  sextier, measure of Paris, or  $2\frac{1}{3}$  bushels of Bourdeaux.

Eight bushels of Rouen make 1 sextier of Paris, and 2 bushels of Bourdeaux.

The *asnée* of Macon makes  $1\frac{2}{3}$  sextiers of Paris, or  $3\frac{1}{3}$  bushels of Bourdeaux.

The 5 bushels of Avignon make 3 sextiers of Paris, and 6 bushels of Bourdeaux.

The sextier of Montpelier is composed of 2 emines, and the emine of 2 quarters. The sextier, weighing between 60 and 95 lb. weight of that town, being between 75 and 80 lb. marc weight; so that 100 sextiers make 1 last 22 muds of Amsterdam.

The sextier of Castres is composed of 2 emines, and the emine of 16 bushels. The sextier weighs about 200 lb. weight of that place, which is about 170 lb. marc weight; so that it may be reckoned that 100 sextiers of Castres make 4 lasts of Amsterdam.

The sextier of Abbeville is composed of 16 bushels, and is equal to that of Paris.

The sextier of Amiens weighs from 50 to 52 lb. and 5 sextiers.

The sextier of Bologne weighs 270 lb. small weight; and 8 sextiers of that place render 5 of Paris.

The sextier of Calais weighs 260 lb. and 12 of them render 13 of Paris.

Which

Which sextier of Paris renders

At	St Valery	- - - - -	1 sextier.
	Dieppe	- - - - -	18 mines.
	Havre de Grace	- - - - -	5 $\frac{1}{4}$ bushels.
	Amboise	- - - - -	14 bushels.
	Saumur	- - - - -	1 bushel.
	Tours	- - - - -	14 bushels.
	Blois	- - - - -	20 bushels.
	Aubeterre	- - - - -	5 bushels.
	Barbesieux	- - - - -	5 bushels.
	Perigueux	- - - - -	5 bushels.

The sextier of Arles weighs only 93 lb. marc weight, and the load is 360 lb. weight of that country.

The load of Beaucaire is 2 *per cent.* greater than that of Arles.

The load of Marseilles is composed of 4 emines, and weighs 300 lb. weight of Marseilles, or thereabout, which make 243 lb. marc weight; 100 lb. of which make 123 $\frac{1}{2}$  lb. weight of Marseilles; so that the emine weighs 75 lb. weight of Marseilles.

The load of St Giles's is 18 or 20 *per cent.* greater than that of Arles.

The load of Tarson is 2 *per cent.* less than that of Arles.

The load of Toulon is composed of 3 sextiers of that place, and the sextier contains 1 $\frac{1}{2}$  emines, 3 of which make 2 sextiers of Paris; or, otherwise, they reckon that the bushel weighs 31 lb. and that 7 $\frac{1}{4}$  bushels make one sextier of Paris.

The tun of Auray in Britany is reckoned 2200 lb.

That of Audierne 2300 lb.

That of Brest is 2240 lb.

That of Hennebon 2950 lb.

Port Lewis the same.

Quinpercotentin the same.

The tun of Nantes is composed of 10 sextiers, and the sextier of 16 bushels: it weighs between 2200 and 2250 lb. the measure being heaped, and 18 or 20 *per cent.* less, if otherwise.

The tun of Rennes weighs 2400 lb.

That of St Malo the same.

The tun of Brieux 2600 lb.

That of Rochelle and Maran 42 bushels.

## Of SPAIN.

At Seville they reckon 4 cahies to a last, each cahy consisting of 12 anegras.

The fanegue of Cadiz weighs  $93\frac{1}{4}$  lb. weight of Marseilles,  $3\frac{1}{2}$  lb. of which make the load of 300 lb. weight of Marseilles aforesaid, or 243 lb. marc weight.

## Of PORTUGAL.

At Lisbon they reckon 4 alguiers to the fanegue, 15 fanegues to the muid, and 4 muids to the last of Amsterdam.

## Of ITALY.

Grain is sold at Genoa by the mine.

Two sacks of wheat, at Leghorn, make 288 lb. weight of Marseilles.

Corn is sold at Venice by the sextier, or staro, which is the ordinary measure, two of which make a load of Marseilles.

Of the chief MEASURES of CONSTANTINOPLE, and of the EAST-INDIES in general.

There being but about 3 *per cent.* difference betwixt the aunes of Amsterdam and picos of Constantinople, 100 aunes of Amsterdam make 103 picos of Constantinople; 100 picos of Constantinople make 97 aunes of Amsterdam.

## MEASURES of FORT ST GEORGE, or MADRASS.

## GRAIN MEASURES.

1 measure weighs about - - - 2 lb. 10 oz. Avoir.

8 ditto is 1 mercal - - - 21 —

3200 ditto is 400 ditto, or 1 garfe 8400 —

1 Madrass rupee weighs 7 dwts. 11 gr. Troy, and is better than English standard 14 dwts. 10 gr. in 1 lb.: it is country-touch  $9\frac{7}{8}$ , China-touch  $9\frac{3}{4}$ .

## LIQUID and DRY MEASURES.

1 measure is equal to  $1\frac{1}{2}$  pint English of 423 cubic inches.

8 ditto are equal to 1 mercal of 3384 cubic inches.

400 mercals are equal to 1 garfe of 1,353,600 cubic inches.

1 covid is equal to  $18\frac{9}{10}$  inches.

N. B.

N. B. One measure weighs about 2 lb. 8 oz. Avoirdupoise.

Eight ditto weigh about 21 lb. or 22 lb.

3200 ditto is 400 mercals, or 1 garse, which weighs 8400 lb. which is  $3\frac{3}{4}$  tuns, or 100 Bengal bazaar maunds of 82 lb. 2 oz. 2 dr. each.

#### BENGAL MEASURES.

One measure is five feer.

Eight ditto are forty feer.

The covid (in cloth measure) is nine inches.

#### OF MALACCA MEASURES.

A Malacca quoing is 3200 chupas, or 800 cantins, equal to 5000 Dutch pounds, or 5475 lb. English, or Canton peculs, (according to the Dutch calculation of 125 lb. to a pecul), 40 peculs.

A last is 2000 chupas, 500 cantins, 3000 Dutch pounds, 24 peculs, 3285 lb. English.

#### ANJENGO MEASURE.

One Anjengo covid is eighteen inches English.

#### CALICUTT AND TELLICHERRY MEASURE.

One covid is eighteen inches English; and the Callicutt guz, made use of in measuring timber, is equal to  $28\frac{2}{3}$  inches English.

They likewise, sometimes at Callicutt, measure their timber by the covid and borrebl; twelve borrebls is one covid when the timber is sawed, and 24 borrebls is one covid when unsawed: the price generally is one Callicutt fanam per solid covid.

#### CARWAR MEASURE.

One covid is eighteen inches English.

#### SURAT MEASURES

Are the larger and lesser covid, *viz.*

One covid of 36 inches, and one covid 27 inches.

By the latter all things are sold, except broad cloth, velvet, and satin, which are sold by the large covid, or English yard.

GOMBROON

## GOMBROOM LONG MEASURE.

93 guz are equal to 100 yards English.

## MOCHA MEASURES.

Rice, and other grain, are sold by the kalla and tomard; forty kallas is one tomard, and weighs about 165 lb. but the governor's custom (of half a kalla *per* tomard upon all grain sold) being deducted, and the intolerable cheat in the measuring, together with the pilferage from the water-side home, being allowed for, the Bengal maund will not come out above nineteen kallas; whereas one bag, or Bengal maund, ought to hold out more than a tomard; but, for the foregoing reasons, two Bengal maunds seldom come out above thirty-eight kallas, and rarely that.

Oil is sold by the cudda, noosfia, and vakia.

Sixteen vakias is one noosfia.

Four noosfias, or measures, one cuddy poise, about 18 lb.

Of late years the price has been from three to five noosfias *per* Mocha dollar; and, computing the dupper of two Bengal factory maunds to hold out about 67 or 68 measures each, at which rate, the noosfia, or measure, weighs about  $2\frac{1}{4}$ .

Cotton is sold by the hearf, and nine hearfs is  $11\frac{1}{2}$  Mocha dollars: it generally sells from 30 to 40 hearfs *per* bahar.

## LONG MEASURE.

The guz is twenty-five inches English.

The covid is nineteen inches English.

## CHINA.

## CANTON MEASURE.

Ten punts are one covid in piece-goods, equal to  $14\frac{1}{2}$  inches."

6. Time is a mode of duration, marked and ascertained by certain unerring periods and measures, whereof the apparent motion and revolution of the sun seems to be the principal. Hence the interval of time elapsed between the centre of the sun's appearance on the meridian, and its return after one revolution to the same meridian again, hath been concluded on by all nations to be one day. Again, from the instant that the sun is in the vernal equinox, or first degree

of

of Aries, till it revolve round the ecliptic to the same point again, hath been found from repeated observations to contain 365 days and near  $\frac{1}{4}$ , and the time of this revolution is called a *tropical year*; which, as it did not amount to an exact number of entire days, but the fraction in 4 years would come little short of a day; therefore every fourth year, called *bisextile*, or leap-year, was made to consist of 366 days, and the common years of 365 days.

The year, being thus established, was divided as follows.

Seconds.

60= 1 minute.

3600= 60= 1 hour.

86400= 1440= 24= 1 day. h. min. sec.

$31556937 = 525949 = 8765 = 365 + 5 + 48 + 57 = 1$  trop. year.

In almanacks, the year is divided into 12 calendar months, the names of which, and days they respectively contain, are immediately subjoined.

Months.	Days.	Months.	Days.	Months.	Days.
January,	31	May,	31	September,	30
February,	28	June,	30	October,	31
In leap year	29	July,	31	November,	30
March,	31	August,	31	December,	31
April,	30				

For some particular business, such as payment of wages in the royal navy, they use the following table.

7 days = 1 week, 4 weeks = 1 month, and 13 months = 1 year.

## C H A P. VII.

### A D D I T I O N O F A P P L I C A T E N U M B E R S.

#### R U L E.

WHEN the numbers to be added are of one denomination, they must be placed and added as before: but when the denominations are different,

rent, like, or homogeneal denominations, must stand in one column, so that there must be as many columns as there are denominations given, decreasing from the left hand to the right, as in the subsequent examples. Then, beginning with the lowest denomination, find its sum as in whole numbers, out of which carry to be added with the next column, the units belonging thereto, and note what remains in its proper place; proceed through the whole in this manner, till you come to the integers, or highest place, which are added as before.

## EXAMPLES IN MONEY.

L.	s.	d.	I am indebted as follows; to how much will it amount?	L.	s.	d.
57456	15	6 $\frac{1}{2}$		To A.	74568	19
6478	19	7 $\frac{1}{4}$		To B.	54789	18
5745	17	11 $\frac{1}{2}$		To C.	4900	17
6785	14	10 $\frac{1}{3}$		To D.	578	18
598	11	11 $\frac{1}{2}$		To E.	489	14
678	10	10 $\frac{1}{2}$		To F.	584	15
57	11	10 $\frac{1}{2}$		To G.	674	11
98	14	9 $\frac{1}{4}$		To H.	495	0
5	0	8 $\frac{1}{2}$		To I.	55	0
6	9	6 $\frac{3}{4}$				
7	11	9 $\frac{1}{2}$				
<hr/>						
77919	19	1	Upwards.			
<hr/>						
77919	19	1	Downwards.			

## ILLUSTRATION.

In the place of farthings expressed by the fractions on the right hand, in which  $\frac{1}{2}$  is reckoned 2, I find 24, which is just 6 d.; as there is nothing over to be noted, I carry the 6 d. to the column of pence, in which, including the 6 d. I find 97 pence, or 8 s. 1 d. wherefore the penny falls to be noted down in the column of pence, and the 8 s. carried to be added in with its proper column; in the units place of the column of shillings, the 8 s. included, I find 49; wherefore, as in integers, I note 9, and carry 4 to be added with the 10's place, in which I find 13 tens = 6 twenties, and 1 ten, which is noted before the 9, and both together make 19 s.; the

Chap. VII. ADDITION of applicate Numbers. 49

the L. 6 is carried to be added in with the units place of the pounds, which are added as integers.

O B S E R V A T I O N .

1. If the reason of addition of integers hath been properly attended to, the reason of the last operation will be pretty evident. For since 12 pence is equal to 1 shilling, it is plain that 97 pence is equal to 8 shillings, and 1 penny; wherefore, as it would be not only inconvenient, but really absurd, to write shillings in the place of pence, no more falls to be noted in the place of pence, but the penny which is over the shillings, being truly a part of the next column, &c.

2. From the addition of the last example, it will appear very necessary that the foregoing tables be committed to memory, otherwise it will be impossible to add at all.

3. It will be found very convenient, in adding those denominations, in whose units place you cannot stop by ten, as you did in the place of shillings of the last example, to make a table of the nature of those subjoined, which is effected by multiplying the number in question by 2, 3, 4, 5, &c. and commit these several products to memory, especially in such cases as more frequently occur in practice.

4. Though the method of performing addition is perhaps as easily discovered as any other rule in arithmetic; yet, to add with accuracy, and at the same time with dispatch, requires a considerable practice: I would therefore advise the young arithmetician, to take frequent exercises by himself of this kind, beginning at first but with a few lines, and increasing that number as he becomes more expert. He should at first add slowly figure by figure, and repeat the same column again and again, till he can take in 2, 3, or 4 figures at once; and by thus accustoming himself to addition, he will be able to perform in a few minutes, with absolute certainty, what would otherwise take him up for hours.

TABLE of grains Troy. TABLE of pounds Avoirdupoise.

Grains. dwt.

Pounds. quarters.

24	=	1	28	=	1
48	=	2	56	=	2
72	=	3	84	=	3
96	=	4	112	=	4
120	=	5	140	=	5

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G

Grains

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Grains Troy continued. Pounds Avoirdupois continued.

Grains.	dwt.	Pounds.	quarters.
144	= 6	168	= 6
168	= 7	196	= 7
192	= 8	224	= 8
216	= 9	252	= 9
240	= 10	280	= 10
264	= 11	308	= 11
288	= 12	336	= 12
		364	= 13

The use of these tables is obvious.

I shall now give some examples of weights and measures. Those in which there may appear no difficulty, shall be left for the learner's practice; such as require illustration shall be added.

## Troy Weight.

Pounds. oz. dwts. gr.

472	11	19	22
534	10	14	21
647	11	18	21
348	10	17	18
246	11	19	12
347	11	13	19
618	5	15	17
947	9	17	11
64	8	8	9
9	7	11	15
5	4	10	16

## Dutch Weight.

Stones. lb. ȝ. dr. gr.

475	15	14	13	34
568	14	13	15	23
647	13	12	11	32
954	12	11	13	31
876	9	14	15	29
678	10	11	12	28
578	11	10	13	27
485	14	13	15	24
657	11	11	7	28
6	4	0	6	0
5	9	9	0	19

5937 1 13 15 23

## Apothecaries Weight.

Pounds. ȝ. ȝ. ȝ. gr.

57	11	7	2	19
66	10	4	1	18
57	8	6	2	17
8	9	7	0	5
9	11	4	1	18
8	10	0	2	0
5	11	5	1	13
0	6	0	2	0
9	3	4	1	16
5	2	2	2	12
3	1	3	1	15

I. L. L. U.

Chap. VII. ADDITION of applicate Numbers. 51

ILLUSTRATION.

In the example of Dutch weight, I find in the column of grains, whereof  $36 = 1$  drop, the sum of  $275$ ; wherefore, discovering at once from my memory, or a mental division, that  $252$  grains  $= 7$  drops, the remaining  $23$  is noted down, and  $7$  carried to be added in with the drops in the next column, &c.

*Note.* One that is not much versant in addition, had better add the denominations, consisting of two places, at twice, till he can do that very quickly; a little practice will enable him afterwards to add up two places of any denomination at once with the greatest ease.

Avoirdupoise weight.

Tuns.	cwt.	qu.	lbs.
8745	19	2	27
4856	18	3	26
5479	14	1	25
3568	17	2	24
594	16	1	23
55	11	2	21

Wool weight.

Lbs.	sa.	w.	t.	sto.	c.	lbs.
87	11	1	$6\frac{1}{4}$	1	1	6
43	5	1	$3\frac{1}{2}$	1	1	4
6	4	1	$2\frac{1}{4}$	1	1	3
8	6	1	$3\frac{1}{2}$	1	1	5
17	2	1	$5\frac{1}{4}$	1	1	3
2	0	0	0	0	0	4

Cloth measure.

Yds.	qrs.	nls.	parts.
45	3	3	15
30	2	1	12
67	1	2	13
8	3	1	14
9	2	2	11
7	2	3	12
8	1	1	10
9	2	0	9

English square measure.

Acres.	r.	p.	yds.	feet.	inch.
373	3	37	$27\frac{1}{4}$	6	115
485	2	38	$22\frac{1}{2}$	7	116
574	2	19	29	8	114
648	1	18	$18\frac{1}{2}$	5	125
741	2	17	$19\frac{3}{4}$	4	130
874	3	14	$24\frac{1}{2}$	3	134
878	1	18	30	5	116
6	3	17	15	6	117

4583    1    24    10    5    103

*Note.* In the above example of square measure, I find the sum of square yards, including what I carried, to be  $191\frac{1}{2}$ , I then consider that  $6 \times 30 = 180 + \frac{5}{4} = 181\frac{1}{2}$ , and the remainder easily occurs to be 10.

52 OF ARITHMETIC. Part I.

Scotch square measure.

Acre. rood. fall. ell.

42	1	38	10
57	3	39	35
43	2	27	24
8	3	15	27
65	2	24	17
37	3	31	32
45	2	15	18

Long measure.

Miles. fur. yds. feet. inch.

3467	5	219	2	10
4567	7	184	1	11
5678	6	62	2	8
78967	4	9	0	9
56789	3	84	2	11
24608	2	147	1	10
35791	1	210	2	6

209871 0 80 0 5

In the example of long measure, the perches are neglected; and as there is no other intermediate denomination betwixt yards and furlongs, I carry at 220.

Wine measure.

Tun. hhd. gall. qu. pints.

436	3	61	2	1
678	2	61	3	0
569	1	48	1	1
456	0	29	3	1
789	1	36	2	0
987	2	54	1	1
672	3	46	3	1

4591 1 24 1 1

Ale and beer measure.

Hhd. bar. kil. fir. gal.

176	1	1	1	8 $\frac{1}{4}$
374	1 $\frac{1}{4}$	1	1	7 $\frac{1}{2}$
842	1	0	1	6
374	1 $\frac{1}{4}$	1	0	5 $\frac{1}{4}$
516	0 $\frac{1}{2}$	1	0	5 $\frac{1}{2}$
637	1	1	1	6 $\frac{3}{4}$
591	1	1	1	8

Scotch dry measure.

Ch. b. f. p. l.

875	15	2	1	1
43	14	3	3	3
87	13	2	2	2
6	15	3	2	2
67	14	2	3	2

English dry measure.

Q. b. p. g.

875	7	3	1
43	4	2	1
67	5	3	1
54	4	2	1
475	5	3	1

Chap. VIII. SUBTRACTION of applicate Numbers. 53

C H A P. VIII.

S U B T R A C T I O N of applicate Numbers.

R U L E.

PLACE the numbers, or denominations, homogeneal under homogeneal, and borrow according to the division of the integer, as illustrated in some of the following examples.

Money.			Troy weight.				Apothecaries weight.				
L.	s.	d.	lbs.	oz.	dwt.	gr.	lbs.	ʒ.	ʒ.	ʒ.	gr.
From 54	13	11 $\frac{1}{4}$	87	7	13	21	83	7	4	1	13
Take 35	15	8 $\frac{1}{2}$	43	10	15	17	42	5	7	2	18
Rests	18	18	2 $\frac{3}{4}$	43	8	18	4	41	1	4	15

I L L U S T R A T I O N.

In the example of money, I cannot take  $\frac{1}{2}$  from  $\frac{1}{4}$ , and therefore I take it from an unit of the next higher denomination, and to the remainder add the given  $\frac{1}{4}$ ; thus, 4 farthings —  $2 + 1 = \frac{3}{4}$ . I replace the unit I had thus borrowed by adding it to 8 d. whence I had taken it. And indeed it would have answered the end to have taken the difference betwixt  $8\frac{1}{2}$  d. and  $11\frac{1}{4}$  d. at once, if it could upon all occasions have been recommended to practice, as it would have brought out the same remainder.

Subtraction is so easy an operation, and the memory is so little burthened with it, that a farther illustration would be needless, and therefore I shall only subjoin a few examples, for practice.

Dry measure.				Long measure.				Cloth measure.			
Lft.	wey.	qu.	b. p.	M.	f.	p.	y. feet.	Yds.	qrs.	nails.	
87	1	2	4 2	47	5	34	3 $\frac{1}{2}$	1	57	1	0
43	1	3	5 3	13	6	37	4 $\frac{1}{4}$	2	14	3	1

Avoirdupoise

## 54 OF ARITHMETIC. Part I.

Avoirdupoise weight. Wine measure. Ale measure.

Tun. cwt. qrs. lbs. Tuns. p. hhds. gal. Bar. f. gal. q. p.

75	7	3	17	75	1	0	35	43	1	5	2	0
69	14	3	19	68	1	1	50	29	1	7	3	1

---

Yarn.				Time.					
Sp.	ha.	be.	c.	tb.	Y.	d.	b.	m.	f.
572	1	4	1	100	7415	199	14	50	40
69	1	5	1	115	897	270 $\frac{1}{4}$	19	50	52

---

## Questions for practice in addition and subtraction.

1. A merchant, in balancing his books, finds he hath in ready money, L. 456, 17 s.; in goods, L. 1749: 19: 6; his stock in a company-trade was L. 199: 19: 6 $\frac{1}{2}$ ; due him in open accounts L. 2977: 19: 7 $\frac{3}{4}$ ; in bills, L. 647, 16 s.; in ships and houses L. 1976: 14: 7 $\frac{1}{2}$ ; and in consignments, L. 479: 19: 7. He owes to A, L. 1456: 18: 7 $\frac{1}{2}$ ; to B, L. 99: 19: 11; to C, L. 497: 17: 10; and to the bank, L. 490. What is his nett stock?

	L.	s.	d.		L.	s.	d.
Cash,	456	17	0	He owed to A,	1456	18	7 $\frac{1}{2}$
Goods,	1749	19	6	To B,	99	19	11
Company,	199	19	6 $\frac{1}{2}$	To C,	497	17	10
Accounts,	2977	19	7 $\frac{3}{4}$	To the bank,	490	0	0
Bills,	647	16	0				
Ships, &c.	1976	14	7 $\frac{1}{2}$	Sum of his debt	2544	16	4 $\frac{1}{2}$
Consign.	479	19	7				
<hr/>							
Gross stock,	8489	5	10 $\frac{3}{4}$				
Sum of debt	2544	16	4 $\frac{1}{2}$				
<hr/>							
Nett stock,	5944	9	6 $\frac{1}{4}$	Answer.			

2. A merchant hath a bill to pay of L. 500, for which he had prepared in cash L. 197, 19 s.; he hath a bill on Edinburgh

## Chap.VIII. SUBTRACTION of applicate Numbers. 55

burgh for L. 120 ; for how much must he draw on the bank to retire the bill of L. 500 ?

Answer, L. 182, 1 s.

3. A farmer paid of yearly rent for his possession L. 156, 17 s. 8 d. ; at the expiration of three years, when he was called to settle accounts with the landlord, he could produce receipts only for L. 376 : 19 : 7 $\frac{1}{2}$ . How much must he pay to even the account ?

Answer, L. 93 : 13 : 4 $\frac{1}{2}$ .

4. Bought 8 hogsheads of raisins, each weighing gross 5 cwt. 1 quarter, and 11 pounds ; upon each hogshead whereof I am allowed a deduction of 3 quarters and 21 lbs. What will be the nett weight ?

Answer, 35 cwt. 1 quarter, 4 pounds.

5. A merchant sent his clerk to a fair, where he bought linen to the amount of L. 47 : 19 : 7 ; stockings to the amount of L. 184 : 16 : 11 ; he recovered of accounts due the merchant to the amount of L. 64, 10 s. ; and got payment of a bill for L. 139, 19 s. ; he paid some few demands, amounting in all to L. 19 : 19 : 11 d. ; his account of petty charges came to L. 1 : 14 : 6 ; and he gave back to his master L. 27 : 11 : 10. How much money had he got from the merchant before he set out ?

Answe, L. 135 : 7 : 1.

6. A castle was built in the year 1459 ; how old is it in the year 1763 ?

7. Received from Jamaica 103 tuns, 13 cwt. of logwood, by the Happy Janet ; of which I sold to A, 15 tuns, 10 cwt. 3 qrs. 17 lb. ; to B, the double of what I sold to A ; and to C as much as I had sold to A and B together. How much have I on hand ?

## C H A P. IX.

### MULTIPLICATION of APPLICATE NUMBERS.

#### R U L E.

WHEN these numbers are of different denominations, beginning the multiplication at the right

## 56 OF ARITHMETIC. Part I.

right hand, carry, as in addition, one from each denomination to another, for as many as make an unit of the next superior order, and place the remainder under its proper denomination. A few examples will render this extremely plain.

*Quest.* 1. What cost 8 pieces of broad cloth, at L. 5:17:8 per piece?

$$\begin{array}{r} L. \quad 5 \quad 17 \quad 8 \\ \quad \quad \quad 8 \\ \hline L. \quad 47 \quad 1 \quad 4 \end{array}$$

Answer.

2. What cost 72 bags of cotton at L. 7:14:8 per bag?

$$\begin{array}{r} L. \quad 7 \quad 14 \quad 8 \\ \quad \quad \quad 8 \\ \hline L. \quad 61 \quad 17 \quad 4 \\ \quad \quad \quad 9 \\ \hline L. \quad 556 \quad 16 \quad 0 \end{array}$$

Answer.

3. What cost 34 pieces of lutestrings, at L. 9:18:8 per piece?

$$\begin{array}{r} L. \quad 9 \quad 18 \quad 8 \times 2 \\ \quad \quad \quad 8 \\ \hline 79 \quad 9 \quad 4 \\ \quad \quad \quad 4 \\ \hline 317 \quad 17 \quad 4 \quad \text{price of } 32 \text{ pieces.} \\ 19 \quad 17 \quad 4 \quad \text{price of } 2 \text{ pieces.} \\ \hline 337 \quad 14 \quad 8 \quad \text{price of } 34 \text{ pieces. Ans.} \end{array}$$

4. What

Chap. IX. Multiplication of applicate Numbers. 57

4. What cost 76 cwt. of ship-biscuit, at 13 s. 6 d. per cwt?

$$\begin{array}{r}
 \text{s.} \quad \text{d.} \\
 13 \quad 6 \times 4 \\
 \hline
 12 \\
 L. \quad 8 \quad 2 \quad 0 \quad \text{price of } 12 \\
 \hline
 6 \\
 \hline
 48 \quad 12 \quad 0 \quad \text{price of } 6 \times 12 = 72 \\
 2 \quad 14 \quad 0 \quad \text{price of } 4 \times 1 = 4 \\
 \hline
 51 \quad 6 \quad 0 \quad \text{price of } \quad 76 \text{ Answer.}
 \end{array}$$

5. Sold 174 ingots of silver, each ingot weighing 15 lb.  
11 oz. 19 dwt. 21 gr.; what is the weight of the whole?

lb. oz. dwt. gr.

$$\begin{array}{r}
 15 \quad 11 \quad 19 \quad 21 \\
 \hline
 10
 \end{array}$$

$$\begin{array}{r}
 159 \quad 11 \quad 18 \quad 18 \quad \text{weight of } 10 \\
 \hline
 10
 \end{array}$$

$$\begin{array}{r}
 1599 \quad 11 \quad 7 \quad 12 \quad \text{ditto of } 10 \times 10 = 100 \\
 1119 \quad 11 \quad 11 \quad 6 \quad \text{ditto of } 7 \times 10 = 70 \\
 63 \quad 11 \quad 19 \quad 12 \quad \text{ditto of } 4 \times 1 = 4 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 2783 \quad 10 \quad 18 \quad 6 \quad \text{ditto of } \quad 174 \text{ Answer.}
 \end{array}$$

6. Bought 574 pounds of tobacco, at 7½ d. per pound;  
what cost the whole?

$$\begin{array}{r}
 L. \quad 0 \quad 0 \quad 7\frac{1}{2} \\
 \hline
 10
 \end{array}$$

$$\begin{array}{r}
 0 \quad 6 \quad 5\frac{1}{2} \quad \text{value of } 10 \text{ pounds.} \\
 \hline
 10
 \end{array}$$

$$\begin{array}{r}
 3 \quad 4 \quad 7 \quad \text{ditto of } 10 \times 10 = 100 \\
 \hline
 5
 \end{array}$$

$$\begin{array}{r}
 16 \quad 2 \quad 11 \quad \text{ditto} \quad 5 \times 100 = 500 \\
 2 \quad 5 \quad 2\frac{1}{2} \quad \text{ditto of } 7 \times 10 = 70 \\
 0 \quad 2 \quad 7 \quad \text{ditto of } 4 \times 1 = 4 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 L. 18 \quad 10 \quad 8\frac{1}{2} \quad \text{ditto of } \quad 574 \text{ Answer.}
 \end{array}$$

These examples are sufficient for exhibiting all the necessary varieties in multiplication of mixed numbers, the more especially as the learner will find, in the rules of practice, methods that are in general much more expeditious, and less burthensome to the memory, for solving all questions of this kind; though in some cases, where a single multiplication, or perhaps two, are only necessary, this may be used with great propriety.

## C H A P. X.

## DIVISION of APPLICATE NUMBERS.

## R U L E.

**I**N dividing different denominations, the remainder of the integral part must be brought to the quality of the next inferior denomination; and, if any of that denomination was given, it must be added to the product: then find how oft the divisor is contained therein, and the quotient will be of that denomination: if any thing still remain, proceed in the same way, as illustrated in the following examples.

*Quest.* 1. L. 34, 16 s. is to be divided among 5 men equally. What will fall to each?

$$\begin{array}{r}
 5)34 : 16(6 : 19 : 2\frac{2}{5} \\
 \underline{30} \\
 4 \times 20 + 16 = 96 \\
 \underline{5} \\
 46 \\
 \underline{45} \\
 1 \times 12 = 12 \\
 \underline{10} \\
 \underline{2}
 \end{array}$$

*Note,*

I.

Chap. X. DIVISION of applicate Numbers. 59

*Note.* After a little practice, the last question, or any other where the divisor is small, may be expeditiously done by a mental multiplication and subtraction. When the divisor is great, the memory may be helped in the more burthensome part of the work, by using a piece of waste paper, or still better by abbreviating the terms when possible.

The last example resumed.  $5)34 : 16 : 0$

$$\underline{6 : 19 : 2\frac{2}{3}}$$

*Note.* When L. 34 is divided, the remaining L. 4, 16 s. will readily occur to be = 96 s.; which, being partially divided, quotes 19 s. with 1 remaining = 12 d. of which  $\frac{1}{3} = 2\frac{2}{3}$ .

2. Bought 48 pieces of cloth for L. 256, 18 s. what did it cost per piece?

$$\begin{array}{r} 8)256 \quad 18 \quad 0 \\ \hline 6)32 \quad 2 \quad 3 \\ \hline 5 \quad 7 \quad 0\frac{1}{2} \text{ Answer.} \end{array}$$

3. Bought 375 pieces of Irish linen, for L. 701 : 15 : 3; freight and other charges came to L. 4 : 17 : 11. What did it cost per piece?

$$\begin{array}{r} L. \quad 701 \quad 15 \quad 3 \\ \quad 4 \quad 17 \quad 11 \\ \hline 5)706 \quad 13 \quad 2 \\ \hline 5)141 \quad 6 \quad 7\frac{2}{3} \\ \hline 5)28 \quad 5 \quad 4 \\ \hline 3)5 \quad 13 \quad 0\frac{4}{3} \\ \hline 1 \quad 17 \quad 8\frac{1}{3} \end{array}$$

*Note.* It will be sufficiently exact to take the quotient of the fractions that is nearest the truth; and, if you are above the just quotient at one time, be below it at another time, as in the last example.

## 60 OF ARITHMETIC. Part I.

4. If 60 gallons of water fall into a cistern that will contain 200 gallons, in the space of an hour, and by a pipe in the same cistern there run out 45 gallons in the same time; how long will it take to be full in this case?

$$60 - 45 = 15) 200$$

Hours 13, 20 minutes. Answer.

5. There is a legacy of L. 4753, 19 s. to be divided among A, B, C, D, and E, in such a manner, that for every shilling E takes up, D shall have two, C 3, B 4, and A 5; what will fall to each?

$$1+2+3+4+5=15) 4753 \quad 19 \quad 0$$

316	18	$7\frac{1}{3}$	E's share.
633	17	$2\frac{2}{3}$	D's share = 2 E's share.
950	15	$9\frac{1}{3}$	C's share = D+E.
1267	14	$4\frac{4}{5}$	B's share = C+E.
1584	13	0	A's share = B+E.

4753 19 0 Proof.

6. A gentleman, on his death-bed, leaving his wife pregnant, and an estate of L. 6666 : 13 : 4, ordered by his testament, that, if his wife bore a son,  $\frac{2}{3}$  of the fortune should go to that son, and the other  $\frac{1}{3}$  to the widow; but, if she bore a daughter, the widow should enjoy  $\frac{2}{3}$ , and the daughter the remainder: she had a son and a daughter at the same time: In this case, how will the fortune be divided?

By the testament the son was secured in double the widow's share, and the widow in double the daughter's; therefore, the daughter's share must be to the widow's as 1 to 2, and the widow's to the son's as 2 to 4.

Upon these principles it will be divided thus:

$$1+2+4=7) 6666 \quad 13 \quad 4$$

952      7       $7\frac{1}{7}$  the daughter's share.  
              2

1904      15       $2\frac{6}{7}$  the widow's share.  
              2

3809      10       $5\frac{5}{7}$  the son's share.

6666      13      4 as before.

7. A

I.

Chap. X. DIVISION of applicate Numbers. 61

7. A privateer takes a prize, value L. 20,000: the crew consisted of the captain, the lieutenant, the master, master's mate, surgeon, surgeon's mate, purser, 4 midshipmen, and 100 men: by the ship's regulations, the private men shared equally, a midshipman had as much again as a private man, each mate the double of a midshipman's share, the surgeon and purser drew each the double of a mate's share, the lieutenant and master had each as much as the surgeon and purser, and the captain as much as both lieutenant and master: According to these regulations, how will the prize be divided?

$$100 + 8 + 8 + 16 + 32 + 32 = 196) 20,000$$

$$\begin{array}{r} L. 10204 \quad 1 \quad 3 = 100 \times \dots \quad 102 \quad 0 \quad 9\frac{3}{4} \\ \hline \end{array} \text{to each p. man.}$$

2

$$\begin{array}{r} 816 \quad 6 \quad 6 = 4 \times \dots \quad 204 \quad 1 \quad 7\frac{1}{2} \\ \hline \end{array} \text{to each midship.}$$

2

$$\begin{array}{r} 816 \quad 6 \quad 6 = 2 \times \dots \quad 408 \quad 3 \quad 3 \\ \hline \end{array} \text{to each mate.}$$

2

$$\begin{array}{r} 1632 \quad 13 \quad 0 = 2 \times \dots \quad 816 \quad 6 \quad 6 \\ \hline \end{array} \text{to the surgeon}$$

2 and purser each.

$$\begin{array}{r} 3265 \quad 6 \quad 0 = 2 \times \dots \quad 1632 \quad 13 \quad 0 \\ \hline \end{array} \text{to the lieutenant}$$

2 and master each.

$$\begin{array}{r} 3265 \quad 6 \quad 0 = 1 \times \dots \quad 3265 \quad 6 \quad 0 \\ \hline \end{array} \text{to the captain.}$$

0 0 9 lost with the remainder.

$$20000 \quad 0 \quad 0 \text{ as before.}$$

The

The APPLICATION of MULTIPLICATION and DIVISION to square and solid measures.

*Quest.* 1. What is the square contents of a room, 18 feet 8 inches by 14 feet 6 inches?

$$\begin{array}{r}
 2) 18 \quad 8 \\
 \quad \quad 7 \\
 \hline
 130 \quad 8 \\
 \quad \quad 2 \\
 \hline
 261 \quad 4 = 18 \quad 8 \times 14 \\
 \quad \quad 9 \quad 4 = 18 \quad 8 \div 2 \\
 \hline
 270 \quad 8 \text{ Answer.}
 \end{array}$$

2. What is the square contents of a room, 12 feet 7 inches long and 11 feet 5 inches broad?

$$\begin{array}{r}
 3) 12 \quad 7 \\
 \quad \quad 11 \quad 5 \\
 \hline
 138 \quad 5 = 11 \times 12 \quad 7 \\
 4) 4 \quad 2\frac{1}{3} = 12 \quad 7 \div 3 \text{ for } 4 \text{ inches.} \\
 \quad \quad 1 \quad 0\frac{7}{3} = 4 \quad 2\frac{1}{3} \div 4 \text{ for } 1 \text{ inch.} \\
 \hline
 143 \quad 7\frac{1}{3} \text{ Answer.}
 \end{array}$$

3. What is the solid content of a box, 5 feet 6 inches thick, 4 feet 4 inches broad, and 7 feet 3 inches long?

$$\begin{array}{r}
 7 \quad 3 \\
 4 \quad 4 \\
 \hline
 29 \quad 0 = 7 \quad 3 \quad 4 \\
 2 \quad 5 = 7 \quad 3 \div 3 \text{ for four inches.} \\
 \hline
 31 \quad 5 \text{ square content.} \\
 5 \quad 6 \\
 \hline
 157 \quad 1 = 31 \quad 5 \times 5 \\
 15 \quad 8\frac{1}{2} = 31 \quad 5 \div 2 \text{ for six inches.} \\
 \hline
 172 \quad 9\frac{1}{2} \text{ solid content.}
 \end{array}$$

4. There

4. There is a box 6 feet 6 inches long, 7 feet 9 inches broad, at one end, and 3 feet 7 inches at the other, and 4 feet 6 inches thick; what is its solid content?

$$\begin{array}{r}
 7 \quad 9 \text{ one way.} \\
 3 \quad 7 \text{ the other way.} \\
 \hline
 2) 11 \quad 4 \text{ sum of the breadths.} \\
 \hline
 5 \quad 8 \text{ mean breadth.} \\
 6 \quad 6 \\
 \hline
 34 \quad 0 \\
 2 \quad 10 \\
 \hline
 36 \quad 10 \text{ square content.} \\
 4 \quad 6 \\
 \hline
 147 \quad 4 \\
 18 \quad 5 \\
 \hline
 165 \quad 9 \text{ solid content.}
 \end{array}$$

## CHAP. XI. REDUCTION.

**R**EDUCTION converts one denomination, or species, into another, without altering the value.

## RULE I.

To reduce numbers of a higher denomination to numbers of the same kind of an inferior denomination, multiply by as many of the inferior denomination as makes one of the greater.

## RULE II

To reduce numbers of a lower denomination to numbers equivalent of a higher denomination, divide by as many of the inferior as makes one of the greater.

EXAMPLE

## EXAMPLES.

(1.) Reduce £. 760 to farthings.  
 $\frac{20}{15200}$  farthings.

$$\begin{array}{r} 15200 \text{ shillings.} \\ \frac{12}{182400} \text{ pence.} \\ \frac{4}{729600} \text{ farthings.} \end{array}$$

(2.) Reduce 729600 farthings to pounds.  
 $\frac{4}{182400}$  pence.  
 $\frac{20}{15200}$  shillings.  
 $\frac{760}{760}$  pounds.

In 74 11 15 Troy, how many  
 $\frac{12}{899}$  ounces.  
 $\frac{20}{17995}$  dwts.  
 $\frac{24}{431880}$  grains.

(3.) In 431880 grains, how  
 $\frac{4}{107970}$  many pounds Troy?  
 $\frac{20}{17995}$  dwts.  
 $\frac{12}{899 15}$  oz.  
 $\frac{74}{74 11 15}$  Anf.

More examples of this kind of reduction would be unnecessary, as the reason and manner of the operation must be obvious.

## RULE III.

To reduce one species into its equivalent of another, when the one is no even part of the other,— multiply the given number by the value of an unit of the same species expressed in the lowest name mentioned in the question, and divide that product by the value of an unit of that species which is required, expressed in the same name.

Quest.

*Ques<sup>t</sup>.* 1. I have sent 345 bank-notes, each 20 s. to be exchanged for guineas; how many guineas will compensate the notes?

$$\begin{array}{r} 345 \\ 20 \\ \hline 3) 6900 \\ \hline 7) 2300 \\ \hline \end{array}$$

328 guineas and 12 s. or  $\frac{1}{4}$  of a guinea.

2. I want to exchange for guineas 360 five pound notes, 150 ten pound notes, and 27 ten shilling notes; what number of guineas ought I to receive?

$$\begin{array}{r} 360 \times 5 = 1800 \\ 150 \times 10 = 1500 \\ 27 \div 2 = 13 \text{ } 10 \\ \hline 21 ) 3313 \text{ } 10 \\ \quad \quad \quad 157 \text{ } 15 \\ \hline \end{array}$$

Guineas 3155 15 shillings. Answer.

3. I have 1478 moidores, worth 27 s. each, which I would exchange for pistoles at 16 s. 6 d. each; how many pistoles should I have in return?

First, 27 s. = 54 sixpences, the lowest name mentioned, and 16 s. 6 d. = 33 sixpences.

But 54 and 33 are each commensurable by 3, and quote 18 and 11.

Therefore, 1478 moidores

$$\begin{array}{r} 18 \\ \hline 11 ) 26604 \\ \quad \quad \quad 11 \\ \hline \end{array}$$

2418  $\frac{6}{11}$  pistoles. Answer.

4. How many French crowns, at  $31\frac{1}{2}$  d. may I have in exchange for 37542 guilders, at  $21\frac{1}{4}$  d.?

$$21 \times 4 + 1 = \frac{37542}{85} \text{ farthings in a guilder.}$$

$$\begin{array}{r} 187710 \\ 300336 \\ \hline 3191070 \\ 21 ) 531845 \text{ the quotients of both factors by 6.} \\ 7 ) 177281\frac{1}{3} \text{ ditto. — Ditto by 3.} \end{array}$$

$25325\frac{3}{7}$  French crowns. Answer.

5. In 87547 ducats, at 3 s. 9 d. how many Hamburg marks, at 1 s. 6 d.?

$$\begin{array}{r} s. \quad d. \\ 3 \quad 9 \\ 4 \\ \hline 3) 15 \text{ threepences.} \\ \quad 5 \text{ ninepences.} \quad 87547 \\ \hline 2) 437735 \\ \quad 218867\frac{1}{2} \text{ marks of Hamburg.} \end{array}$$

After a little practice, multipliers and divisors may be abridged mentally, without any formal multiplication or division: for, in the last instance, it will easily occur that there are 5 times 9 pence in 3 s. 9 d. and 2 ninepences in 1 s. 6 d.

6. If I had 4754 livres in France, which I rate at  $10\frac{1}{2}$  d. each, how many milrees of Portugal, at 5 s. 3 d. would they answer for?

$10\frac{1}{2}$  d. is contained in 5 s. 3 d. just 6 times.

Therefore  $6) 4754$

$792\frac{2}{3}$  milrees. Answer.

7. In

7. In L. 4754, 19 s. how many guineas, crowns, shillings, and sixpences, of each an equal number?

$$\begin{array}{r}
 4754 \quad 19 \\
 \underline{-} \qquad \underline{20} \\
 95099 \\
 \underline{\quad \quad \quad 2} \\
 42+10+2+1=55) \quad \underline{190198} \\
 \underline{\quad \quad \quad 11} \qquad \underline{\quad \quad \quad 38039\frac{1}{2}} \text{ abridged by 5.} \\
 3458\frac{8}{3} \text{ of each species.}
 \end{array}$$

8. If I had guineas 90 score, and crowns just 92; In place of 30 hundred pounds, what money would be due?

$$\begin{array}{r}
 90 \times 21 = 1890 \text{ l.} \\
 92 \div 4 = \underline{23}
 \end{array}$$

$$\begin{array}{r}
 \text{Take L. } 1913 \text{ in the guineas and crowns.} \\
 \text{From } 3000 \\
 \hline \\
 \text{Remains } 1087 \text{ due.}
 \end{array}$$

In the country, particularly in the north of Scotland, they keep their accounts in Scotch money, the pound of which is  $\frac{1}{2}$  of a pound Sterling, and divided in the same manner. In making bargains, and sometimes in granting bonds, they express their money in merks, one of which is 13 s. 4 d. or  $\frac{2}{3}$  of their pound. Hence, to reduce pounds Scotch to merks, multiply by 3, and divide by 2; and to reduce merks to pounds, multiply by 2, and divide by 3. Again, since the merk is to the pound Scotch as 2 to 3, and the pound Scotch to the English as 1 to 12, the Scotch merk will be to the pound Sterling as 1 to 18.

These distinctions are never used among merchants.

9. I have a bond for 5000 merks Scotch, a bill for L. 559 Scotch, and 24 crown-pieces; what are these worth in Sterling?

$$\begin{array}{r}
 2) 5000 \text{ merks} \\
 \hline
 9) 2500 \\
 \hline
 L. 559 \text{ Scots.} \\
 \hline
 12 = & 277 & 15 & 6\frac{2}{3} \text{ value of the bond.} \\
 24 \div 4 = & 46 & 11 & 8 \text{ ditto of the bill.} \\
 & 6 & 0 & 0 \text{ ditto of the crowns.} \\
 \hline
 330 & 7 & 2\frac{2}{3} & \text{value of the whole.}
 \end{array}$$

10. Two merchants, A and B, had been long in a company-trade; A's share of the concern was to B's as 4 to 1: When circumstances rendered it necessary for them to wind up and separate, the state of their affairs was as follows. Their cash and other effects, by an inventory, amounted to L. 5000; bills and open accounts, in Britain, to L. 1300; Holland was indebted to them in nett proceeds of tobacco, for 7485 guilders, at  $21\frac{1}{8}$  d.; and Dunkirk in crowns 7456, at  $31\frac{1}{4}$  d.; they were due in Britain L. 3754, and in Hamburg 7315 marks, at 1 s. 7 d.: Required their nett stock, and a partition thereof, according to each partner's original input.

They had effects, *per* inventory,

$$\begin{array}{r}
 \text{valued at} & - & - & L. 5000 \\
 \text{Bills and open accounts, in Bri-} \\
 \text{tain, for} & - & - & 1300 & 0 & 0 \\
 7485 \text{ guilders, at } 21\frac{1}{8} \text{ d.} = & & & 658 & 16 & 8\frac{1}{2} \\
 7456 \text{ crowns, at } 31\frac{1}{4} = & & & 970 & 16 & 8 & L. \\
 \hline
 & & & & & & 7929 & 13 & 4\frac{1}{2} \\
 \text{They were due in Britain} & & & 3754 & 0 & 0 \\
 \text{In Hamburg 7315 marks, at 19d.} & & & 579 & 2 & 0 & 4333 & 2 \\
 \hline
 \text{Nett stock} & 4+1=5) & 3596 & 11 & 4\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{B draws} & 719 & 6 & 3\frac{3}{5} \\
 \text{A draws} & 2877 & 5 & 1\frac{2}{5}
 \end{array}$$

12. In 1000 ale-gallons Scotch measure, how many English?

103 solid inches in a Scotch pint.  
8

$$282) 824000$$

2922 nearly.

Suppose

Suppose the distance betwixt Glasgow and London 330 miles, how oft will a coach-wheel turn round, whose circumference is 6 yards, in driving that distance?

Miles. Fur.

$$\frac{330 \times 8 \times 220}{6} = 96800 \text{ times. Answer.}$$

Questions of this nature might be multiplied to any length, but it is presumed there is a sufficient variety in the foregoing for the improvement of the ingenious.

## CHAP. XII. Of NUMBERS.

**N**UMBER in general may be defined, A collection or assemblage of several units, or several things of the same kind. Sir Isaac Newton conceives number to consist, not in a multitude of units, as Euclid defines it, but in the abstract ratio of a quantity of any kind to another quantity of the same kind, which is accounted as unity; and, in this view, he divides number into three different kinds, integer, fraction, and surd.

Integers, whole or natural numbers, are all the various assemblages of unity, or the ideas we have of several multitudes.

Fractions are the divisions or subdivisions of unity, to which they refer, as a part doth to the whole.

Rational numbers are such as are commensurable with unity, or whereof unity is an aliquot part.

Surd, or irrational numbers, are such as are incommensurable with unity.

Even numbers are those which may be divided into two equal parts, without any remainder, as the numbers 6, 8, 10, 12, &c. which have likewise these properties, that the sum, difference, and product of even numbers, will each be an even number.

Uneven numbers, or such numbers as cannot be divided into two equal parts without a fraction, have these properties, that the sum or difference of two uneven numbers will be an even number, but the product will be an uneven number. If an even number be added to an uneven number,

or

## 70 OF ARITHMETIC. Part I,

or if the one is subtracted from the other, the sum in the one case and the remainder in the other will be uneven; but if they are multiplied, the product will be even.

A prime number is such as hath no measure but itself and unity, and which of consequence can be the product of no other numbers, as 2, 3, 5, 7, 11. Numbers prime to one another are such as have no measure common to both but unity, as 12 and 19; for though both 3 and 4, or 2 and 6, will measure 12, yet neither of these will measure 19.

Composite numbers are such as are divisible by some number besides unity, as 8 divisible by 4, and by 2.

Numbers composite to one another have some measure common to each besides unity, as 12 and 15 divisible by 3.

A number that can divide several numbers exactly is called a *common measure*, as in the last instance 3 is a common measure to 12 and 15.

*Prob. 1.* To find the greatest common measure to any numbers proposed, add all the proposed numbers, excepting the least, which is reserved for a divisor. If there is no remainder, after the sum of the greater numbers hath been divided by the least, then that divisor is the common measure; but, if there is a remainder, it becomes a new divisor, and the last divisor becomes a new dividend, &c. the last divisor, which left no remainder, is the common measure.

What is the greatest common measure of 8, 16, 24, and 32?

$$\begin{array}{r} 16 \\ 24 \\ 32 \\ \hline 8)72 \end{array}$$

9 without a remainder: So 8 is the greatest common measure for  $\frac{8}{8}=1$ ,  $\frac{16}{8}=2$ ,  $\frac{24}{8}=3$ , and  $\frac{32}{8}=4$ .

What is the greatest common measure of 12, 16, 24, 36, and 52?

$$\begin{array}{r}
 16 \\
 24 \\
 36 \\
 52 \\
 \hline
 12)128 \\
 \hline
 10 \text{ rem. } 8 | 12 \\
 \quad | 1 \text{ rem. } 4 | 8 \\
 \quad \quad | 2 \text{ and } 0 \text{ remains.}
 \end{array}$$

Consequently 4 is the greatest common measure.

*Note.* If only two numbers were proposed, the greater is divided by the lesser, &c. Hence any number not exceeding the least of several numbers that can measure their sum, will measure them all severally.

*Prob. 2.* To find the least common multiple of several numbers. Write the numbers in a line, and divide them by 2, 3, or any other numbers which will measure two or three of them exactly; place the quotients with the numbers undivided below, and divide them continually till their quotients be 1: Then the product of all the divisors multiplied continually, is the least common multiple required, which, if they were primes to one another, will be the continual product of the given numbers.

What is the least common multiple of 6, 12, 24, 36?

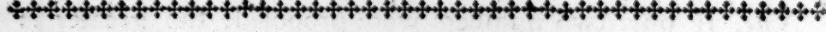
6	6	.	12	.	24	.	36
2	I	.	2	.	4	.	6
2			I	.	2	.	3
3				I	.	3	
				I			

There are other problems which might have got a place here, but these I reckon sufficient for my purpose.

THE  
UNIVERSAL ACCOUNTANT.

P A R T II.

O F F R A C T I O N S.



I. O F V U L G A R F R A C T I O N S.

I N T R O D U C T I O N.

**A**VULGAR FRACTION is a part, or parts of an integer arising from division, and stands to an unit in the relation that a part doth to the whole. — Of those parts the numerator expresseth the number, as  $\frac{3}{4}$ , and the denominator the quality, as  $\frac{4}{4}$ .

Hence the denominator supposeth the integer to be divided into 4 equal parts; for instance, 1 yard into 4 quarters; and the numerator ascertaineth the number of those parts to be 3; and the fraction is accordingly read three fourths, or quarters, of 1.

Since the denominator represents all the parts into which the integer is divided, and the numerator the number of those parts expressed by the fraction, it must follow, that, if the numerator be less equal to, or greater than the denominator, the quantity represented by the fraction must be less equal to, or greater than the integer accordingly. Hence, if the numerator is less than the denominator, the fraction is called *proper*, and represents something less than the integer, as  $\frac{3}{4}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ , &c.

If the numerator is equal to, or greater than the denominator, the fraction is called *improper*, and represents something equal to, or greater than the integer, as  $\frac{4}{4} = 1$ ,  $\frac{5}{4} = 1\frac{1}{4}$ , &c.

Since an integer may be divided into any number of parts, each of these parts may be again subdivided, and each of these

these subdivisions again, *ad infinitum*; as a pound is divided into 20 shillings, each of these shillings into 12 pence, and each of these pence into 4 farthings. Hence, 3 farthings would be expressed fractionally  $\frac{3}{4}$  of  $\frac{1}{12}$  of  $\frac{1}{20}$ , and such fraction would be called a *compound fraction*.

Since any of the denominations, or parts of an integer, can be expressed fractionally, a fraction annexed to an integer will express the same thing as its equivalent denomination. Hence L. 4, 10 s. since 10 s. represents 10 of 20 parts of a pound, may be wrote  $4\frac{10}{20}$ , and so called a *mixed number*.

*Observation 1.* Those fractions are equal to each other, whose numerators have the same relation to their denominators,  $\frac{3}{8} = \frac{6}{16} = \frac{9}{24} = \frac{12}{32} = \frac{15}{40} = \frac{1}{2}$ . For as all fractions arise from remainders in division, when the divisor can no longer measure the dividend, so every fraction may be considered as the two given terms of a division, the numerator as the dividend, and denominator as the divisor: Consequently, if the numerator and denominator of a fraction be either multiplied or divided both by the same number, the products or quotients will still remain in the same proportion, and the numerator of the new fraction bear the same relation to its denominator as it did in its former state.

2. Fractions having a common denominator, are greater or less as their numerators, as  $\frac{4}{3}$  represents a greater part of a quantity than  $\frac{3}{4}$ .

3. Of fractions whose numerators are equal, that which hath the least denominator represents the greatest part, as  $\frac{3}{4}$  of a yard represents 3 quarters, and  $\frac{3}{8}$  only 3 nails.

4. If two fractions are equal, the numerators multiplied into each other's denominator respectively, will make the products equal. — Suppose  $\frac{2}{4} = \frac{1}{2}$ , then  $2 \times 2 = 4$  and  $4 \times 1 = 4$ .

2

## C H A P. I.

## REDUCTION of VULGAR FRACTIONS.

*Prob. 1.* To express a whole number fractionally.

The given integer will be the numerator, and unity the denominator: Thus  $\frac{1}{1}$ ,  $\frac{2}{1}$ , &c.; because to divide 5 by 1, the quotient will be 5, &c.

*Prob. 2.* To reduce a mixed number to an improper fraction.

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To

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To the product of the integer and denominator multiplied add the numerator; the sum shall be the numerator of the improper fraction, whose denominator shall be that of the fraction given.

$$\text{Exam. } 4\frac{4}{5} \quad 4 \times 5 + 4 = 24, \text{ and } 5\frac{5}{6} = 5 \times 6 + 5 = 35.$$

*Observ.* The reason of this operation is evident; for the multiplication of the integer into the denominator + the numerator, expresses in the product all the parts contained in both, and the same denominator being again applied, the quality of those parts are the same.

*Prob. 3.* To reduce an improper fraction to a whole, or mixed number.

Divide the numerator by the denominator, and to that quotient annex the remainder, if any, with the divisor for the fractional part.

$$\text{Exam. } 24 = 4\frac{4}{5}, \text{ and } 35 = 5\frac{5}{6}.$$

This is the reverse, and consequently an additional proof of the former problem.

*Cor.* Hence it will be obvious, that, to reduce an integer to an improper fraction of an assigned denominator, we have only to multiply the integer into the assigned denominator, and the product will be the numerator required. For instance, to change 8 into a fraction whose denominator is 7,  $8 \times 7 = 56$ , &c.

*Prob. 4.* To reduce a compound fraction to its equivalent simple one.

The continued product of all the numerators will be the numerator, and the continued product of all the denominators will be the denominator required.

$$\text{Ex. } \frac{3}{4} \text{ of } \frac{4}{5} \text{ of } \frac{7}{3} = \frac{3 \times 4 \times 3}{4 \times 5 \times 7} = \frac{36}{210}, \text{ and } \frac{5}{2} \text{ of } \frac{3}{4} \text{ of } \frac{5}{8} = \frac{5 \times 3 \times 5}{2 \times 4 \times 8} = \frac{75}{64}.$$

*Observation.* The continued multiplication, of the numerators and denominators respectively, brings each to the quality of the lowest name; for, to express 9 d. as the fraction of a pound, it would be  $\frac{9}{240}$ , because there are 240 pence in a pound; if we consider it as a compound fraction, as it really is, it will be expressed  $\frac{9}{12}$  of  $\frac{1}{10}$ , which, by the rule,  $= \frac{9}{240}$ , as before.

*Cor.* Hence all the known subdivisions of an integer may be expressed in compound or simple fractions at pleasure.

*Prob.*

# Chap. I. Of VULGAR FRACTIONS. 75

*Prob. 5.* To reduce fractions having different denominators to other equivalent fractions, having a common denominator.

The continued product of each numerator into all the denominators but its own, will give correspondent numerators, and the continued product of the denominators will give a common denominator.

*Exam. 1.*  $\frac{3}{5}$ ,  $\frac{1}{2}$ ,  $\frac{4}{7}$ , thus       $\begin{array}{l} 3 \times 2 \times 7 = 42 \\ 1 \times 5 \times 7 = 35 \end{array} \left. \begin{array}{l} \\ \end{array} \right\}$  Numerators.

Therefore  $\frac{3}{5} = \frac{42}{70}$        $\begin{array}{l} 4 \times 2 \times 5 = 40 \\ 5 \times 2 \times 7 = 70 \end{array}$  Common denominator.  
 $\frac{1}{2} = \frac{35}{70}$   
 $\frac{4}{7} = \frac{40}{70}$ , as required.

*Exam. 2.*  $\frac{3}{5}$ ,  $\frac{5}{8}$ , and  $\frac{1}{4}$  of  $\frac{1}{2}$ . First,  $\frac{1}{4}$  of  $\frac{1}{2} = \frac{1}{8}$  in Ex. 2.

Then  $\frac{3}{5}$ ,  $\frac{5}{8}$ ,  $\frac{1}{8}$ , the given fractions, which by the rule will produce  $\frac{144}{40} = \frac{3}{5}$ ,  $\frac{200}{40} = \frac{5}{8}$ , and  $\frac{30}{40} = \frac{1}{8}$ , or  $\frac{1}{4}$  of  $\frac{1}{2}$ .

*Exam. 3.*  $\frac{5}{9}$ ,  $\frac{6}{15}$ , and  $6\frac{3}{5}$ . When  $6\frac{3}{5}$  is made an improper fraction  $\frac{33}{5}$ ,  $\frac{5}{9}$  by the rule will be  $= \frac{1}{4}, \frac{6}{15} = \frac{2}{5}, \text{ and } \frac{33}{5} = \frac{2970}{450}$ .

*Obj.* The reason of this problem is obvious; for, since the numerator and denominator of each fraction is equally multiplied, *viz.* by the denominators of all the other fractions, consequently the fractions produced must be equivalent.

*Prob. 6.* To reduce a given fraction to another equivalent, having an assigned denominator when possible.

This is only an abbreviation of the last problem in possible cases, and may be done by multiplying the numerator by the assigned denominator, and dividing that product by the old denominator, the quotient, if there is no remainder, will be the numerator required.

*Exam. 4.* to a fraction whose denominator is 6. Thus,  
 $\frac{2 \times 6}{3} = \frac{4}{6}$ .

*Note.* In addition of money, the different reckonings by Scotch and English will occasion fractions of different denominators, to be annexed in the column of pence, as 4ths, 3ds, and 6ths; but these are very expeditiously added, by considering 12 as the common denominator; in which case,  $\frac{1}{4} = \frac{3}{12}$ ,  $\frac{1}{3} = \frac{4}{12}$ ,  $\frac{1}{6} = \frac{2}{12}$ , &c.

*Prob. 7.* To reduce a fraction to lower terms when possible.

K 2

Divide

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Divide both numerator and denominator by any number that will measure both without a remainder; but, if nothing but unity is the same measure of both, the fraction is already in its lowest terms.

Hence even numbers may be divided by 2 continually, while possible. Numbers with ciphers may have an equal number cut off from each, and afterwards divided when possible.

Numbers ending in 5s, or 5 and cipher, may be divided by 5.

By a little practice, a common measure will readily occur for divisible numbers.

$$\text{Exam. } 5 \mid \frac{355}{455} = \frac{71}{91}. \quad 2 \mid \frac{76}{72} = \frac{19}{18}. \quad 3 \mid \frac{1500}{1800} = \frac{5}{6}. \quad \text{and } 9 \mid \frac{18}{18} = \frac{2}{1}.$$

*Prob. 8.* To bring a fraction of a higher denomination to an equivalent fraction of a lower.

Reduce the numerator to the name required for the numerator of the new fraction, the denominator will be the same as before.

*Exam.*  $\frac{5}{7}$  l. to the fraction of a sixpence; thus,

s. 6ds.

$$5 \times 20 \times 2 = 200.$$

s. d. g.

$\frac{5}{7}$  of a guinea to the fraction of a farthing,  $5 \times 21 \times 12 \times 4 = 3024$ .

*Prob. 9.* To reduce the known parts of a relative unit to the equivalent fraction of that unit,

This was formerly taken notice of in the corollary of prob. 4. and it is only resumed here for further illustration, to those who may find it still necessary.

Reduce all the given parts to the lowest mentioned, for a numerator, and the integer into the same name for a denominator.

*Exam.* 5 s.  $7\frac{1}{2}$  d. to be expressed fractionally,  $5 \times 12 \times 7 + 2 + 1 = 355$

$$20 \times 12 \times 2 = 480$$

*Prob. 10.* To value fractions in the known parts of the integer.

This is the converse of the last problem, and hath been exemplified in division, but still it may not be improper to give the rule.

Multiply the numerator by the parts of the next inferior denomination, and divide the product by the denominator; the

the quotient shews the parts of that denomination, and the remainder becomes a new numerator, which must be valued as before, &c. till the fraction is brought to the lowest known name of the integer.

*Exam.* Value  $\frac{135}{480}$  l. Value  $\frac{7}{12}$  of cwt.

$\frac{135}{20}$ $\frac{2400}{300}$ $\frac{12}{3600}$ $\frac{3360}{240}$ $\frac{4}{960}$ $\frac{960}{0}$	<i>s. d.</i> as above $\frac{7}{4}$ $\frac{24}{28}$ $\frac{4}{112}$ $\frac{108}{16}$ $\frac{64}{60}$ $\frac{60}{4}$	$\frac{7}{4}$ $\frac{24}{12}$ $\frac{112}{108}$ $\frac{108}{16}$ $\frac{64}{60}$ $\frac{60}{4}$	<i>Qr. lb. 3.</i> $(\frac{1}{2} \text{ or } \frac{1}{3})$ $\frac{12}{3}$
---	--	--	--

## C H A P. II.

### ADDITION of VULGAR FRACTIONS.

#### R U L E.

**R**EDUCE all the given fractions to simple fractions of the same integer and denominator, if not so already; then the sum of the numerators with the common denominator, will be the fractional sum required, which may be reduced to a mixed number, valued or expressed in shorter terms, as seems most expedient, or the case will admit.

E X A M-

## EXAMPLES.

Add  $\frac{3}{7} + \frac{4}{7} + \frac{5}{7}$ .  $3+4+5=\frac{12}{7}=1\frac{5}{7}$ . Here 7 is a common denominator.

Add  $\frac{3}{7}$ ,  $\frac{5}{7}$ , and  $\frac{2}{7}$ .

$$3 \times 2 \times 3 = 18$$

$$1 \times 5 \times 3 = 15$$

$$2 \times 5 \times 2 = 20$$

$$\frac{53}{5 \times 2 \times 3} = 1\frac{2}{6}$$

53 numerator.

5  $\times$  2  $\times$  3 = 30 denominator.

Add  $\frac{3}{7}$  s. to  $\frac{3}{7}$  l. First,  $\frac{3}{7}$  s. =  $\frac{3}{7}$  of  $\frac{1}{10} = \frac{3}{10}$ .

$$\text{Then } 3 \times 100 + 5 \times 3 = 315 = \frac{63}{100}.$$

$$\text{And } 5 \times 100 = 500.$$

Add L. 4 : 15 : 6, and L. 3,  $\frac{7}{8}$ . First, 15 s. 6 d. =  $\frac{3}{4}$ .

$$\text{Therefore, } 7 \times 40 = 280$$

$$3 \times 8 = 24$$

$$\frac{528}{8 \times 40} = 1\frac{2}{5}$$

$$8 \times 40 = 320$$

$$\begin{array}{r} 3 \\ 4 \\ \hline 8 \end{array} \frac{2}{5}$$

*Obs.* If reduction is well understood and remembered, the addition of fractions will be very easy; the reason of which will be obvious, if we consider that the given fractions being such, or reduced to such a state, that all the numerators represent things of the same denomination, both absolute and relative; their sum must therefore be a number of such parts as the common denominator expresses of the same common integer.

## C H A P. III.

## SUBTRACTION OF VULGAR FRACTIONS.

## R U L E.

**R**EDUCE the given fractions to simple ones of the same integer and denominator, as in addition, and

## Chap. IV. Of VULGAR FRACTIONS. 79

and the difference betwixt the numerators, with the common denominator, will be the fractional difference required.

### E X A M P L E S.

$$\begin{array}{lll}
 (1.) & (2.) & (3.) \\
 4 - \frac{3}{5} = \frac{1}{5} & 5 - \frac{3}{4} = \frac{5 \times 4 - 20}{6 \times 3 - 18} = \frac{2}{6} = \frac{1}{3} & 3 l. - \frac{3}{4} s. = \frac{57}{80} \\
 & & \text{For } \frac{60}{80} - \frac{3}{80} = \frac{57}{80} \\
 & & 6 \times 4 = 24
 \end{array}$$

(4.)

$$L. 4\frac{1}{2} - L. 3\frac{1}{2} = 1\frac{1}{2}. \quad \text{For } 4\frac{1}{2} - 3\frac{1}{2} = 1\frac{1}{2}, \text{ &c.}$$

The reason of this rule will be evident from the last observation.

## C H A P. IV.

### MULTIPLICATION OF VULGAR FRACTIONS.

#### R U L E.

**R**EDUCE the given fractions to their simple expression, if not so before, and to improper fractions, if mixed numbers, then will the product of the numerators and denominators respectively give the product required.

### E X A M P L E S.

$$\begin{array}{ll}
 (1.) & (2.) \\
 \frac{3}{5} \times \frac{5}{6} = \frac{15}{30}, \text{ or } \frac{1}{2}. & 5\frac{1}{2} \times 3\frac{1}{4} = \frac{11}{2} \times \frac{13}{4} = \frac{143}{8}.
 \end{array}$$

$$\begin{array}{l}
 (3.) \\
 \frac{3}{4} \text{ of } \frac{5}{6} \times \frac{1}{4} \text{ of } \frac{1}{3} = \frac{15}{24} \times \frac{1}{12} = \frac{15}{288}.
 \end{array}$$

$$\begin{array}{l}
 (4.) \\
 \frac{3}{8} s. \times \frac{1}{2} l. = \frac{3}{80} \times \frac{1}{2} = \frac{3}{160}.
 \end{array}$$

### O B S E R.

## OBSERVATION.

The operations in this rule are so simple, that more examples would be unnecessary.

It is worth while however to observe, that multiplication of fractions at first sight, would seem to contradict the definition of multiplication given in the first part. But this difficulty will vanish, if we consider that the more any integral number is increased, the farther is the figure in the highest place removed from unity, and the more any part of an integer is decreased, the farther will its value also be removed from its relative unit; consequently, as it is the nature of integers to increase, and of fractions to decrease, the purpose of multiplication is equally answered in both cases. But, to be plainer still, we had instances in the application of multiplication and division to square and solid measure, that a part of the multiplicand was taken when we wanted to multiply by a part of a foot; for this reason, To multiply by 1, would give the multiplicand for the product, and to multiply by any part of 1, can give but that part of the multiplicand for the product which the fraction expresseth.

## C H A P. V.

## DIVISION OF VULGAR FRACTIONS.

## R U L E.

**P**REPARE the fractions as in multiplication, then place the denominator of the divisor above the separating line, and the numerator below it, work as in multiplication, and the products so found will be the quotient.

## E X A M P L E S.

$$(1.) \text{Divide } \frac{3}{5} \text{ by } \frac{2}{3}. \text{ Thus, } \frac{3}{5} \frac{1}{2} \left( \frac{9}{10} \right). \quad (2.) \frac{4}{7} \text{ by } \frac{4}{3} \text{ of } \frac{2}{5} \left( \frac{8}{25} \right).$$

$$(3.) \frac{4}{5} \text{ by } 2\frac{2}{5} \left( \frac{14}{25} \right). \quad (4.) 4\frac{3}{7} \text{ by } 1\frac{1}{4} \text{ of } \frac{8}{11} \left( \frac{24}{11} \right).$$

Obſ.

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*Obs. 1.* Mixed numbers may be divided by mixed numbers, with as much brevity, and perhaps more intelligibly, especially in applicate cases, by the following rule.

Reduce the divisor to an improper fraction, whose numerator will be a new divisor, the quotient arising therefrom multiplied into the old numerator, will give the fractional part of the quotient; and, being multiplied into the denominator, will give the quotient due to unity.

An example will illustrate this.

Suppose L. 478 : 19 : 6 were to be divided among 3 men, who were to have equal shares, and a boy who was to get only  $\frac{1}{3}$  of what any one of the men got; what would fall to each?

$$\begin{array}{r}
 \text{First, } 3 \times 3 + 1 = 10 \\
 ) 478 \quad 19 \quad 6 \\
 \hline
 47 \quad 17 \quad 11\frac{2}{3} \text{ to the boy, for the} \\
 \qquad \qquad \qquad 3 \text{ old numerator was 1.} \\
 \hline
 143 \quad 13 \quad 10\frac{1}{3} \text{ to each man, for the} \\
 143 \quad 13 \quad 10\frac{1}{3} \text{ denominator is 3.} \\
 143 \quad 13 \quad 10\frac{1}{3} \\
 \hline
 478 \quad 19 \quad 6 \text{ as before.}
 \end{array}$$

*Obs. 2.* The reason of division of fractions in general will be pretty obvious, if we consider, that, were the divisor and dividend reduced to one common denominator, the dividend would contain the divisor as oft as its numerator did that of the divisor; for, having one denominator, they are in the same state with respect to one another, as whole numbers. Now, though no mention is made in the rule of a common denominator, yet the operation is manifestly the same, as that by which fractions are reduced to a common denominator. Hence we have this general consequence, that if any two numbers be divided the one by the other, the quotient of the one by the other is the reciprocal of the quotient of that one by the other. — The method in observation 1. for dividing by mixed numbers in applicate cases, is so plain, that it needs no demonstration. For since all the parts are equal to the whole, the divisor, I mean the numerator by which we divided, expresseth all the parts contained in it when a mixed number, of which the quotient represents so many as the dividend containeth, considered as a mixed number; the quotient therefore, when multiplied by the nu-

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merator of the fractional part of the divisor, in its first state, must undoubtedly give the quotient due to that fraction; and, when multiplied by the denominator, must give the quotient due to unity; the sum of all the units, and the fractional part being for proof always equal to the dividend.

*Application of vulgar fractions to questions.*

1. A merchant had bonds for L.  $774:13:4\frac{1}{3}$ , bills amounting to L.  $199:19:7\frac{5}{8}$ , goods valued at L.  $785:15:5\frac{4}{5}$ , and open accounts for L.  $570:19:6\frac{3}{4}$ ; his debts amounted to L.  $999:19:9\frac{5}{8}$ ; how much was he worth?

First,	L. $774$	$13$	$4\frac{1}{3} = \frac{3}{8}$	$\frac{5}{8} + \frac{3}{8} = 1\frac{1}{8} = 1\frac{10}{8}$
	$199$	$19$	$7\frac{5}{8} = \frac{5}{8}$	
	$785$	$15$	$5\frac{4}{5} = \frac{16}{5}$	$\frac{16}{5} + \frac{15}{5} = 1\frac{11}{5} = 1\frac{11}{5}$
	$570$	$19$	$6\frac{3}{4} = \frac{27}{8}$	Therefore I carry 2, and note $\frac{4}{8}$ for the fraction.
From	<hr/>	<hr/>	<hr/>	
Take	$2331$	$8$	$0$	
	<hr/>	<hr/>	<hr/>	
	$999$	$19$	$9$	
	<hr/>	<hr/>	<hr/>	
Nett stock,	$1331$	$8$	$3$	$\frac{17}{80}$

To one versant in additions of this kind, the summation of such fractions as can be brought to one assigned denominator, is as easy as any other.

2. What will  $17\frac{1}{4}$  yards of cloth come to at 15 s. 8 d. per yard?

First,  $17\frac{1}{4} = \frac{71}{4}$ . Then,  $\frac{71}{4} \times \frac{188}{80} = \frac{13148}{800} = L. 15:10:5\frac{1}{4}$ , and 15 s. 8 d.  $= \frac{188}{80}$ .

3. Bought  $17\frac{1}{4}$  yards of cloth for L.  $15:10:5\frac{1}{4}$ , what did it cost per yard?

First,  $17\frac{1}{4} = \frac{71}{4}$ , and L.  $15:10:5\frac{1}{4} = \frac{13148}{800}$ . Then  $\frac{4}{71} \times \frac{13148}{800} = 15$ s. 8d.

4. What is the solid content of a bale 5 feet 6 inches long, 4 feet 5 inches broad, and 3 feet 3 inches thick?

f. i.  
First,  $5:6 = \frac{11}{2}$  least terms. Then  $\frac{11 \times 53 \times 13}{2 \times 12 \times 4} = \frac{7579}{96} = 77:10\frac{7}{96}$   
 $4:5 = \frac{5}{4}$   
 $3:3 = \frac{3}{4}$  least terms.

5. What

5. What is the tunnage of a bale 77 feet  $10\frac{7}{8}$  inches solid, at 40 per tun?

$$\text{f. i.} \\ 40 ) 77 \ 10\frac{7}{8} = 77\frac{7}{8} (\overline{-} 7\frac{7}{8} = 1\frac{7}{8}\frac{7}{8}$$

6. Divide L. 578, 19 s. among A, B, C, and D; let A, B, and C have equal shares, and each  $\frac{1}{4}$  more than D.

$$\begin{array}{r} L. \quad s. \quad d. \\ 3\frac{3}{4} = \frac{15}{4} \left[ \begin{array}{r} 578 \quad 19 \quad 0 \\ \hline 38 \quad 11 \quad 11\frac{1}{5} \\ \hline 3 \end{array} \right] \\ \begin{array}{l} 115 \quad 15 \quad 9\frac{3}{5} \text{ to D.} \\ 154 \quad 7 \quad 8\frac{4}{5} \text{ to A.} \\ 154 \quad 7 \quad 8\frac{4}{5} \text{ to B.} \\ 154 \quad 7 \quad 8\frac{4}{5} \text{ to C.} \\ \hline 578 \quad 19 \quad 0 \text{ Proof.} \end{array} \end{array}$$



## II. OF DECIMAL FRACTIONS.

### INTRODUCTION.

**I**N decimal fractions, an unit is supposed to be divided into 10 equal parts, each of these into 10 other equal parts, and each of these into 10 other equal parts, if necessary, and so on *ad infinitum*.

A decimal fraction is distinguished from an integer by a comma or point prefixed to it, whose denominator, though seldom or never expressed, is easily known by the distance of the first figure to the right hand, from the separating point, counted as 1, at the same distance in the numeration-table, from the unit's place. Hence .5 is expressed  $\frac{5}{10}$ , .75 as  $\frac{75}{100}$ , .05 as  $\frac{5}{100}$ , .005 as  $\frac{5}{1000}$ , &c. Hence it will be obvious, that, in a series of decimal parts, the decrease will be in proportion as the increase in a series of whole numbers,

and that, in consequence, ciphers immediately after the point diminish the value of the figures succeeding them, in the same proportion, as the figures considered as integers would be increased by the same number of ciphers annexed.

## C H A P. I.

## REDUCTION OF DECIMALS.

*Prob. 1.* To reduce a vulgar fraction to a decimal.

Divide the numerator by the denominator; and as ciphers must be annexed to the numerator before the division, the quotient must consist of as many places as the numerator had ciphers annexed to it, which will be the decimal required.

## E X A M P L E S.

(1.)

(2.)

Reduce  $\frac{4}{5}$  to a decimal.  $5) \overset{1}{\cancel{4}} \underset{0}{\cancel{0}}$ . Red.  $\frac{3}{4}$  to a decimal.  $8) \overset{1}{\cancel{3}} \underset{0}{\cancel{0}}$ .

(3.)

Reduce  $\frac{1}{20}$  to a decimal.  $20) \overset{1}{\cancel{0}} \underset{0}{\cancel{0}}$ .

(4.)

Reduce  $\frac{1}{80}$  to a decimal.  $80) \overset{1}{\cancel{0}} \underset{0}{\cancel{0}}$ .

*Note.* It will often happen, that, in the division, there will continually be a remainder, and the quotient repeat the same figure, or figures, *ad infinitum*; in which case, it will be unnecessary to carry on the division farther, when you have once got the repetend, which may be single, repeating always the same figure; or compound, always repeating or circulating the same figures. A single repetend may be marked or distinguished with a point above it, and compounds with a point above the first and last figures of the circulation.

## EXAMPLES OF SINGLE REPETENDS.

(1.)

Reduce  $\frac{1}{3}$  to a decimal.  $3) \overset{1}{\cancel{0}} \underset{3333}{\cancel{3}} \text{ ad infinitum.}$

(2.)

Reduce  $\frac{5}{6}$  to a decimal.  $6) \overset{5}{\cancel{0}} \underset{833333}{\cancel{3}} \text{ ad infinitum.}$

(3.)

(3.)

Reduce  $\frac{7}{12}$  to a decimal.  $12) \overline{70} \quad .58333, \text{ &c.}$

(4.)

Reduce  $\frac{1}{18}$  to a decimal.  $36) \overline{50} \quad .13888, \text{ &c.}$

## EXAMPLES of COMPOUND REPETENDS.

(1.)

Reduce  $\frac{2}{18}$  to a decimal.  $18) \overline{20} \quad .181818 \text{ ad infinitum.}$

(2.)

Reduce  $\frac{5}{27}$  to a decimal.  $27) \overline{50} \quad .185185, \text{ &c.}$

## O B S E R V A T I O N.

The reason of this reduction is manifest: for, to add ciphers to the numerator, is in effect the very same thing as to multiply by 10, 100, 1000, &c.; consequently the quotient will be so many 10's, 100's, 1000's, &c. as the places contained in it represent; the numerator and denominator being increased in the same proportion: hence, if the quotient is found without a remainder, it will be equal to the vulgar fraction, from which it was taken; and if it terminated in any repeating figure or figures, it will be short of the fraction from which it was taken, by a fraction of which the remainder is the numerator, and the divisor the denominator, in the last place of the quotient; that is, the remainder 1, in the first example of single repetends, =  $\frac{1}{9}$  of  $\frac{1}{181818}$ , and the remainder 2, in the second example, =  $\frac{2}{9}$  of  $\frac{1}{185185}$ . If therefore, in any decimal reduction, the division is carried on to 4 or 5 places, the remainder will be too minute to cause any sensible error in calculations of business; though, at the same time, for the satisfaction of the curious, particular rules are given here for the management of circulating decimals.

*Prob. 2.* To reduce any of the known parts of an integer to a decimal.

This

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This can be done by reducing the given parts to their equivalent vulgar fraction, and thence to a decimal, as above; but much more expeditiously by the following

## R U L E.

Annex ciphers to the given number of the lowest denomination, and divide it by such a number of that, as is equal to an unit of the next superior; to that quotient prefix the given number of units of the next superior, and divide by as many of that as will make an unit of the next superior; proceed in the same manner throughout the whole, and the last quotient is the decimal required.

## E X A M P L E S.

(1.)

Reduce 7 s. 6 d. to a decimal.  $\underline{12)60}$ 

(2.)

Reduce 4 s.  $6\frac{1}{2}$  d. to a decimal.  $\begin{array}{r} \underline{2)75} \\ \underline{12)65} \\ \underline{2)45416} \\ .227083 \end{array}$ 

*Note 1.* The reason of this manner of reduction is the same with the foregoing, as it is only dividing by the component parts of the denominator of a vulgar fraction equivalent to the given parts.

2. In reducing to decimals, it is unnecessary to mind any ciphers on the right of the divisor, as ciphers must be annexed to the remainders to bring out the quotient.

3. When in any division the quotient is repeated, the next division is carried on, not by ciphers, but by that repetend as in the second example, and that quotient will always terminate in a repetend.

By the last rule, it will likewise be found, that

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s. d. L. oz. drwt. gr. lb.  
 10 8 $\frac{1}{3}$  = .534583; that 11 18 20 Troy=.9951383; that  
 in. feet. c. q. lb. tuns.  
 4 $\frac{1}{8}$  = .34375, and that 19 3 14 = .99375, &c.

O B S E R V A T I O N.

After this manner are all decimal tables constructed, and they who think such tables contribute any thing to dispatch, may easily construct them. But I cannot help thinking that any decimal parts may be found as quickly, as they can be picked out of a table, if not more so, by one much versant in figuring; and with this further advantage, that, by calculating them upon all occasions, one gains a certain practice which renders figuring easy and familiar, at the same time that he avoids the meanness of seeing with the eyes of others.

*Prob. 3.* To value decimals in the known parts of the integer.

*Case 1.* If the decimal was finite, multiply by as many of the next inferior denomination as make one of the integer, and point off a number of places from the right of the product, equal to those in the given decimal; which will be a new decimal, to be multiplied by as many of the next inferior denomination, as made one of the former; proceed thus till the decimal places become ciphers, or no denomination be left for a multiplier, and the figures to the left hand of the point give the value required.

E X A M P L E S.

(1.)	s. d.	(2.)
Value .375 l.	375=7 6	Value .99375 tuns.
	20	cwt. q. lb.
	—	.99375=19 3 14
7.500		20
12		—
—		19.87500
6.000		4
		—
		3.50000
		28
		—
		14.00000

*Note,* This is only the converse of the last problem.

*Cafe*

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*Case 2.* If the given decimal terminate in a repetend, carry 1 for every 9 in the product of the first figure of the multiplication.

## EXAMPLES.

L.	s.	d.	cwt.	q.	lb.	Troy,	oz.
.833=16	8	.666	tun=13	1	9 $\frac{1}{3}$	.533 = 10	
	20		20				12
							—
16.660		13.33				10.00	
12		4					—
8.00		1.33					
		28					—
		9.33					

*Note.* In multiplying, the ciphers to the right need not be noted, but they must be remembered in pointing the product.

## OBSERVATIONS.

1. The reason of carrying at 9 in multiplying repetends, will be pretty obvious, if we consider, that a series of 9's infinitely continued is equal to one in the left-hand place; for it is evident that .9, or  $\frac{9}{10}$ , is only  $\frac{1}{10}$  short of 1; that .99, or  $\frac{99}{100}$ , wants  $\frac{1}{100}$  of 1, and that .999, or  $\frac{999}{1000}$ , is only  $\frac{1}{1000}$  of 1; so if the series was carried on to infinity, the difference between that series of nines and an unit would be equal to unity divided by infinity, which would quote nothing.

2. The first three places in decimals will give the value exact enough in business; and as, in mercantile calculations, the value of the decimal of a pound Sterling is more generally wanted than any other, it may be found by inspection thus.

Double the figure next to the point, makes shillings; if the next figure be 5, or above, add one to the shillings; the figure in the second place, if below 5, or the surplus above 5, added to the third figure, make farthings; but, if their sum be 25, or under 25, one must be taken off, to make them farthings; and if their sum be 26, or above, two must be taken off.

As the last part of this rule differs from the rules commonly given, it will perhaps be necessary to give some illustration of it.

The difference between a pound in thousand parts, and a pound

pound in farthings, is 40; consequently, as 1000 is to 40, so is any other number of decimal parts to what must be taken off to make them farthings; but 1000 : 40 :: 500 : 20, so is 250 to 10, so is 25 to 1; by which it is plain, that one taken from 25 1000 parts, make them 24 farthings exactly.

As the rules commonly given for finding the value by inspection, give an answer frequently not precisely the same with the other rule of multiplying by 20, 12, &c. and dividing by 1000, it seems necessary to show that both rules give the same answer precisely as in the margin.

$$\begin{array}{r} .507 \quad .776 \\ 20 \quad 20 \\ \hline 10.140 \quad 15.520 \\ 12 \quad 12 \\ \hline 1.68 \quad 6.24 \\ 4 \quad 4 \\ \hline 2.72 \quad 0.96 \end{array}$$

In the first example, there is seven thousand parts, from which I take one, conform to the rule, and there remains 6 farthings, or 1 penny 2 farthings; and, to find the .72 that is over, I say, as 25 to 1, so 7 to .28, which must be taken from the 7, as in the margin.

In the second example there is 26 over the shillings from which take 2, conform to the rule, and there remains 24 farthings, or 6 pence; and to find the 96, I say, as 25 to 1, so is 26 to 1.04, which must be subtracted from the 26.

$$\begin{array}{r} 7 \\ .28 \\ \hline 6.72 \end{array}$$
  

$$\begin{array}{r} 26 \\ 1.04 \\ \hline 24.96 \end{array}$$

## CHAP. II. ADDITION OF DECIMALS.

*Case 1.* HAVING placed the numbers to be added, whether pure decimals or mixed numbers, successively below one another, in such a manner as the several points may be in one column, tenths under tenths, hundreds under hundreds, &c.; if the given decimals are finite, add them as integers, and mark the separating point in the sum directly under the points of the given decimals, or point off as many for decimals as were in any of the given numbers which had most places.

E	X	A	M	P	L	E	S.
75.436			59 $\frac{1}{4}$	=	59.25		
47.324			67 $\frac{1}{2}$	=	67.5		
3.21			48 $\frac{3}{4}$	=	48.75		
6.7547			8 $\frac{1}{8}$	=	8.125		
.307			9 $\frac{1}{20}$	=	9.05		
.005							
			192 $\frac{27}{40}$	=	192.675		

133.0367

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*Case*

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*Case 2.* When all, or any of the decimals, repeat a single digit, make the repetends conterminous, and add 1 to the sum of the first, or right-hand column, for every nine that is contained in it.

E X A M P L E S.
$475\frac{2}{7} = 475.\overline{6666}$
$397\frac{4}{7} = 397.\overline{1333}$
$475\frac{5}{7} = 475.\overline{8333}$
$99\frac{1}{5} = 99.\overline{1111}$
$8\frac{5}{9} = 8.\overline{5555}$
<hr/>
$1456\frac{1}{7} = 1456.3$
$245\frac{1}{18} = 245.18\overline{4}$
<hr/>

## O R B: S E R V A T I O N.

The finite value of a pure circulate is a fraction whose numerator is the repetend, and denominator a number of as many places of nines, with a number of ciphers on the right, equal to the places betwixt the point and repetend. Hence, where the conterminous repetends of several circulates are added, their sum is a numerator to the common denominator; and if one for every nine in the sum is added thereto, it is reduced to the finite expression.

*Case 3.* If the decimals are compound repetends, from the place where all the repetends begin together, continue each decimal to a number of places, equal to the least common multiple of those several numbers, which represent the places of figures in the said repetends; then add, and to the last place add as many units as there are tens in the place, where the repetends all begin together, and the figures in these two places are the first and last of the repetend.

## E X A M P L E S.

$175.\overline{3724}$	$14.\overline{472956}$	$121.\overline{47237}$
$84.\overline{5634}$	$12.\overline{307243}$	$80.\overline{27555}$
$126.\overline{4526}$	$9.\overline{020763}$	$64.\overline{90834}$
$79.\overline{3279}$	$11.\overline{912375}$	$80.\overline{07444}$
$105.\overline{7105}$	$8.\overline{457813}$	$9.\overline{83333}$
<hr/>	<hr/>	<hr/>
$571.\overline{4270}$	$56.\overline{171152}$	$356.\overline{56405}$
		O B S E R-

## Chap. III. Of DECIMAL FRACTIONS. 91

### O B S E R V A T I O N.

Any repetend whatever is to the same number complete, as 10 to 9, if of one place; as 100 to 99, if of two places; and as 1000 to 999, if of three places, &c. Hence any number multiplied by 1, with as many ciphers as it contains figures, and the product thereof divided by as many nines, will give the same number perpetually circulating; but to add 1 for every ten contained in the sum, is the same as to multiply that sum by 10, and divide by 9.

### C H A P. III.

### S U B T R A C T I O N O F D E C I M A L S.

*Case 1.* PLACE the numbers homogeneous under homogeneous, so as point may be under point; then, when the decimals are finite, subtract as in integers, and let the point in the remainder stand directly under those of the factors.

#### E X A M P L E S.

$$\begin{array}{r} \text{From } L. 74 \ 19 \ 6 = 74.975 \quad L. 171 \ 13 \ 6 = 171.675 \\ \text{Take} \quad 18 \ 11 \ 9 = 18.5875 \quad \quad 97 \ 18 \ 9 = 97.9375 \\ \hline 56 \ 7 \ 9 = 56.3875 \quad \quad 73 \ 14 \ 9 = 73.7375 \end{array}$$

*Case 2.* If a single digit is repeated, borrow 9 in the first repeating place when necessary.

#### E X A M P L E S.

$$\begin{array}{r} \text{From } L. 7849 \ 6 \ 8 = 7849.333 \quad L. 17 \ 11 \ 7 = 17.5761\dot{6} \\ \text{Take} \quad 6979 \ 13 \ 4 = 6979.666 \quad \quad 14 \ 16 \ 8 = 14.83333 \\ \hline 869 \ 13 \ 4 = 869.666 \quad \quad 2 \ 14 \ 11 = 2.7428\dot{3} \end{array}$$

*Case 3.* If the decimals be compound repetends, order them as in addition of compound repetends; and if it is necessary to borrow one, where both repetends begin together, add one to the right-hand place of the subtrahend, and that

M 2 figure

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figure in the remainder, where both repetends begin together, will be the first, and the right-hand figure the last of the repetend.

## EXAMPLES.

$$\begin{array}{r}
 47.4178178 \\
 -15.5656565 \\
 \hline
 31.8521612
 \end{array}
 \quad
 \begin{array}{r}
 47.8540060 \\
 -40.9259259 \\
 \hline
 6.9280801
 \end{array}
 \quad
 \begin{array}{r}
 153.92749 \\
 -142.85353 \\
 \hline
 11.07395
 \end{array}$$

## ANALYSIS CHAP. IV.

## MULTIPLICATION OF DECIMALS.

*Case 1.* **W**HEN the decimals are finite, find the product as in integers, from which point off as many for places as were in both factors; if the whole product doth not count so far, supply that defect by prefixing ciphers.

## EXAMPLES.

$$\begin{array}{r}
 368.5 \\
 -2.75 \\
 \hline
 18425
 \end{array}
 \quad
 \begin{array}{r}
 672.5 \\
 -365 \\
 \hline
 33625
 \end{array}
 \quad
 \begin{array}{r}
 .246 \\
 .125 \\
 \hline
 1230
 \end{array}
 \\
 \begin{array}{r}
 25795 \\
 7370 \\
 \hline
 10.13375
 \end{array}
 \quad
 \begin{array}{r}
 40350 \\
 20175 \\
 \hline
 245.4625
 \end{array}
 \quad
 \begin{array}{r}
 492 \\
 246 \\
 \hline
 .930750
 \end{array}$$

## OBSERVATION.

If we conceive the two given numbers as fractions, it will be plain that the numerators and denominators are multiplied together respectively, since as many places are taken from the product, as there are in the denominators of both factors; which likewise accounts for prefixing ciphers, when there are otherwise not so many places in the product as were in both factors.

CON-

## CONTRACTIONS.

All the contractions in the first part, which regard multiplication, may be applied here, but the following seem peculiarly adapted to multiplication of decimals.

1. To multiply by 10, 100, 1000, &c. remove the decimal point so many steps further to the right, as there are ciphers in the multiplier.

As,  $47.565 \times 100 = 4756.5$ , and  $.45 \times 1000 = 450$ , &c.

2. When the places of decimals run far in both factors, the work may be contracted to as few places of decimals as may be thought sufficient for the purpose, by the following rule.

Set the units place in the multiplier directly under that figure of the decimal part in the multiplicand, whose place you would preserve in the product; invert all the other figures of the multiplier, and, in multiplying, begin with the figure of the multiplicand, which stands over the figure wherewith you are then multiplying, and set down the first figure of every particular product directly under each other, remembering at the same time to add the increase which would arise from the multiplication of the two next right-hand figures of the multiplicand, to the first figure of every product; that is, if the product of the next right-hand figure, with as many units added to it as there are tens in the product of the second right-hand figure, be any number betwixt 5 and 15, carry 1; if 15, or any number below 25, carry 2, and so in proportion.

## EXAMPLES.

Multiply 54.321711 into 3.12321, and preserve 4 decimal places in the product.

$$\begin{array}{r}
 54321711 \\
 12321.3 \\
 \hline
 543217 \times 3 = 1629651 \\
 54321 \times 1 + 1 = 54322 \\
 5432 \times 2 = 10864 \\
 543 \times 3 + 1 = 1630 \\
 54 \times 2 + 1 = 109 \\
 5 \times 1 = 5 \\
 \hline
 1696581
 \end{array}
 \qquad
 \begin{array}{r}
 \text{Multiply } 231.3121 \text{ into } 21.32, \\
 \text{and save 3 decimal places.} \\
 231.3121 \\
 21.32 \\
 \hline
 4626242 \\
 231312 \\
 69394 \\
 4626 \\
 \hline
 4931.574
 \end{array}$$

Cafe

94 OF ARITHMETIC. Part II.

*Case 2.* If the multiplicand terminate in a single repetend, and the multiplier is only a single digit, carry at 9 in the first figure of the multiplication on the right hand.

E	X	A	M	P	L	E	S.
10.7016		9.305		476.05		74.86	
5		7		.08		.6	
—		—		—		—	
53.5083		65.138		38.0844		44.920	

*3.* If the multiplier consists of several digits, or figures, make the products conterminous before addition.

E	X	A	M	P	L	E	S.
748.65				158.83			
.634				123			
—				—			
299462				47650			
2245966				1906000			
44919333				19536.50			
—				—			
474.64762							

*Case 3.* If the multiplier be a repetend, multiply the product found, as before, by 10, and divide that product by 9, which will quote the true product, if the division is continued, till the quotient terminate in a single or compound repetend.

E	X	A	M	P	L	E	S.
724.35				251.43		48.754	
5.04				8.74		2.13	
—				—		—	
9)289.740				9)100.572		9)14.6263	
—				—		—	
32.193				11.1746		1.625148	
3621.75				176001		4875444	
—				201144		97508888	
Product 3653.943				Product 2198.6156		104.009481	

*Cafe*

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*Case 4.* If the multiplicand be a compound repetend, and the multiplier only a single digit, to the product of the first figure on the right hand add as many units as there are tens in the product of the left-hand place of the repetend.

E X A M P L E S.

$$\begin{array}{r} 582.347 \\ \times 8 \\ \hline 4658.7\dot{8} \end{array} \qquad \begin{array}{r} 5924.378 \\ \times .03 \\ \hline 177.73135 \end{array}$$

*Case 5.* If there are no repetends in the multiplicand, and the multiplier be a compound repetend, add the product found, as in finite decimals, to itself in this manner. Set the first left-hand figure so many places forward, as exceeds the number of places in the repetend by one, and the rest of the figures in order after it, and proceed thus till the highest figure of the product stand directly below, or be removed beyond the lowest figure in the first position. Then, beginning with the lowest figure of the product in its first position, add it with all the figures that are below it, and do the same by the rest in their order; then point off as many places for a repetend as the multiplier consists of.

E X A M P L E S.

$$\begin{array}{r} 235.0\dot{1} \\ \times 3.2\dot{6} \\ \hline 141006 \\ 47002 \\ 70503 \\ \hline 766.1326 \\ 7661326 \\ 7661, & \text{etc.} \\ \hline 766.8994 \end{array} \qquad \begin{array}{r} 42710.36 \\ \times .20403 \\ \hline 12813108 \\ 17084144 \\ 8542072 \\ \hline 8714.1947508 \\ 87141947, & \text{etc.} \\ 871 \\ \hline 8714.2818938 \end{array}$$

All

All these cases of circulating decimals might be effected as intelligibly, though not so quickly, by managing them as vulgar fractions.

## C H A P. V.

## D I V I S I O N o f D E C I M ' A L S.

*Case 1.* WHEN the decimals are finite, the quotient is found, as in integers, and in all cases pointed or valued, by the following rules.

1. If the places in the divisor and dividend are equal, the quotient is integral.
2. If the divisor hath most places, annex ciphers to the dividend, to make them equal, and the quotient will still be integral.
3. If the dividend hath most places, point off places for the excess in the quotient.
4. If the whole quotient is not equal to the excess, prefix ciphers for the defect.

*Note.* If, after the quotient is qualified, there be a remainder, the division may be continued at pleasure.

## E X A M P L E S.

(1.)	(2.)
$24.35)78345.15(32\bar{1}7$	$.4725)113.4$
$\underline{7305}$	$\underline{4725}$
$5295$	$113.4000(240$
$\underline{4870}$	$\underline{9450}$
$4251$	$18900$
$\underline{2435}$	$\underline{18900}$
$18165$	....0
$\underline{17045}$	
$(1120)$	

(3.)

$$\begin{array}{r}
 \text{(3.)} \\
 2.87)67.05627(23.364 \\
 \underline{574} \\
 965 \\
 \underline{861} \\
 1046 \\
 \underline{861} \\
 1852 \\
 \underline{1722} \\
 1307 \\
 \underline{1148} \\
 \underline{\quad\quad\quad} \\
 (159)
 \end{array}
 \qquad
 \begin{array}{r}
 \text{(4.)} \\
 .543).0020091(.0037 \\
 \underline{1629} \\
 3801 \\
 \underline{3801} \\
 \underline{\quad\quad\quad} \\
 (0)
 \end{array}$$

## ILLUSTRATION.

In the first example, the places in each factor are two, and the quotient is integral.

In the second example, because the divisor consisted of four places, and the dividend but of one, three ciphers were annexed to the dividend, which made the places in both equal, and the quotient was accordingly integral.

In the third example, because there were five places in the dividend, and but two in the divisor, the excess, 3, was pointed off for decimals.

In the fourth example, as there were seven places in the dividend, and three in the divisor, the quotient required four places of decimals; and as there were but two places in the quotient, two ciphers were prefixed to make up the deficiency.

## OBSERVATION.

The rationale of valuing or qualifying the quotient will appear, if we consider, that the product of the quotient and divisor is equal to the dividend, and consequently the places of the divisor and quotient, counted together, will always be equal to the dividend; or, which is the same thing, the number of places in the quotient must be equal to the difference of the places in the divisor and dividend.

## CONTRACTIONS.

- I. In dividing by 10, 100, 1000, &c. the quotient is  
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found by removing the decimal point in the dividend so many steps towards the left hand, as there were ciphers in the divisor. Hence,  $\frac{34.5}{10} = 3.45$ , and  $\frac{44.5}{100} = .345$ .

2. The work of division may be contracted in the same manner as multiplication, by the following

## R U L E.

Having considered in what place the first figure of the quotient ought to stand, and so found its value, or denomination, take as many of the left-hand figures as you intend to have figures in the quotient, for the first divisor, and then take as many figures of the dividend as will answer them; in dividing, omit, or point off one figure at each operation, at the same time judging as exactly as possible what would be the increase, arising from the figure, or figures, so omitted.

## E X A M P L E S.

384.672158)	14169.2066238510(36.8345
11540	16474
26290	4188.
23080	3295.
3210	0893..
3077	3772..
132	7121...
115	4016...
17.3	105....
15	3869....
192	36.....
192	34.....
	519
	468
	51
	47
	(4)

All

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All the other contractions proposed in the division of integers may be very properly applied here.

*Case 2.* When the dividend contains a single repetend, and the divisor is either a single terminate digit, or any number of terminate digits, the quotient will either repeat a single digit or compound repetend, commencing generally where the repetend is first taken down.

E X A M P L E S.

$$8)79.2\dot{6}$$

$$6)3076.\dot{1} \dots$$

$$7)51.2$$

$$\underline{9.908\dot{3}}$$

$$\underline{512.6\dot{8}5\dot{1}}$$

$$\underline{7.31746\dot{0}}$$

2. When the divisor is only a single repetend, and the dividend either a terminant number, or contains a repetend, multiply the dividend by 9, and point off the same number of integral places, which were first given, the product thus qualified will be a new dividend.

*Exam.* 378.45 by .6. Or thus, 572.4 by .8  

$$\begin{array}{r} 9 \\ \hline .6)340.605 \end{array}$$

$$\begin{array}{r} 57.24 \text{ subtract } \frac{1}{6} \\ \hline .8)515.16 \end{array}$$

$$\underline{567.675}$$

$$\underline{643.95}$$

3. If the divisor hath terminant numbers joined to the repetend, subtract the terminant part of the divisor from the whole divisor, — prepare the dividend as before, and work by the new factors.

100 OF ARITHMETIC. Part II.

$$\begin{array}{r}
 \text{Exam.} \quad 48.6 \left) \begin{array}{r} 8567.28 \\ - 48 \\ \hline 856.728 \end{array} \right. \quad 3.3 \left) \begin{array}{r} 728.5 \\ - 3 \\ \hline 72.85 \end{array} \right. \\
 \hline
 \begin{array}{r} 4.38 \left) \begin{array}{r} 7710.552(1760.4 \\ - 438 \\ \hline 3330 \\ - 3066 \\ \hline 2645 \\ - 2628 \\ \hline 1752 \\ - 1752 \\ \hline \end{array} \right. \end{array} \quad 3.0 \left) \begin{array}{r} 655.65 \\ - 2185.5 \\ \hline \end{array} \right. \\
 \end{array}$$

4. When both factors, or the divisor only, consists of compound repetends, set the divisor and dividend under themselves, so many places forward, as there are places in the repetend of the divisor; subtract them, and the remainders will be respectively a new divisor and dividend.

E X A M P L E S.

$$\begin{array}{r}
 111.98 \left) \begin{array}{r} 243.306 \\ - 243 \\ \hline 243.063(2.172 \\ - 22374 \\ \hline 19323 \\ - 11187 \\ \hline 81360 \\ - 78309 \\ \hline 30510 \\ - 22374 \\ \hline 8136'' \end{array} \right. \quad 587645 \left) \begin{array}{r} 411351.9 \\ - 58.7 \\ \hline 58705.8 \end{array} \right. \quad 411.3 \left) \begin{array}{r} 410940.6(7 \\ - 410940.6 \\ \hline \end{array} \right. \\
 \hline
 \end{array}$$

APPLI-

Chap. V. Of DECIMAL FRACTIONS. 101

APPLIQUE QUESTIONS in DECIMAL FRACTIONS.

1. What is the decimal difference betwixt L. 1, and  
13 s. 4 d.?

s. d. 1.000

$$13 \text{ } 4 = 0.\overline{666}$$

.333 Answer.

2. What is the square content of a room, 15 feet 6 inches  
by 14 feet 9 inches?

$$\begin{array}{r} \frac{1}{2}) 14.75 \\ \quad 15\frac{1}{2} \\ \hline 221.25 \\ \quad 7.375 \\ \hline 228.625 = 228 \text{ } 7\frac{1}{2} \end{array}$$

3. What is the solid content of a box 5 feet 4 inches  
long, 4 feet 10 inches broad, and 3 feet 8 inches thick?

$$\begin{array}{r} 4.8333 \\ 5\frac{1}{3} \\ \hline 24.1666 \\ \quad 1.\overline{6111} \\ \hline 25.7777 \\ \quad 3\frac{2}{3} \\ \hline 77.3333 \\ \quad 8.5925 \\ \quad 8.5925 \\ \hline 94.4185 = 94 \text{ } 5 \end{array}$$

4. What

## 102 OF ARITHMETIC. Part II.

4. What is the tunnage of a bale 9 feet 9 inches long, 7 feet 3 inches broad, and 5 feet 6 inches thick?

$$\begin{array}{r}
 9.75 \\
 7\frac{1}{4} \\
 \hline
 68.25 \\
 2.4375 \\
 \hline
 70.6875 \\
 5.5 \\
 \hline
 353.4375 \\
 3534.375 \\
 \hline
 4.0)388.78125 \text{ solid content.}
 \end{array}$$

9.71953125 tunnage.

5. A piece of cloth, consisting of  $25\frac{1}{2}$  yards, is valued at L. 23 : 17 : 5 $\frac{1}{2}$ ; how must one yard be valued at that rate?

$$\begin{array}{r}
 25.5 ) 23.872916 \\
 \hline
 5.1 \quad 4.774583 \\
 \hline
 .9361927 = 18 s. 8\frac{1}{2} d.
 \end{array}$$

6. What cost 22 cwt. 3 qrs. of sugar, at L. 3 : 17 : 6 per cwt?

$$\begin{array}{r}
 3.875 \\
 22\frac{3}{4} \\
 \hline
 7750 \\
 7750 \\
 \hline
 85.25 = 22 \text{ cwt.} \\
 1.9375 = \frac{1}{2} \\
 96875 = \frac{1}{4} \\
 \hline
 88.15625 = 22\frac{3}{4} = \text{L. } 88 : 3 : 1\frac{1}{2}.
 \end{array}$$

7. Divide

7. Divide L. 875 : 14 : 6, among A, B, and C, whereof let A have  $\frac{1}{2}$ , B  $\frac{1}{4}$ , and C  $\frac{1}{3}$ .

$$\frac{1}{2} = .75$$

$$\frac{1}{4} = .375$$

$$\frac{1}{3} = .6$$

$$\begin{array}{r}
 5) 1.725 \quad \left[ \begin{array}{r} 875.725 \\ - 345 \\ \hline .345 \end{array} \right] \quad \begin{array}{r} 175.145 \\ - 345 \\ \hline .069 \end{array} \\
 \begin{array}{r} 35.029(507.6666 \times \left\{ \begin{array}{l} .75 = 380.75 = A's \text{ share.} \\ .375 = 190.375 = B's \text{ share.} \\ .6 = 304.6 = C's \text{ share.} \end{array} \right. \\ \hline 345 \end{array} \\
 \begin{array}{r} 529 \\ 483 \\ \hline 460 \\ 414 \\ \hline .46 \end{array} \quad \begin{array}{r} 875.725 \text{ Proof.} \\ \hline \end{array}
 \end{array}$$

*ad infinitum.*

lb. 3. dwt.

8. What will 46 8 16 amount to at L. 7 : 8 : 6 per lb?

$$\begin{array}{r}
 40) 46.73 = \text{the weight.} \\
 \quad \quad \quad 7.4 = \text{L. } 7, 8 \text{ s.} \\
 \hline
 \end{array}$$

For 6 ds.      1 1683

$$346.995 = \text{L. } 346 : 19 : 10\frac{3}{4}.$$

*Demonstration of the last contraction after division of integers, promised in this place.*

The sum of 1, 10, 100, 1000, &c. continued to any number of terms = 1, the first term, as often repeated as the number of terms to be added: Thus,  $1+10=11$ , and  $1+10+100=111$ ; and  $1+10+100+1000=1111$ , as is obvious from the nature of numerical notation. Therefore, if any number,

number, as  $N$ , is multiplied by a series of equal figures, as  $fff$ , &c. the first product by  $f$ , in the place of units, will be  $fN$ , or  $1\ fN$ ; the next product by  $f$ , in the place of tens, will be  $10fN$ , the next  $100fN$ , &c.; and the total product of the whole multiplication will be  $111fN$ , according to the number of places in the multiplier. If 1, 10, 100, 1000, is divided decimaly by 9, the quotient-figures will be  $1111$ , &c. continued to the number of periods in the dividend, thus,  $\frac{1}{9}=1111$ , and  $\frac{10}{9}=1.1111$ , and  $\frac{100}{9}=11.1111$ , still leaving 1 of a remainder, and the quotient continually repeating 1. Therefore,  $f\frac{N}{9}=1111fN$ ; and, if we compare the two expressions,  $111fN$  and  $1111fN$ , &c. we will find they may be made equal, by placing the decimal point after the third figure, in the last, thus,  $111.111$ , &c. and then subtracting 111 from the decimal parts, the last, as well as the first, will be equal to the total product required. That is,  $f\frac{N}{9}-1111fN=111fN=T$ , the total product.

For let the multiplicand  $N=784$

The multiplier  $f = \underline{\hspace{2cm}} 333$

Therefore  $fN = \underline{\hspace{2cm}} 2352$

$111fN=f\frac{N}{9} = \underline{\hspace{2cm}} 261.\underline{3}3333$ , &c.

$111fN$  the difference deduced from the right hand =  $\underline{\hspace{2cm}} 2613$

$261072=T$ , the total product.

Q. E. D.

*Note.* The figure 3 repeating equally in both on the right hand, and making no difference, is not to be subtracted.

T H E  
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P A R T III.

THE ELEMENTS OF ALGEBRA.

I N T R O D U C T I O N.

**A**LGEBRA is the art of reckoning by symbols, or a short and concise method of reasoning on the various relations of numbers and quantities to each other, by the help of letters, or such other characters, as the algebraist thinks most convenient to adopt.

By algebra are resolved, in a method of reasoning which is clear and connected, and founded on self-evident principles, many obscure questions in arithmetic and geometry, and from thence theorems deduced, which hold universally with respect to problems of the same nature. Hence it is called by some *the new analysis*, by others *specious arithmetic*, or *numerical geometry*, because it is equally connected with both, and extends not only to abstract numbers, but also to lines, surfaces, and solids. It is not intended in this place to consider algebra in its full extent; what is precisely necessary for conducting the arithmetician scientifically through the whole of mercantile computations, shall be reduced to a few plain and intelligible rules; by the help of which we shall be able to demonstrate the reason of the most complicated operation that can occur in arithmetic, with greater perspicuity and elegance, than from principles purely arithmetical. At the same time, in the course of this work, we will not make use of algebraic demonstrations, but where we cannot otherwise do justice to the subject, that the reader may have an alternative, if his time should not permit him to study this part of the treatise.

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CHAP.

## C H A P. I.

## NOTATION OF ALGEBRA.

A NY simple number is represented by a single letter, and, for distinction's sake, numbers that are known, and given in a question, are generally expressed by consonants, and numbers that are required are expressed by some of the small vowels. Compound quantities are represented by two or more letters joined together; the sum of two or more numbers is expressed by the sign + connecting two or more letters together, as  $a+b$  means  $a$  and  $b$  added together;  $a+e+b$  means the sum of  $a$  and  $e$  and  $b$  added together. The difference of two numbers is noted by the sign - : thus  $a-b$  signifies the difference of  $a$  and  $b$ , or what  $b$  wants of being equal to  $a$ . The product of two or more letters is expressed by connecting them together in the form of a word, as  $ab=a\times b$ ;  $abc=a\times b\times c$ ; that is, the continual product of  $a$ ,  $b$ , and  $c$ . The quotient of two numbers by division is expressed like a vulgar fraction, with one letter representing the dividend above the line, and another below it representing the divisor, as  $\frac{a}{b}$  signifies the quotient arising from  $a$  divided by  $b$ , or  $a\div b$ . The square of any number or quantity is represented by a letter doubled, as  $aa$ , or its power noted above it thus  $a^2$ , each of which notations express the square of  $a$ , or  $a$  square. The cube of any number or quantity is represented by tripling the letter, or noting its power, as  $aaa$ , or  $a^3$ ; the biquadrate thus,  $aaaa$  or  $a^4$ , &c. The square root of any number is thus noted  $\sqrt{2}$ , the cube root  $\sqrt[3]{3}$ , the biquadrate  $\sqrt[4]{4}$ , &c.; as  $\sqrt[2]{186}$  is to be understood the square root of 186;  $\sqrt[3]{27c}$  is to be understood the cube root of 27c.

## O B S E R V A T I O N.

The characters + and - in algebra import something more than simple addition and subtraction, as in arithmetic: + is an affirmative or positive sign, expressive of real existence; and when it is put before any quantity, it is to be understood that that quantity is of a real and positive nature; whereas - is the sign of negation, or negative existence, denoting the quantity to which it is prefixed to be less than nothing.

Though no real body or substance can properly be supposed

sed to be less than nothing, yet quantities, whereby the different degrees of qualities are estimated, may easily be conceived to pass from an existence through nothing into negation. Thus a merchant's stock was at first 1000*l.* incumbered only with 50*l.* of debt, then his neat stock was 1000—50; but in the course of trade he met with misfortunes, which raised his debts to 1000*l.* and his subject when collected together amounted to no more than 1000*l.* and in this case he is just worth nothing: unwilling, however, to give up his trade, he struggles hard to preserve his credit for another year, and then, upon examining his books, finds he was still 1000*l.* in debt, and no subject to pay it, and in this case he is 1000*l.* worse than he was the preceding year, and at that time he was worth nothing, consequently he will be at this time 1000*l.* worse than nothing. Wherefore, as affirmative and negative quantities are contrary in their nature, they are likewise contrary in their effects; a consideration, which, if properly attended to, will obviate all the difficulties that may occur in the application of these signs; for the result of working by affirmative quantities is, in every operation, obvious, and therefore like operations, where the quantities are negative, may be known by the rule of contraries.

Where there is no sign prefixed to a quantity, the quantity is always understood to be affirmative, and if there is no numerical coefficient before it, unity must always be understood; thus  $3a$  signifies  $+3a$ , and  $a$  signifies  $+1a$ ; but the sign of a negative quantity is never omitted, nor indeed the sign of an affirmative one, excepting when such an affirmative quantity is considered by itself, or happens to be the first in a series of quantities succeeding one another.

## C H A P. II.

## ADDITION OF ALGEBRAIC QUANTITIES.

- I. ADD homogeneous quantities according to their number.

## E X A M P L E S.

(1.)	(2.)	(3.)
$a+a+a=3a$	$27a+3a=30a$	$4b+3b+5b=12b$

O 2

2. Heterogeneous

2. Heterogeneous or different quantities, expressed by different letters, are added by connecting them with the sign +. Thus  $a$  and  $b$  and  $c$  are added  $a+b+c$ , &c.

3. When the quantities to be added have like signs, the same sign is to be retained in the sum.

## E X A M P L E S.

(1.)	(2.)	(3.)
$+3a - 4b$	$15c + 5b - 4a$	$43a + 56$
$+7a - 5b$	$16c + 3b - 3a$	$37a + 13$
<hr/>	<hr/>	<hr/>
$10a - 9b$	$31c + 8b - 7a$	$80a + 69$

In this case it is plain, that one affirmative increased by the accession of another affirmative, must make a greater affirmative, in the same sense that a stock of 1000*l.* increased by a legacy of 500*l.* would in this new state be worth 1500*l.*; and that one negative increased by the addition of another negative must make a greater negative, in the same sense that the loss of 1000*l.* followed by another loss of 500*l.* would increase the loss to 1500*l.*

4. If the quantities to be added have contrary signs, the one affirmative and the other negative, the sign of the greater quantity must be prefixed to their difference.

## E X A M P L E S.

$+86$	$+2a$	$+37d$	$+a$
$-36$	$-9a$	$-14d$	$-a$
<hr/>	<hr/>	<hr/>	<hr/>
$+50$	$-7a$	$+23d$	$o$

In this case it is plain, that the sum of the quantities must be found by subtracting the lesser coefficient from the greater, and prefixing the sign of the greater quantity; for let  $+37a$  represent 37*l.* which a man hath in his pocket, and  $-19a$  19*l.* which he must pay out of it, he will after all have  $+18a$ : but reverse the case, and suppose he hath only  $+19a$  to pay  $-37a$ , he will then have  $-18a$  or 18*l.* short of his sum.

From these four cases may be deduced one general rule for addition of algebra, viz. Add like signs, and retain the same sign in the sum; but subtract contrary signs, and keep the sign of the greater.

## E X A M .

## EXAMPLES.

$$\begin{array}{r} 4a - 8b \\ a + 9b \\ \hline 5a + b \end{array}
 \quad
 \begin{array}{r} 4b + 3c \\ 2b - c \\ \hline 6b + 2c \end{array}
 \quad
 \begin{array}{r} 3a + b \\ a + 2b \\ \hline 4a + 3b \end{array}
 \quad
 \begin{array}{r} b - c \\ b - 2c \\ \hline 2b - 3c \end{array}$$

## C H A P. III.

## SUBTRACTION OF ALGEBRAIC QUANTITIES.

1. **H**omogeneous quantities are subtracted by taking the difference of their coefficients, as

$$5a - 3a = 2a. \quad 7b - 6 = 7b \text{ and } 8b - b = 7b$$

2. Heterogeneous quantities admit of no other subtraction but the sign  $-$ , as the difference betwixt  $a$  and  $b$  would stand  $a - b$  upon the supposition that  $a$  is the greatest quantity.

3. If quantities have like signs, their difference retains the same sign.

4. If a greater quantity must be taken from a lesser, to the difference betwixt the quantities prefix the contrary sign. Thus if it were required to subtract  $+5b$  from  $+2b$ , the remainder would be  $-3b$ . For suppose a man owes me  $2l.$  or  $2b$ , and I owe another man  $5l.$  or  $5b$ ; it is plain, that the difference betwixt these sums is a debt upon me, for which I have nothing equivalent, and therefore I am  $3l.$  or  $3b$  worse than nothing: for had the sums been equal, I should have had nothing in reversion, but then the first  $3l.$  or  $3b$  I ever got should have been my own; but whilst I am  $3l.$  or  $3b$  in debt, I must get  $3l.$  or  $3b$  before I am even with the world, and therefore in every acceptation in this case the remainder is a negative quantity. Besides, for a proof of subtraction,  $+5b - 3b$  being added together will be found  $=+2b$ .

5. When the quantities to be subtracted have different signs, add their coefficients, and to that sum prefix the sign of the minuend, which will express the difference. From  $+5a$  if it was required to subtract  $-3a$ , the remainder would be  $+8a$ : but if it had been required to take  $+5a$  from  $-3a$ , the remainder would be  $-8a$ . Hence to subtract a negative is the same thing as to add an affirmative, and to subtract an affirmative

affirmative is the same thing as to add a negative. For in the first instance, if I am in possession of  $5a$ , and indebted in  $3a$ , should that debt be forgiven, I am certainly  $3a$  better than when it stood against me; and in like manner, if I am indebted in  $5a$ , represented by  $-5a$ , and have only  $+3a$  to pay it, should these  $3a$  be lost or taken from me, I am certainly  $3a$  worse than I was in my former situation.

## EXAMPLES of COMPOUND QUANTITIES.

$$\begin{array}{r} \text{From } 12x+6a-4b-12c \quad -7e-5f \\ \text{Take } 2x-3a+4b \quad 5c+6d-7e \\ \hline \text{Rem. } 10x+9a-8b \quad 7c-6d \quad -5f \\ \hline \text{ } \quad 12x+6a-4b-12c \quad -7e-5f \end{array}$$

If never a member of the subtrahend is found to be of the same denomination with any member of the minuend, change the sign of every member of the subtrahend, and then add it to the other; as if  $5c-6d$  was to be subtracted from  $3a-4b$ , the remainder would be  $3a-4b-5c+6d$ .

## C H A P. IV.

## MULTIPLICATION OF ALGEBRAIC QUANTITIES.

**I**N multiplying quantities, it must be observed, that like signs, whether affirmative or negative, produce  $+$ , but different signs produce  $-$ .

## EXAMPLES.

$$+bx+c = +bc \quad -bx-c = -bc \quad -bx+c = -bc$$

## O B S E R V A T I O N S.

1. That  $+b$  into  $+c$  must produce  $+bc$ , is self-evident: for instead of the letters let us take  $+4$  and  $+3$  for a multiplicand and multiplier, the product in this case must be  $4+4+4=+12$ ; wherefore  $+3$  into  $+4$  must likewise give  $+12$ .

2.  $-b$  into  $-c = +bc$ . For assume the same figures 3 and 4 with negative signs, then multiply  $-4$  into  $3,0$  and  $-3$  successively, and the products will be in arithmetical progression;

## Chap. IV. MULTIPLICATION OF QUANTITIES. III

tion; but the two first products are  $-12$  and  $0$ , and the third product will be  $+12$ .

3.  $+b$  into  $-c = -bc$ . For if the product were  $+bc$ , it would be the same with the product of  $+b$  into  $+c$ , which would be absurd. But further, suppose a man  $10l.$  in debt, and without any subject to pay it, his worth may be expressed by  $-10$ , i. e.  $10$  worse than nothing; the double of  $-10$  would be  $-20$ , the triple would be  $-30$ , &c. according to any multiplication whatever. Therefore  $-10 \times +2+3$ , &c. by any multiplication whatever must give a negative product.

### E X A M P L E S.

(1.)

$$\begin{array}{r} 4 a+3c \\ a-b \\ \hline -4ba-3bc \\ +3ac+4aa \\ \hline 4aa+3ac-4ba-3bc \end{array}$$

(2.)

$$\begin{array}{r} a-b \\ a-b \\ \hline -ab+bb \\ aa-ab \\ \hline aa-2ab+bb \end{array}$$

(3.)

$$\begin{array}{r} a+b \\ a-b \\ \hline -ab-bb \\ +aa+ab \\ \hline +aa \dots -bb \end{array}$$

(4.)

$$\begin{array}{r} a+b \\ a+b \\ \hline +ab+bb \\ +aa+ab \\ \hline +aa+2ab+bb \end{array}$$

(5.)

$$\begin{array}{r} 6x-7a-8b \\ 2x-3a+4b \\ \hline 12xx-14ax-16bx+21aa+24ab-32bb \\ \quad -18ax+24bx \quad -28ab \\ \hline 12xx-32ax+8bx+21aa-4ab-32bb \end{array}$$

A dash over two or more quantities imports, that all those quantities are to be considered as constituting one compound quantity.

O B S E R.

## O B S E R V A T I O N.

From this rule there may be very naturally deduced the following theorems.

1. Exam.  $a+b$  into  $a-b$  produceth  $aa-bb$ . Hence the sum and difference of any two numbers being multiplied together will give the difference of these squares, and vice versa.

For  $4 \times 4 = 16$

$2 \times 2 = 4$

Diff. 2      12 the difference of the squares.

Sum 6

Product 12 of their sum and difference.

2. Exam. 4  $a+b$  into  $a+b$  produceth  $aa+2ab+bb$ . Hence, if a number be resolved into any two parts whatever, the square of the whole will be equal to the square of each part, and the double rectangle, or product of the multiplication of those parts added together. Thus if the number 10 be resolved into 7 and 3, 100 the square of 10, the whole will be equal to  $7 \times 7 + 3 \times 3 + 7 \times 3 \times 2$ , which is the square of each part, and double the product of the parts. *Eucl. book 2. prop. 4.*

3. Exam. 2  $a-b$  into  $a-b$  produceth  $aa-2ab+bb$ . Hence, if from the sum of the squares of any two numbers be subtracted the double product of those two numbers, there will remain the square of their difference. Thus in the numbers 7 and 3,  $7 \times 7 + 3 \times 3 = 58$  and  $7 \times 3 \times 2 = 42$  but  $58 - 42 = 16 = 7 - 3 \times 7 - 3$ , i. e.  $4 \times 4$ . *Eucl. 2. 7.*

## C H A P. V.

## DIVISION OF ALGEBRAIC QUANTITIES.

1. **T**O divide simple algebraic quantities, divide the coefficient of the dividend by that of the divisor, when possible, and then put down after the quotient all the quantities in the dividend that are not in the divisor; and for the sign to be prefixed to the quotient observe the rules in multiplication, since the quotient ought to contain a quantity, which being multiplied into the divisor, will produce the dividend.

E X A M-

## E X A M P L E S.

(1.)

$$+4b) -8bc \left( \begin{array}{r} -2c \\ +4b \\ \hline \end{array} \right)$$

 $-8bc$  Proof.

(2.)

$$4a) 24abc \left( \begin{array}{r} 6bc \\ 4a \\ \hline \end{array} \right)$$

 $24abc$  Proof.

(3.)

$$3b) 18bc \left( \begin{array}{r} 6c \\ 3b \\ \hline \end{array} \right)$$

 $18bc$  Proof.

(4.)

$$4a) 12abc \left( \begin{array}{r} 3bc \\ 4a \\ \hline \end{array} \right)$$

 $12abc$  Proof.

2. When the coefficients are incommensurable, or the quantities heterogeneous, place the divisor below the dividend in the manner of a vulgar fraction; as

$$\frac{b}{a}$$

$$\frac{17b}{9a}$$

$$\frac{5ab}{7a}$$

$$\frac{9xm}{7xm}$$

## C H A P. VI.

## A L G E B R A I C F R A C T I O N S.

**F**Ractions in algebra are managed in the same manner as in arithmetic, in their reduction, addition, subtraction, multiplication, and division; with this difference only, that the operations in the one are numerical, and in the other algebraical.

## E X A M P L E S.

1. Reduce  $\frac{4ab}{6bc}$  to their lowest terms.

Here  $2b$  is a common measure for both numerator and denominator, and therefore when they are both divided by  $2b$ , the quotient will be  $\frac{2a}{3c} = \frac{4ab}{6bc}$ , but expressed in lower terms.

Hence it follows, that whenever a common letter or factor is to be found in every member both of the numerator and denominator, it may be cancelled every where without affecting the value of the fraction; thus the fraction  $\frac{ac+bc}{cd+ce}$ , expunging  $c$ , becomes  $\frac{a+b}{d+e}$ ; but if there be any one member wherein the factor is not concerned, it must not be expunged at all.

2. Reduce  $\frac{a}{2}, \frac{b}{3}, \frac{c}{4}$  to one common denominator.

$$\left. \begin{array}{l} a \times 3 \times 4 = 12a \\ b \times 2 \times 4 = 8b \\ c \times 3 \times 2 = 6c \end{array} \right\} \text{Numerators.}$$

$2 \times 3 \times 4 = 24$  Common denominator.

3. Add  $\frac{2b}{3a}$  and  $\frac{3b}{4a}$

$$\begin{array}{r} 3a \times 3b = 9ab \\ 4a \times 2b = 8ab \\ \hline 3a \times 4a = \frac{17ab}{12aa} = \frac{17b}{12a} \end{array}$$

4. Subtract  $\frac{a}{2d}$  from  $\frac{4a}{5d}$

$$\begin{array}{r} a \times 5d = 5da \\ 4a \times 2d = 8da \\ \hline 2d \times 5d = \frac{3da}{10dd} = \frac{3a}{10d} \end{array}$$

5. Multiply  $\frac{4a}{9}$  into  $\frac{4ba}{7d}$

$$\begin{array}{r} 4a \times 4ba = 16baa \\ 9 \times 7d = 63d \end{array}$$

6. Divide  $\frac{a}{4d}$  by  $\frac{4b}{c}$

$$\frac{c}{4b} \left( \frac{a}{4d} \right) \left( \frac{ca}{16bd} \right)$$

## CHAP. VII.

### INVOLUTION NUMERICAL and ALGEBRAICAL.

**I**nvolution is the raising of a quantity or number to any assigned power, and is performed like multiplication, the multiplier always continuing the same. Hence, when a quantity or number is said to be raised to a certain power, it is multiplied into itself as often as the power to which it is raised expresseth.

Thus when 4 is multiplied into itself, it becomes 16, which is the square or 2d power of 4; when the square 16 is multiplied into 4, it will produce 64, the cube or 3d power; and when 64 is multiplied into 4, it will produce 256, the biquadratic or 4th power; and that multiplied into 4 will produce 1024, the fursolid or 5th power, &c.

In like manner,  $a \times a = aa$  or  $a^2$ ;  $a^2 \times a = aaa$  or  $a^3$ ;  $a^3 \times a = a^4$ ; and  $a^4 \times a = aaaa$  or  $a^5$ , &c. To express the power by a digit

is no doubt the best method of notation, as it is shortest, and the power discovered by its index at once, without the trouble of numbering the repetitions, which in the other case is unavoidable.

*Examples of roots raised to the 3d power.*

$$\begin{array}{r} 4a \\ 4a \\ \hline 16a^2 \text{ square:} \\ 4a \\ \hline 64a^3 \text{ cube.} \end{array} \qquad \begin{array}{r} 5b \\ 5b \\ \hline 25b^2 \\ 5b \\ \hline 125b^3 \end{array}$$

$a+b$  a binomial root.

$a+b$

$$\begin{array}{r} a^2+2ba+b^2 \\ a+b \\ \hline a^3+3ba^2+3bba+b^3 \end{array}$$

$a-b$  a residual root.

$a-b$

$$\begin{array}{r} a^2-2ba+b^2 \\ a-b \\ \hline a^3-3ba^2+3bba-b^3 \end{array}$$

## C H A P. VIII.

### EVOLUTION OF ALGEBRAIC QUANTITIES.

**E**volution reduces any given power back to its original quantity when possible. To do which, it must be observed, that some quantities are rational and have roots; but there are others surd or irrational, whose just root cannot be extracted. In the first species of quantities, consider what quantity involved or multiplied into itself as often as the index represents will produce a quantity equal to that given, and the quantity found in this manner will be that required. Thus, suppose it was required to extract the root of  $a^2$ , it would be  $a \times a = a^2$ , consequently  $a$  is the root required. In the same manner  $a$  is found to be the root of  $a^3$ , for  $a \times a \times a = a^3$ . In the same manner  $81d^2$  will be found to proceed from  $9d$ , for  $9d \times 9d = 81d^2$ .

Thus likewise will  $a$  be found to be the cube root of  $a^3$ , for  $a \times a \times a = a^3$ ,  $4a$  the cube root of  $64a^3$ , and  $a$  the biquadratic root of  $a^4$ .

If the square root of  $a^2+2ab+b^2$  was required, it would be found to be  $a+b$ ; for  $a+b \times a+b = a^2+2ab+b^2$ . In the same manner will  $a-b$  be found to be the square root of  $aa-2ab+b^2$ .

In this manner likewise may be found the roots of higher powers, as  $a^3+3ba^2+3bba+b^3$ , whose root is  $a+b$ .

In surd or irrational quantities, the roots are signified or denoted by the radical sign prefixed thus,  $\sqrt{z}g$ .  $\sqrt[2]{12B}$ ,  $\sqrt[2]{a^2-G}$  is the square root of those quantities to which the sign  $\sqrt{z}$  is prefixed; and  $\sqrt[3]{B+H}$   $\sqrt[3]{24S+a^3}$  is the cube root of the quantities to which the sign  $\sqrt[3]{}$  is prefixed.

## C H A P. IX.

## OF SURD QUANTITIES.

1. If the given surd quantities are heterogeneous, they are added by connecting them with the sign +, and subtracted with the sign -; as  $\sqrt{z}B+\sqrt{z}C$  expresseth the sum of these two surd quantities,  $\sqrt{z}G-\sqrt{z}ab$  expresseth the difference of the surd quantities on each side of the negative sign.

2. When they are homogeneous, they are added and subtracted according to their number; as  $4\sqrt{z}B+8\sqrt{z}B=12\sqrt{z}B$ , and  $9\sqrt{z}G-3\sqrt{z}G=6\sqrt{z}G$ , &c.

3. Surds are sometimes multiplied and divided by the signs  $\times$  and  $\div$  connecting them,  $\sqrt{z}b \times \sqrt{z}c$ , or  $\sqrt{z}b \div \sqrt{z}c$ ; and sometimes by multiplying and dividing the quantities themselves, when it can be done, and prefixing the radical sign to the product, or quotient, as  $\sqrt{z}ab \times 2\sqrt{z}12b=\sqrt{z}12ab^2$ ; or  $\sqrt{z}b \times \sqrt{z}a=\sqrt{z}ab$ , and  $\sqrt{z}12ba \div \sqrt{z}4a=\sqrt{z}3b$ ; and  $\sqrt{z}bb+\sqrt{z}b=\sqrt{z}3b$ , &c.

4. Surds are involved merely by removing the radical sign; thus to square  $\sqrt{z}G$ , take away the sign  $\sqrt{z}$ , and there is left  $G$  = the square of  $\sqrt{z}G$ . To cube  $\sqrt{z}G$ , take away the radical sign  $\sqrt{z}$ , and there will remain  $G$  = to the cube of  $\sqrt{z}G$ ; and this holds good in all cases, whatever power is represented by the radical sign.

## C H A P. X.

## RULES OF EQUATION.

**B**Y equation is to be understood in general one expression of a number or quantity made equal to another; thus,  $4+6=10$ , and  $7-3=4$ ; or  $9\times6=54$ ; or  $12\div4=3$ . And in the same manner may these quantities  $a+b=G$ , or  $a-b=D$ , or  $ab=P$ , or  $\frac{a}{b}=Q$ , &c.

There is no question can occur in numbers or quantities, but some equation will arise from the very terms of the question, if they are intelligibly put: for instance, suppose a number was required, whereof  $\frac{1}{2}$  and  $\frac{1}{3}$  being added, the sum would be 18. Let this number be represented by  $a$ , then will  $\frac{1}{2}a + \frac{1}{3}a = 18$ . Wherefore the equation here is  $\frac{1}{2}a = 18$ , and from this equation the number is required. Again, let the question be to find two numbers, the sum whereof is 24, and difference 12. In this case, one equation will be  $a+c=24$ , and another  $a-c=12$ ; and so there are two equations given, and two numbers required. Hence arises this general

## R U L E.

When the number of equations given in any question are just equal to the number of quantities required, and the one equation no way inconsistent with the other, the question is truly limited, and capable of a solution; but when the number of equations given are not so many as the numbers required, the question is ambiguous, and admits of various answers. If the number of equations given exceed the number of quantities required, and if any one of these equations is found inconsistent with the other, the question is impossible, and can have no rational answer.

*Rules for resolving simple equations.*

- To letter a question, or to represent quantities by proper characters.

As this is in a great measure arbitrary, or matter of choice, we need only observe, that in general given quantities are represented either by consonants or absolute numbers, as they lie

lie in the question ; and quantities required, by some of the small vowels, one or more as there is occasion.

2. To state a question, or, in other words, to raise an equation.

The nature of every question must be well considered ; given and known quantities distinguished from such as are required ; numbers are to be added, subtracted, multiplied, divided, or involved exactly, according to the conditions of the question ; and then one side of an equation compared with the other, which being done, the equation is truly stated. To make this still plainer, by an example : Suppose it was required to find a number to which  $\frac{1}{4}$  of itself being added, 5 subtracted from the sum, the remainder multiplied by 10, and the product divided by 4, gives 25 in the quotient.

For the unknown number put  $a$ , and for the given quotient put  $Q$ . The other members, being but small, may be taken absolutely as they lie, or represented by letters ; but in this instance, let them be taken absolutely as they lie. Then by considering the question, the following equation will occur,  $a + \frac{1}{4}a - 5 \times 10 \div 4 = Q$ ; which is stating the question at large. And the same may be abridged in the manner which the signs of addition, subtraction, multiplication, and division require ; thus,  $a + \frac{1}{4}a - 5 = \frac{1}{4}a$ , by subtracting 5, it becomes  $\frac{1}{4}a - 5$ , which, multiplied into 10, it becomes  $\frac{5}{2}a - 50$ ; and this being divided by 4, it becomes  $\frac{12}{4}a - 50 = Q$ . And thus will the equation be stated and properly abridged.

3. To reduce an equation.

After the question hath been properly stated and abridged, the equation must be reduced, by bringing the unknown quantity  $a$ , and its coefficients, to one side, and the known quantities to the other : for this purpose, observe the following cases.

*Case 1.* When  $a - b = G$ , reduce by addition, since equal quantities added to equal quantities give equal sums ;  $b$  added to both sides gives  $a = G + b$ .

*Case 2.* When  $a + b = G$ , reduce by subtraction, since equal quantities taken from equal quantities give equal remainders ; thus, by subtracting  $b$ , it will be  $a = G - b$ .

*Case*

*Case 3.* When  $\frac{a}{b}=G$ , reduce by multiplication, since equal quantities multiplied by equal quantities give equal products; by multiplying both sides by  $b$ , it will be  $a=Gb$ .

*Case 4.* When  $ab=G$ , reduce by division, since equal quantities divided by equal quantities give equal quotients; therefore, dividing both sides by  $b$ , we have  $a=\frac{G}{b}$ .

*Case 5.* When  $\sqrt{a}=G$ , reduce by involution; for since equal roots have equal squares, cubes, biquadrates, &c.; wherefore, by squaring, cubing, &c. both sides, we will have  $a=GG$ , or  $a=G^2$ .

*Case 6.* When  $a^2=G$ , reduce by evolution; for since equal squares, cubes, biquadrates, &c. have equal roots, by extracting the root of each, we will have  $a=\sqrt{G}$ .

By these six cases, founded on their respective axioms, all equations whatever may be reduced; but to set the application in a more practical point of view, let us resume the equation  $12\frac{1}{2}a-50=2Q$ .

Since one side of the equation, where  $a$  the unknown quantity lies, must be divided by 4, in order to remove the fraction, multiply both sides by the denominator 4; and then you will have  $12\frac{1}{2}a-50=4Q$ ; for not to divide  $12\frac{1}{2}a-50$  by 4, is the same thing as to multiply  $Q$  by 4.

Then seeing 50 is a known number, affected with the sign  $-$ , remove it by case 1. to the known side of the equation, and then we will have  $12\frac{1}{2}a=4Q+50$ ; and, in this case likewise, not to subtract 50 from  $12\frac{1}{2}a$ , is the same thing as to add it. But still there is a fraction upon one side; wherefore multiply both sides of the equation by the denominator 2, and we will have  $25a=8Q+100$ . Lastly, dividing both sides by the coefficient of  $a$ , viz. 25, we will have  $a=\frac{8Q+100}{25}$ ; by which means the equation is not only reduced, but resolved; for the unknown quantity  $a$ , by itself alone, is found equal to such as are known, namely  $8Q+200=8\times25$ , for  $Q$  was  $=25$ ; then  $\frac{200+100}{25}=\frac{300}{25}=12$ , the number required, or the value of  $a$ .

## C H A P. XI.

## PROBLEMS producing SIMPLE EQUATIONS.

**A** Simple equation is that wherein the unknown number or quantity  $a$  is found equal to known numbers, by multiplication or division only, without the extraction of roots, as in the above example; or, if  $\frac{a}{4}=12$ , then  $a=4 \times 12=48$ ; or if  $4a=12$ , then, by division,  $a=\frac{12}{4}=3$ , and it will be the same in letters for  $\frac{a}{b}=g$ , therefore  $a=bg$ ; and  $ab=g$ , therefore  $a=\frac{g}{b}$ , &c.

*Prob. 1.* A man being asked how many bank-notes he had in his pocket-book, made answer, If to what I have you add as many,  $\frac{1}{2}$  as many, and  $7\frac{1}{2}$  more, I would then have 130; required the number he had?

In this problem there is only one number required, for which put  $a$ , and there are two given, viz. 130, for which put  $S$ , and  $7\frac{1}{2}$ , for which put  $B$ .

Then, according to the conditions of the question,  $a+a+\frac{1}{2}a+B=S$ ; which, being properly abridged, will be  $2\frac{1}{2}a+B=S$ .

Then remove  $B$  to the known side of the equation by subtraction, and it will be  $2\frac{1}{2}a=S-B$ .

Take away the fraction by multiplication, and it will be  $5a=2S-2B$ . But by division  $a=\frac{2S-2B}{5}$ ; wherefore  $2=S-260$ , from which take  $2B=15$ , and there will remain 245, which being divided by 5, quotes 49, the answer.

*Prob. 2.* A man speaking of his money, said, that with one half of his money he could buy a certain horse, and had he 30 l. more, he could purchase three horses of the same value; how many pounds had he?

Here it appears that  $\frac{1}{2}a$  is the value of the horse, and  $\frac{3}{2}a$  the value of 3 such horses, which, by the question, is  $=a+30$ . Therefore, to remove the fraction by multiplication, it will be  $2a+60=3a$ , and by subtracting  $2a$  from both sides,  $60=a$ , the answer required.

*Prob.*

*Prob. 3.* A gentleman being asked how many horses he kept, made answer, For want of room in my own stable, I must put 5 horses in my neighbour's stable; but I am now building a stable twice as large, and then I can accommodate my own horses and 5 of my neighbour's: You may find out the number of horses I keep.

Here it is plain that  $a-5$ = the number the stable contains, and  $2a-10$ = the number a stable twice as large will contain, which being 5 more than the number the gentleman had, the equation will stand  $a+5=2a-10$ ; then, by adding 10 to both sides,  $a+15=2a$ , and by subtracting  $a$ , we have  $15=a$ , the answer.

*Prob. 4.* A man meeting some beggars, found, that by giving 4 d. to each, he would be 4 d. short of serving them all, and by giving 3 d. to each, he should have 4 d. over; required the number of beggars, and how many pence the man had in his pocket.

Here two numbers are required, which let be represented by  $a$ = the number of beggars, and  $e$ = the number of pence.

By the question it appears, that  $\frac{e+4}{a}=4$ , and  $\frac{e-4}{a}=3$ , the two given equations. By the first  $e+4=4a$ , and  $e=4a-4$ ; but by the second  $e-4=3a$ , and  $e=3a+4$ ; therefore, by comparing both equations of  $e$ , it will be  $4a-4=3a+4$ , and  $4a=3a+8$ , and  $a=8$ , and  $e=28$ . *Answer.*

*Prob. 5.* Two persons, *A* and *B*, were talking of their ages; says *A* to *B*, 7 years ago I was just three times as old as you were, and 7 years hence I shall be twice as old as you will be. I demand their present ages.

Let  $a$  and  $e$  represent the present ages of *A* and *B*, that is, let  $a=A$  and  $e=B$ ; then their ages 7 years ago were  $a-7$  and  $e-7$ , and their ages 7 years hence will be  $a+7$  and  $e+7$ ; whence, and from the conditions of the problem, may be derived the two following fundamental equations:

$$a-7=e-7 \times 3 = 3e-21, \text{ and}$$

$$a+7=e+7 \times 2 = 2e+14$$

From the former of these two equations, *viz.*  $a-7=3e-21$ , we have  $a=3e-14$ ; and from the second equation  $a+7=2e+14$ , we have  $a=2e+7$ ; therefore  $3e-14=2e+7$ , since both are equal to  $a$ ; whence  $e=21$ , and  $2e+7$ , or  $a=49$ .

*A*'s age therefore was 49 years, and *B*'s 21 : For 7 years before *A*'s age was 42 and *B*'s 14, and  $42=14\times 3$ ; but on the other hand, 7 years after *A*'s age would be 56 and *B*'s 28; but  $56=28\times 2$ .

*Prob. 6.* A certain company at a tavern found, that when they came to pay their reckoning, if there had been 3 more in company to the same reckoning, they might have paid 1 shilling apiece less than they did; but had there been 2 fewer in company, they must have paid 1 shilling apiece more than they did: what did each pay, and how many were there in company?

Represent the number in company by  $a$ , and the number of shillings every one actually paid by  $e$ ; now, if 4 persons were to pay 5 shillings apiece, the whole reckoning would be  $4\times 5$ : therefore if  $a$  persons are to pay  $e$  shillings apiece, the whole reckoning will be  $ae$  shillings. This being premised, suppose them now to be 3 more in company, then will the number of persons be  $a+3$ ; and to find what every particular person ought to pay in this case, the whole reckoning  $ae$  must be divided by  $a+3$ , and the quotient will be  $\frac{ae}{a+3}$  = every one's particular reckoning: but, according to the problem, every one's particular reckoning in this case would have been 1 shilling less than it actually was, i. e.  $e-1$ ; therefore  $\frac{ae}{a+3}=e-1$ . In like manner, the second condition of the problem gives us this equation, viz.  $\frac{ae}{a-2}=e+1$ . The first of these equations, viz.  $\frac{ae}{a+3}=e-1$ , being reduced, gives  $a=3e-3$ ; and the second equation, viz.  $\frac{ae}{a-2}=e+1$ , being likewise reduced, gives  $a=2e+2$ ; therefore  $3e-3=2e+2$  and  $e=5$ ; wherefore  $2e+2$ , or  $a=5\times 2+2=12$ .

Hence there were 12 in company, their reckoning came to  $5\times 12=60$  shillings.

*Prob. 7.* One lays out 2 shillings and 6 pence on apples and pears, buying his apples at 4 and his pears at 5 a penny, and afterwards accommodates his neighbour with half his apples and  $\frac{1}{4}$  of his pears for 13 pence, which was the price he bought them at; I demand how many he bought of each sort?

Put  $a$  for the number of apples, and  $s$  for the number of pears.

Then

Then if 4 apples cost 1 penny,  $a$  will cost  $\frac{a}{4}$ ; and for the same reason  $e$  will cost  $\frac{e}{5}$  pence, and then we will have  $\frac{a}{4} + \frac{e}{5} = 30$  for one fundamental equation. Again, the price of  $\frac{a}{2}$  half of his apples will be  $\frac{a}{8}$ , and the price of  $\frac{e}{3} = \frac{1}{3}$  of his pears will be  $\frac{e}{15}$ ; hence we will have  $\frac{a}{8} + \frac{e}{15} = 13$  for another fundamental equation.

$$\begin{aligned} \text{Equ. 1. } 5a+4e &= 600 \\ \text{2. } 15a+8e &= 1560 \end{aligned}$$

Subtract the second equation from thrice the first, and it will be

$$\begin{aligned} \text{Equ. 3. } 4e &= 240 \\ \text{4. } e &= 60 \end{aligned}$$

Now, substitute 60 for  $e$ , that is, 240 for  $4e$  in the first equation  $5a+4e=600$ , and we will have  $5a+240=600$ ; whence equation 5.  $a=72$ : therefore the number of apples was 72, and the number of pears 60: for  $\frac{72}{2}=36$  apples at the assigned price = 9 d. and  $\frac{60}{5}=20$  pears at 5 for a penny = 4 d.; but  $9+4=13$ .

*Prob. 8.* Three men had each a certain sum of money;  $A$  and  $B$  together had 16 l.;  $B$  and  $C$  together had 27 l.;  $A$  and  $C$  together had 25 l.; how much had each?

Here three numbers are required,  $a$ ,  $e$ ,  $y$ , and per question  $a+e=16$ , and consequently  $e=16-a$ ; also per question  $e+y=27$ , consequently  $e=27-y$ . By comparing these equations of  $e$ , it will be  $16-a=27-y$ , and  $16-a+y=27$ , and  $y-a=27-16=11$ , and  $y=11+a$ . Lastly, per question  $a+y=25$ , and  $y=25-a$ . By comparing these two equations of  $y$ , it will be  $=11+a=25-a$ , and  $11+2a=25$ ,  $2a=14$ ,  $a=7$ ,  $e=9$ , and  $y=18$ .

## CHAP. XII.

### OF DIRECT PROPORTION.

BY Euclid, four numbers or quantities are proportional, when the first hath the same ratio to the second, as the third hath to the fourth; or, in other words, when the first contains the second, or is contained therein, so oft as the

third contains the fourth, or is contained therein. Thus  $3:9::6:18$ , and  $6:4::12:8$ , because in the first, 3 is often found in 9 as 6 is found in 18; and in the second, 6 contains 4 as often as 12 contains 8. From this idea of proportion it follows, *first*, That if four numbers are proportional, and expressed by these four letters,  $A:B:C:D$ , the quotient of the first divided by the second, is equal to the quotient of the third divided by the fourth: and this expression is universally true  $\frac{A}{B} = \frac{C}{D}$ , as may be observed in any rank of four proportionals whatever: thus, if  $4:5::12:15$ , then  $\frac{4}{5} = \frac{12}{15}$ ; if  $6:2::12:4$ , then  $\frac{6}{2} = \frac{12}{4}$ , &c. *Secondly*, It is also true, that the quotient of the second divided by the first is equal to the quotient of the fourth divided by the third, i. e.  $\frac{B}{A} = \frac{D}{C}$ , for  $\frac{5}{4} = \frac{15}{12}$ , and  $\frac{2}{6} = \frac{4}{12}$ , &c. *Thirdly*, When it happens in any four numbers or quantities that the quotients are equal in this manner, these four numbers or quantities must be proportional: thus  $\frac{A}{B} = \frac{C}{D}$ , therefore  $A:B::C:D$ . *Fourthly*, If four numbers are proportional, the product of the extremes will be equal to the product of the means: for if  $A:B::C:D$ , then as before  $\frac{A}{B} = \frac{C}{D}$ , multiply each side of the equation by B, then  $A = \frac{BC}{D}$ ; multiply each side by D, and then  $AD = BC$ , the product of the extremes = the product of the means. *Fifthly*, If in any four numbers the product of the extremes is equal to the product of the means, these four numbers are proportional: for if  $AD = BC$ , then by division  $A = \frac{BC}{D}$ , and again by division  $\frac{A}{B} = \frac{C}{D}$ ; therefore  $A:B::C:D$ . *Sixthly*, If three numbers are proportional, the product of the extremes is equal to the square of the means: for if  $A:B::B:C$ , it will be  $AC = BB$  by the fourth; and if  $AC = BB$ , then by the fifth it will be  $A:B::B:C$ .

*Prob. I.* Three numbers being given to find a fourth direct proportional. Let the number sought be  $a$ , and the three given numbers  $B,C,D$ . Then  $B:C::D:a$ ; therefore the product of the extremes will be equal to the product of the means  $Ba = CD$ . Divide both sides of the equation by B, and then  $a = \frac{CD}{B}$ , the product of the second and third divided by the first.

*Prob.*

*Prob. 2.* Two numbers being given, to find a third proportional, put BC the two given numbers, and  $a$  the third proportional, then  $B:C::C:a$ ; and as before  $Ba=CC$ ; dividing both sides by  $B$  we have  $a=\frac{CC}{B}$ , the square of the second divided by the first.

## C H A P. XIII.

## Alternation, Inversion, Composition, and Division of Proportion.

What Euclid demonstrates, book 5. concerning proportional quantities, as far as relates to numbers and quantities that are commensurable, may be demonstrated algebraically from what hath been said in the last chapter. *First,* If  $A:B::C:D$ , then by alternation it will be  $A:C::B:D$ ; because in the last as well as the first the product of the extremes is equal to the product of the means, *viz.*  $AD=CB$ . *Secondly,* It will be by inversion  $D:C::B:A$ ; for in this also  $DA=CB$ . *Thirdly,* It will be by composition  $D+C:C::B+A:A$ ; for since  $AD=CB$ , add  $AC$  to both, and then  $AD+AC=CB+AC$ . But the first of these is the product of  $D+C\times A$ , and the second is the product of  $B+A\times C$ . *Fourthly,* It will also be by division,  $D-C:C::B-A:A$ ; for  $AD=CB$ , and by subtracting  $AC$ , it will be  $AD-AC=CB-AC$ . But the first is the product of  $D-C\times A$ , and the second is the product of  $B-A\times C$ . In like manner we may demonstrate proportion converted, alternately compounded or divided, &c. by the same principles.

## C H A P. XIV.

## COMPOUND PROPORTION.

If  $A:B::C:D$  and  $E:D::F:a$ , then it will be as  $AE$ : the product of the first two antecedents, : $B$  the first consequent; :: $CF$  the product of the second two antecedents, : $a$  the second consequent. For by the first  $A:B::C:D$ , hence  $AD=BC$ ; and by the second,  $E:D::F:a$ , hence  $Ea=DF$ : therefore the products of these equal quantities will

will be equal; that is,  $AD \times Ea$ , or  $ADEa = BC \times DF$ , or  $BCDF$ : divide both sides by D, and then  $AЕa = BCF$ ; therefore  $AЕ:B::CF:a$ . Q. E. D.

If therefore five numbers are given,  $A:B::C$ , to find  $a$ , the

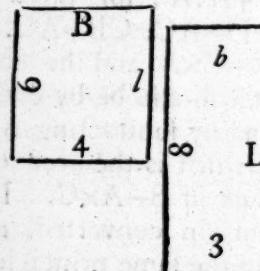
$E \quad F$

sixth direct proportional or answer required, it is plain, that  $a$  is found by multiplying the antecedents together, then multiplying the product of the second two antecedents by B, the second term of the proportion, and dividing by the product of the two first: for since  $AЕ:B::CF:a$ , it will be  $AЕa = BCF$  and  $a = \frac{BCF}{AE}$ ; and however many antecedents there may be, the operation will still be the same, which might be easily demonstrated from the same principles.

## C H A P. XV.

### RECIPROCAL, OR INVERSE PROPORTION.

**I**F two rectangles, as those in the margin, are equal the one to the other, but differing in length and breadth; or, which is the same thing, if any two products are equal the one to the other, as  $8 \times 3 = 6 \times 4$ , in letters  $Lb = lB$ : for, as the length of the longest rectangle 8, or  $L$ , is to the breadth of the shortest, 4, or  $b$ ; so reciprocally is the length of the shortest 6, or  $l$ , to the breadth of the longest, 3, or  $b$ : Hence as  $L:B::l:B$ . Here the first antecedent, and the last consequent, are found in one and the same rectangle, or product, as are the first consequent and last antecedent found in the other. If this proportion is inverted in such a manner, as to take the first antecedent and first consequent in one number, and second antecedent and second consequent in another, instead of direct proportion, we will then have inverse proportion, and no longer will the product of the extremes be equal to that of the means; but, instead thereof, the product of the first and second terms shall be equal to that of the third and fourth. Thus directly  $L:B::l:b$ , but, inversely,  $L:b::l:B$ . Therefore, to find a fourth



fourth inverse proportional, we must always multiply the two first terms, and divide their product by the last. For as  $L:b::l:B$ , and  $Lb=lb$ , and  $\frac{Lb}{b}=l=B$ . Hence it is obvious, that the inverse rule of proportion may be made direct, or the direct rule of proportion may be made inverse, only by changing the order of the terms.

## C H A P. XVI.

To turn Equations into Analogies; or, The application of Algebra to the varieties in Proportion.

*Prob. 1.* TWO merchants, A and B, had each a certain stock; A's stock was to B's as 2 to 3. By trading A gained 3000 *l.* and B 3500 *l.* and then A's stock was to B's as 4 to 5. Required their respective stocks at first?

For 3 put  $r$ , and for 4 put  $R$ ; let A's gain be represented by  $g=3000$ , and B's by  $G=3500$ , 4 in the second proportion by  $q$ , and 5 by  $Q$ , and let the numbers required be  $a$  and  $e$ .

First then  $a:e::r:R$ , per question, and by multiplying extremes and means,  $aR=er$ , and dividing both by  $r$ ;  $e=\frac{aR}{r}$ , to both add their respective gain, and we have, per question,  $a+g:\frac{aR}{r}+G::q:Q$ , and by multiplying extremes and means,  $aQR+gQR=aRq+Gqr$ , and by transposition  $gQR-Gqr=aRq-aQR$ ; and dividing by the coefficients of  $a$ , it will be  $\frac{gQR-Gqr}{qR-Qr}$ . Therefore, to find the value of  $a$  by the rule of proportion, take the divisor  $qR-Qr$  for your first number, the coefficients of  $r$ ,  $gQR-Gqr$ , and let  $r$  be the third; so shall the equation be resolved in this analogy,  $2:1000:2::1000$ . A's stock at first, and  $e=1500$  *l.*

*Prob. 2.* If A works a garden in 18 days = $t$ , and B does it in 28 days = $T$ ; in what time will they do it together?

Here time and motion being reciprocally proportional, it will be as  $t$ , A's time, is to  $G$  = the garden; so inversely is  $T$ , B's time, to what he can work in the same time; that is, as  $t:G::$  so inversely  $T:tG$ , in which A's work is  $G$ , and B's work is  $\frac{tG}{T}$ . Wherefore  $G+\frac{tG}{T}$ , both their works in equal time

time :  $t$ , the time in which they work ::  $G$ , the work of the garden :  $a$ , the time required : that is  $G + \frac{tG}{T} : t :: G : a$ ; therefore, by multiplying extremes and means,  $Ga + \frac{tGa}{T} = tG$ ; and

$$TGa + tGa = TtG; \text{ and } a = \frac{TtG}{TG + tG} = \frac{Tt}{T+t}.$$

By which process it appears, that the question is to be resolved by making the product of both given times the dividend, and the sum of both the times the divisor. Hence it will be  $18:28::46:10\frac{2}{3}$  inversely; or directly  $46:18::28:10\frac{2}{3}$ , answer.

*Prob. 3.* Three workmen could finish a certain piece of work, as follows, viz. A once in three weeks, B thrice in eight weeks, and C five times in twelve weeks; it is required to find in what time they would finish it working together?

Here the powers of the agents A, B, C are such as in the times 3, 8, 12, respectively produce effects, and it is required in what time they will produce the effect  $=a$ . It will be

$$a = \frac{1}{3} + \frac{1}{8} + \frac{1}{12} = \frac{8}{24} \text{ of a week} = 6 \text{ days}, 5\frac{1}{2} \text{ hours, &c.}$$

*Prob. 4.* How many acres of brandy, at 30 s. each, may I have in barter for 48 yards of cloth, at 10 s. per yard?

To find by algebra how this question, and all of the same nature, are to be stated by the rule of proportion. Let  $N$  represent the given quantity of any commodity here 48, and  $V$  its given value = 10 s. per yard: let  $P$  represent the value of the other commodity for which the first is to be exchanged, and  $a$  the quantity required. Then it is plain that  $NV = Pa$ , that it is the quantity of the first  $\times$  by its respective value = to the quantity of the second  $\times$  its value.

Therefore,  $\frac{NV}{P} = a$ , and directly  $P:N:V:a$ , or, inversely,  $V:N:P:a$ . In numbers, as  $10:48::$  so inversely  $30:16$ , the answer. And in this manner the arithmetician may examine, and prove the truth of the rule of fellowship, alligation, and other arithmetical operations.

### Prob. 5.

There is a cup of gold, that weighs ten ounces and four score,  
A silver cup, of the same size, weighs fifty and no more;  
To cast a cup of sixty-six, mix'd metal, in that mould,  
How many ounces must I mix of silver and of gold?

a+

$a+e=66=M$ , and  $e=M-a$ .  $G=90S=50M=66$ ; then, as  
<sup>Mould.</sup>  
 $G:1::a:\frac{a}{G}$ , the part of the mould filled with gold, and as  
<sup>Mould.</sup>  
 $S:1::M-a:\frac{M-a}{S}$ , the part of the mould filled with silver; which two parts added must be equal to one whole mould; that is,  $\frac{a}{G} + \frac{M-a}{S} = 1$  mould. This equation multiplied and brought out of fractions will give  $aS - aG + MG = G$ , and by division and transposition,  $a = \frac{MG-SG}{G-S}$ , which gives this analogy for  $a$  1st,  $\frac{G-S}{40} : \frac{M-S}{16} :: \frac{G}{16} : \frac{a}{36}$   
and for  $e$   $\left\{ \begin{array}{l} \frac{G-S}{40} : \frac{G-M}{24} :: \frac{e}{30} \\ \frac{M-S}{16} : \frac{M-G}{30} \end{array} \right.$

## C H A P. XVII. ARITHMETICAL PROGRESSION.

**N**UMBERS in arithmetical progression are such as increase or decrease by the same common difference, as in the margin. In which it is easy to observe, that any three numbers in this progression being equally distant from each other, or exceeding each other equally, the sum of the extremes will be still equal to double the mean or middle term. For, let  $f$  represent the first,  $s$  the second,  $t$  the third, let  $d$  represent the common difference, then  $f+d=s$ , and  $f+2d=t$ , and therefore the three numbers will stand  $f$ ,  $f+d$ ,  $f+2d$ , the sum of the two extremes, will be  $2f+2d=$  double  $f+d$ , the mean. Also, if four numbers are in arithmetical progression, the sum of the extremes will be = the sum of the means. For then the terms will be  $f$ ,  $f+d$ ,  $f+2d$ ,  $f+3d$ , the sum of the extremes,  $2f+3d=$  the sum of the means.

In any number of terms in this progression, the sum of the extremes is equal to the sum of any two means equally distant from the extremes, as  $2+6+8+10+12, 2+12=4+10=6+8$ . For the difference of the first and second being the same with the difference of the third and fourth, the numbers will stand thus:  $F:F+d:T+d$ , in which  $F+T+d=F+d+T$ . And hence the sum of any series of numbers in arithmetical progression is equal to the sum of the first and last added together, and multiplied by  $\frac{1}{2}$  the number of terms: For the sum

of this series being composed of the sum of the extremes and means, it is evident the number of extremes and means will be equal to  $\frac{1}{2}$  the number of terms; also the difference betwixt the first and last term will be equal to the common difference of each two subsequent terms  $\times$  the number of terms - 1; as 4, 7, 10, 13, 16, 19, in which  $19-4=3\times 5$ . From this it follows, that if the number of terms is expressed by  $N$ , the common difference by  $D$ , the first term  $F$ , the last term  $L$ , the total sum of the series  $S$ ; and if any three of these are given, the other two may be easily found, without a continued addition of the terms, as appears in the following examples.

1. A nobleman gives a penny to a beggar the first day of the year, twopence the second day, threepence the third, and a penny more every day to the year's end; required how much he gives the last day of the year, and what is the total amount of the whole?

Here is given  $F=1$  penny,  $D$  also = 1, and  $N=365$ , the days of the year. To find  $L$  and  $S$ . First,  $L=F+D\times N-1$ , and  $L=D\times N-1+F=365$  pence given the last day of the year. Then  $L+F\times \frac{N}{2}=S$ , that is  $365+1=366\times \frac{N}{2}$ , or  $182=66795$  pence, the sum of the whole series =L. 278 : 6 : 3.

2. Suppose 500 eggs laid in a straight line, at one yard's distance from each other, and the first egg one yard's distance from a basket, into which a man must carry them one by one; required the number of yards he must travel?

Here  $F=2$  yards,  $D$  also = 2,  $N=500$ ; consequently, as before,  $L=DN-1+F=2\times 499$ , or  $998+2=1000$  yards, and  $S=L+F\times \frac{N}{2}$ , that is,  $1000+2=1002\times 250=250500$  yards in all.

## C H A P. XVIII.

### GEOMETRICAL PROGRESSION.

**N**umbers in geometrical progression are such as either increase or decrease proportionally by the same common ratio, as 1. 2. 4. 8. 16. 32. 64. &c. where the common ratio is (2), each subsequent term being double the preceding, or 1. 3. 9. 27. 81. 243. &c. where the ratio is (3), each subsequent term being triple the preceding, and the same of any other series, whether increasing or decreasing. In any series

of

of this kind, it is evident from what has been said of proportional numbers in general, *first*, That of any three numbers of this progression, the product of the extremes is equal to the square of the means. *Secondly*, That of four numbers of this progression, the product of the two extremes is equal to the product of the two means. And, *thirdly*, That in any series exceeding four terms, the product of the means is equal to the product of the extremes, equally distant from these means; as in 1. 2. 4. 8. 16. 32. where  $1 \times 32 = 2 \times 16 = 4 \times 8$ ; the reason of which is, because these extremes and means are proportional by the hypothesis, and consequently by the general theorem their products must be equal. It is also evident, that in any series of this kind, as the first antecedent is to the first consequent, so is the sum of all the antecedents to the sum of all the consequents; as in  $2 : 4 : 8 : 16 : 32 : 64$ , whereas  $2 : 4 :: 2+4+8+16+32 = 64 : 4+8+16+32+64 = 124$ ; for it is plain, that as oft as the first antecedent is contained in the first consequent, so oft is the sum of all the antecedents contained in the sum of all the consequents. Hence, if  $F$  represents the first term of any progression,  $L$  the last,  $R$  the common ratio,  $N$  the number of terms,  $S$  the sum of the whole series, if any three of these are given, the other two may be found without a continued multiplication, division, or addition of the terms, as appears by the following problem.

*Problem.* In any series of numbers in geometrical continued progression, the first term  $F$ , the number of terms  $N$ , and the common ratio  $R$  being given, to find the last term  $L$ , and the sum of the whole series  $S$ , let this common question be proposed. A man bought a horse with 7 nails in each shoe, 28 in all, and, by agreement, was to pay 1 farthing for the first, 2 for the second, 4 for the third, 8 for the fourth, in double proportion till all the nails were paid for; required the price of the horse? Here  $F=1$ ,  $R=2$ ,  $N=28$ , required  $L$  the last nail, and  $S$  the sum of the whole series. Since  $F$  represents the first term,  $FR$  will be the second,  $FR^2$  or  $FR^2$  the third,  $FR^3$  the fourth,  $FR^4$  the fifth,  $FR^5$  the sixth, and so  $FR^{27} =$  the twenty-eighth and last term; that is, if the ratio 2 is involved to the 27th power, being less by 1 than  $N=28$ , and this multiplied by  $F=1$ , (which in this question neither multiplies nor divides), you will have  $L$  the last term. Now, the ratio 2 may be briefly raised to the 27th power,

- R 2

thus,

thus,  $R^2=4$ ,  $R^2 \times R^2=R^4=16$ ,  $R^4 \times R^4=R^8=256$ ,  $R^8 \times R^8=R^{16}=65536$ ,  $R^{16} \times R^8=R^{24}=16777216$ ,  $R^{24} \times R^3=R^{27}=134217728=L$ , the last term or number of farthings to be paid for the last nail. Then  $S-L$  = the sum of all the antecedents, and  $S-F$  the sum of all the consequents; and as  $F:FR::S-L:S-F$ , and by multiplying extremes and means  $FS-FF=FRS-FRL$ , and by division  $S-F=RS-RL$ , and by transposition  $RS-S=RL-F$ , consequently  $S=\frac{RL-F}{R-1}$ , in words as follows.

Multiply the last term by the ratio, subtract the first term, divide the remainder by the ratio—unity, the quotient gives the sum of the whole series. In the question proposed, the last term was found 134217728, this  $\times$  by 2=268435456, from which subtract 1, remains 268435455. This divided by the ratio—1, gives the total number of farthings paid for the horse 268435455=279620 l. 5 s. 3 d. 3 qrs.

*Example 2.* An European having taught an Indian prince to play at chess, was desired to ask his reward. In order to give the Indian an idea of numbers increasing in geometrical progression, the European told him, that there being sixty-four little squares in the chessboard, for the first he demanded only 1 square inch of ground, 2 for the second, 4 for the third, 8 for the fourth, &c. and so forward in double geometrical progression. This seeming at first a very moderate demand, was readily complied with; but the prince soon found it was more ground than the whole world could afford. Required the number of square inches it would amount to?

First, to raise the ratio 2 to the 63d power,  $R \times R=R^2=4$ ,  $R^2 \times R^2=R^4=16$ ,  $R^4 \times R^2=R^6=64$ ,  $R^6 \times R^6=R^{12}=4096$ ,  $R^{12} \times R^{12}=R^{24}=16777216$ ,  $R^{24} \times R^6=R^{30}=1073741824$ ,  $R^{30} \times R^3=R^{33}=8589934592$ ; then  $R^{33} \times R^{30}=R^{63}=9223372036854775808$ , which is the last term or number of square inches of ground for the 64th square of the chessboard. This multiplied by  $R^2$ , gives in all 18446744073709551615 square inches. This divided by 144 square inches in 1 square foot, gives 1281024129927441 square feet. This divided by 36000000, the number of square feet in a square mile, reckoning 6000 feet one geographical mile, gives 35584003 square miles, nearly equal to  $\frac{1}{4}$  of the globe, including land and water.

## C H A P. XIX.

EXTRACTION of the S<sup>Q</sup>UARE R<sup>O</sup>OT.

Since the square of any number is the product of that number multiplied into itself, as  $2 \times 2 = 4$ , and  $3 \times 3 = 9$ , and  $9 \times 9 = 81$ , &c. so reversely, the root of a square number is that number, which being multiplied into itself, produceth the square; hence the square root of 4 is 2, the square root of 9 is 3, and the square root of 81 is 9.

T A B L E O F P O W E R S.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729
Biquadrates	1	16	81	256	625	1296	2401	4096	6561
Surfaces	1	32	243	1024	3125	7776	16807	32768	59049
Squ. cubes	1	64	729	4096	15625	46656	117649	262144	531441

Prob. 1. To extract the square root of a whole number.

1. Draw a crooked line on the right of the given square number, as in division, where the figures of the root, or the quotient expressing the root must be placed.

2. Put a point over the unit's place, and omitting one, point every other figure towards the left hand, so shall the given square be pointed into several periods of 2 figures each, the number of which periods discover the number of places of which the root must consist.

3. The root of the first period to the left hand is found by the above table, or without any other table than that of multiplication; wherefore, place the nearest root in the quotient, and subtract its square from the first period; to the remainder annex another period, and double the quotient for a new divisor; then consider how often the new divisor is contained in

in all the figures excepting the unit's place, and quote the number of times; then affixing this new quotient-figure to its own divisor, multiply the divisor thus increased by the new quotient-figure, and subtract the product from the whole dividend as before; bring down another period, which must be managed exactly as the last, and proceed thus till all the periods be brought down; if at last there be no remainder, then will the quotient express the true root; but should there be something still remaining, annex periods of ciphers, and carry on the division decimals, till the root be brought sufficiently near the truth, according to the exactitude required in the solution of any question depending thereon.

## E X A M P L E S.

3. Required the square root of 321489 ( $567 \times 567 = 321489$  Pr.  
 $25 = 5 \times 5$ )

$$6 \text{ annexed to } 5 \times 2 = 106) 714 = 7 \text{ and } 14 \text{ annexed.}$$

$$636 = 6 \times 106$$

$$7 \text{ annexed to } 2 \times 56 = 1127 \Big) 7889 = 78 \text{ with } 89 \text{ annexed.}$$

## ILLUSTRATION.

First, the unit's place 9 is pointed, omitting 8, 4 is pointed, and omitting 1 we point 2: hence, as there are three points, we conclude there will be three places in the root. Then, since 5 is the nearest root of 32, for 6 would be too much, as  $6 \times 6 = 36$ , we place 5 in the quotient, which being squared and set under 32, there remains 7, to which the period 14 is annexed. Then we double 5 for a divisor, which becomes 10; then we say 10 out of 71, (for the units place is not regarded), and that can be found 7 times; yet, as 7 cannot be found once in 4, we take it but 6 times, which 6 being quoted and annexed to the divisor 10, we multiply 106 by 6, and the product is 636; the difference betwixt 714 and 636 is 78, to which the period 89 being affixed, we have a new dividend or resolvend of 7889; the former part of the quotient being multiplied by 2, becomes 112, which is found in

in 788, 7 times, and this being quoted and annexed to the divisor, makes the whole divisor 1127, which exactly corresponds to the number taken, for there is no remainder.

*Exam. 2.* What is the square root of 6968?

$$\begin{array}{r}
 6968. (83.4745, &c. \\
 64 \\
 \hline
 163) \ 568 \\
 \quad 489 \\
 \hline
 1664) \ 7900 \\
 \quad 6656 \\
 \hline
 16687) \ 124400 \\
 \quad 116809 \\
 \hline
 166944) \ 759100 \\
 \quad 667776 \\
 \hline
 1669485) \ 9132400 \\
 \quad 8347425 \\
 \hline
 (784975)
 \end{array}$$

### D E M O N S T R A T I O N.

The reason of this method is founded in geometry, *Eucl. book 2. prop. 4.* and in algebraic multiplication. Let  $G$  represent any given number whose square root is required; let  $R$  represent the first member of that root found by the table, and  $e$  the second member, or part required to complete it: then  $R+e$  is the complete root, and  $RR+2Re+ee=G$  the given number, and by subtracting from both sides  $2Re+ee=G-RR$ , and dividing by the coefficients of  $e$ , it is  $\frac{G-RR}{2R+e}$ ; which in words is the very method above explained: for, from the given number  $G$  subtract the square of the first member of the root represented by  $R$ , and divide the remainder by  $2R$ , or double the root, annexing the quotient-figure or second member of the root, represented by  $e$ , to the said double root or divisor, by which the value of  $e$  is ascertained, the second member required; and for a third member  $y$ , the method

method is plainly the same; for then  $R+e$  is the root, and is regarded as the first member,  $2R+2e$  the double root, and  $2R+2e+y$  the divisor for finding  $y$ , as will be evident from the following operation.

$$\begin{array}{r}
 G = 55225 \quad (2 \dots = R) \\
 2R = 4 \dots RR = 4 \dots \dots \\
 \hline
 +e = 3. \quad 152 \dots \dots 3. = e \\
 2R + 2e = 46. \quad 129 \dots \\
 \hline
 +y = 5 \quad 2325 \dots 5 = y \\
 2R + 2e + y = 465 \\
 \hline
 \end{array}$$

*Exam. 3.* What is the square root of 43623?

43623 (208. 861. &c.  
 4  
 —  
 408) 3623  
 3264  
 —  
 4168) 35900  
 33344  
 —  
 41766) 255600  
 250596  
 —  
 417721) 500400  
 417721  
 —  
 82679, &c.

**Note 1.** If the root of a mixed number is proposed to be extracted, make the number of decimal places even, that a point may fall upon the units place of the integral part of the proposed number, and point off as many places for decimals in the root as there were periods in the fractional part of the square.

### *Exam.*

Chap. XIX. Of the S Q U A R E R O O T. 137

*Exam. 4.* What is the square root of 751417.574560?

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \\
 751417.574560(866.843 \\
 64 \\
 \hline
 166) 1114 \\
 996 \\
 \hline
 1726) 11817 \\
 10356 \\
 \hline
 17328) 146157 \\
 138624 \\
 \hline
 173364) 753345 \\
 693456 \\
 \hline
 1733683) 5988960 \\
 5201049 \\
 \hline
 \end{array}$$

Rem. 787911

*Note 2.* To extract the root of a vulgar fraction; — reduce it to its lowest terms, if it is not so already, then will the root of the numerator and the root of the denominator, severally extracted, be the fractional root required. — Thus  $\frac{9}{16} = \frac{3}{4}$ , and  $\frac{49}{64} = \frac{7}{8}$ , &c.

*Note 3.* To extract the root of any pure decimal, make the places of an even number, and then the operation will be the same as before.

What is the square root of .5625?

$$\begin{array}{r}
 \cdot 5625(.75 \\
 49 \\
 \hline
 145)725 \\
 725 \\
 \hline
 0
 \end{array}$$

What is the square-root of .1250?

$$\begin{array}{r}
 1250(.353 \\
 9 \\
 \hline
 65)350 \\
 325 \\
 \hline
 703)2500 \\
 2109 \\
 \hline
 391
 \end{array}$$

### APPLICATION OF THE SQUARE ROOT.

*Prob. 1.* To find a geometrical mean between any two numbers.

Multiply the given numbers into one another, and the square root of their product is the geometrical mean required.

Required a geometrical mean betwixt 9 and 16?

$$\begin{array}{r}
 16 \\
 9 \\
 \hline
 144(12 \text{ Answer.} \\
 1 \\
 \hline
 22)44 \\
 44 \\
 \hline
 0
 \end{array}$$

*Prob. 2.* To find the side of a square, whose area shall be equal to that of any given surface whatever.—The square root of the given content will be the side of the square required.

*Exam.* There is a piece of ground in the form of a parallelogram, whose longest side is 134 chains, and shortest 80 chains, to be exchanged for a square piece of ground cut out of a large field, of the same area; required the side of the square?

$$\begin{array}{r} 134 \\ 80 \\ \hline \end{array}$$

Area of the parallelogram       $\frac{10720}{203}(103.53)$  Answer.

I

$$\begin{array}{r} 203)0720 \\ 609 \\ \hline \end{array}$$

$$\begin{array}{r} 2065)11100 \\ 10325 \\ \hline \end{array}$$

$$\begin{array}{r} 20703)77500 \\ 62109 \\ \hline \end{array}$$

(15391)

A maltster hath a kiln which he finds too little for his business, its diameter being only 15 feet; the diameter of another is required, which will hold double the quantity of the old one?

$$15 \times 15 = 225 \text{ square of the diameter.}$$

2

$$\begin{array}{r} 450(21.2 \text{ Answer.}) \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 41)50 \\ 41 \\ \hline \end{array}$$

$$\begin{array}{r} 422)900 \\ 844 \\ \hline \end{array}$$

(56)

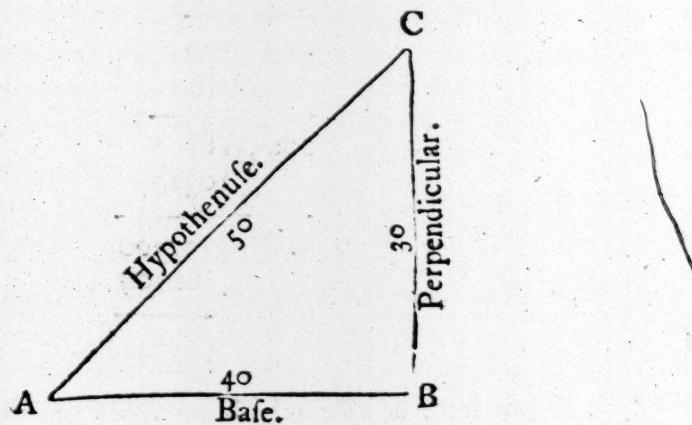
A maltster hath a kiln which he finds too large for his business, its diameter being 21.2 feet; the diameter of another which will hold half the quantity is required?

$$21.2 \times 21.2 = 450 \text{ nearly.}$$

Then  $\frac{450}{2}$ 

$$\begin{array}{r} = 225(15. \text{ Answer.}) \\ 2 \\ \hline \end{array}$$

*Prob. 3.* Having any two sides of a right angled triangle given, to find the third, the sum of the squares made upon the base and perpendicular, is equal to the square made upon the hypothenuse, *per 47. b. i. Euc. Elem.*



Therefore to find A C, extract the square root of the sum of the squares of A B, and B C.

To find A B, or B C, the square root of the difference of the squares of the other two will give that required.

#### E X A M P L E S.

$$\begin{array}{rcl} A B = 40 & \text{to} & \sqrt{A C} \\ B C = 30 & & \end{array}$$

$$\begin{array}{r} 40 \times 40 = 1600 \\ 30 \times 30 = 900 \\ \hline 2500 \end{array}$$

$$\therefore \sqrt{2500} = 50 = A C.$$

$$\begin{array}{rcl} A C = 50 & \text{and} & B C \\ A B = 40 & & \end{array}$$

$$\begin{array}{r} 50 \times 50 = 2500 \\ 40 \times 40 = 1600 \\ \hline 900 \end{array}$$

$$\therefore \sqrt{900} = 30 = B C.$$

A travels north 50 miles, and B travels east 60 miles; how many miles are they distant?

$$\begin{array}{r} 50 \times 50 = 2500 \\ 60 \times 60 = 3600 \\ \hline \end{array}$$

$$\begin{array}{r} 6100 \\ - 49 \\ \hline 1200 \end{array}$$

$$\begin{array}{r} 148) 1200 \\ - 1184 \\ \hline 16 \end{array}$$

The semidiameter of the earth being 3984.58 miles, and the perpendicular height of a mountain 3 miles; how far will it be seen at sea, or on plain ground, supposing the eye of the spectator to be on the surface of the ground or water?

$$\begin{array}{r} 3984.58 \text{ semidiameter.} \\ 3 \text{ height.} \\ \hline \end{array}$$

$$\begin{array}{r} 3987.58 \times 3987.58 = 15900794.2564 \\ 3984.58 \times 3984.58 = 15876877.7764 \\ \hline \end{array}$$

$$\begin{array}{r} 23916.4800(154.64 \text{ miles.} \\ \hline \end{array}$$

$$\begin{array}{r} 25) 139 \\ - 125 \\ \hline 14 \\ - 12 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 304) 1416 \\ - 1216 \\ \hline 200 \end{array}$$

$$\begin{array}{r} 3086) 20048 \\ - 18516 \\ \hline 1532 \end{array}$$

$$\begin{array}{r} 30924) 153200 \\ - 123696 \\ \hline 29504 \end{array}$$

*Prob. 4.* The diameter and capacity of one circle being given, to find the diameter of another circle, that shall have any proportional capacity, greater or lesser than the circle whose diameter and capacity is given.

Since

Since the areas, or capacities of circles are to one another as the squares of their diameters, *per prop. 2. b. 12. of Euc. Elemt.* it will be, as the square of the diameter of any circle is to its area or capacity, so is the square of the diameter of any other circle to its area, and *vice versa.*

*Exam. 1.* If the diameter of a circle be 6, and its area 28.2742; what will be the area of another circle whose diameter is 8?

*Exam. 2.* If the diameter of a circle, whose area is 50.2652, be 8, what is the diameter of that circle whose area is 28.2742?

$50.2652:64::28.2742:36$ , the square of 6, the answer.

C H A P. XX.

## The EXTRACTION of the CUBE ROOT.

**A**S in a square, which is the second power, every second figure from the right hand is to be pointed; so, in a cube, which is the third power, every third figure, beginning with the unit's place, is to be pointed in the like manner. Then find, by the table of cubes, the nearest cube to the first pointed period of the resolvend, the cube root of which place in the quotient to the right hand, and subtract the cube thereof from the first pointed period, bringing down to the remainder the second period.

Secondly, For a divisor to this resolvend, take thrice the square of your first quotient-figure, either annexing, or supposing to be annexed thereunto, a competent number of ciphers, according to the number of points in the given resolvend; by which find the second quotient-figure of the required root.

Thirdly,

*Thirdly,* To your divisor, which was thrice the square of your first quotient-figure, annex thrice the same quotient-figure, multiplied by the second, and also the square of the second; and in this observe, to advance the figures annexed one step further, to the right hand; and then multiply the divisor thus increased by your second quotient-figure, and subtract the product from the resolvend.

*Fourthly,* To your remainder bring down your next period, and find a new divisor, as you did the first; by taking thrice the square of the two figures, of which the root or quotient now consists, and, having divided and noted down your third quotient-figure, annex to your divisor triple the two first figures, and the square of the third, observing, as before, to advance the annexed figures a step to the right hand. Continue this method till all the periods of the resolvend are taken down, and then, if the proposed number be a perfect cube, there will be no remainder; but, if something should remain, annex ciphers, three at a time, and carry on the extraction, decimals, to any proposed degree of exactness, as in the extraction of the square root.

## E X A M P L E S.

1. Required the cube root of 13824?

$$\begin{array}{r} 13824 \text{ (24 the root required.)} \\ 2 \times 2 \times 2 = 8 \\ \hline \end{array}$$

$$2 \times 2 \times 3 = 12 \dots$$

$$2 \times 3 \times 4 = 24 \dots$$

$$4 \times 4 = 16$$

$$5824 \quad \text{For } 24 \times 24 \times 24 = 13824.$$

$$\begin{array}{r} \text{Divisor } 1456 \times 4 = 5824 \\ \hline (0) \end{array}$$

2. Required the cube root of 14886936?

$$\begin{array}{r}
 14886936(246 \\
 8 \\
 \hline
 1456)6886 \\
 5824 \\
 \hline
 177156)1062936 \\
 1062936 \\
 \hline
 (0)
 \end{array}$$

*Illustration of the last example.*

Having pointed the proposed cube, I find the root of the first period to be 2, which is noted in the quotient, and its cube being subtracted from the first period, there remains 6; to which another period being annexed, there is a new resolvend of 6886. To find a divisor for this resolvend, upon a piece of waste paper, I take the triple square of the quotient-figure,  $2=12$ , which I consider as 1200, and note 4 in the quotient. Then,  $3\times 2\times 4=24$   
And  $4\times 4=16$

Divisor 1456 completed.

The divisor being multiplied into 4, and the product subtracted from the last resolvend, the remainder, with another period annexed to it, constitutes the new resolvend 1062936. Then on a piece of waste paper, as before,

$24\times 24\times 3=1728$  which I consider as 172800, and  
 $3\times 24\times 6=432$  note 6 in the quotient,  
Then  $6\times 6=36$

Divisor 177156 completed.

Which being multiplied into 6, and the product subtracted from the resolvend, leaves no remainder, and the root is 246; which, being involved, will be found equal to the given cube.

3. Required the cube root of 27407028375?

$$\begin{array}{r}
 27407028375(3015 \\
 27 \\
 \hline
 270901)407028 \\
 270901 \\
 \hline
 27225475)136127375 \\
 136127375 \\
 \hline
 (0)
 \end{array}$$

## D E M O N S T R A T I O N.

Let  $G$  represent any given cube, the root whereof,  $a$ , is required; let  $r$  represent the nearest root to the first period of the pointed cube, and  $e$  what  $r$  wants of the just root.

Then  $r+e=a$

$$\begin{aligned}
 rr+2re+ee &= aa \\
 r^3+3rre+3ree+e^3 &= a^3 = G
 \end{aligned}$$

By subtracting  $r^3$ .  $3rre+3ree+e^3=G-r^3$   
and dividing by the coefficients of  $e=G-r^3$

$$3r^2+3re+e^2$$

Which being expressed in words, we have this rule. From the given cube, subtract the cube of  $r$ , i. e. the cube nearest to the first period: divide the remainder by thrice the square of  $r$ , the first quotient-figure, annexing thereto thrice the said first figure multiplied by the second, and also the square of the second; which in effect is the rule given above.

## A P P L I C A T I O N of the C U B E R O O T.

The cube root is of singular use in mathematics; but as mercantile computations ingross our present attention, we shall only give a few instances, and therewith conclude this part.

*Prob. 1.* To reduce any solid to a cube. The cube root of the solid content of any figure is the side of a cube, whose solidity shall be equal to that of the figure given.

*Ex.* Suppose a box or chest, whose length is 24, breadth 9, and depth 8 inches; required the side of a cube whose solid content shall be equal thereto?

$$\begin{array}{r}
 \text{Length } 24 \\
 \text{Breadth } 9 \\
 \hline
 & 216 \\
 \text{Depth } 8 \\
 \hline
 & 1728 \\
 & 1 \\
 \hline
 364)728 \\
 & 728 \\
 \hline
 & 0
 \end{array}$$

*Prob. 2.* Having the dimensions of a solid given, to determine the dimensions of a similar, or like solid, that shall be any number of times greater or less than that given.—Cube the given dimensions, and multiply their several cubes by the difference, if the solid required is greater than the given one; or divide the several cubes by the difference, if the required solid be less than that given.

*Ex.* What will be the side of a cube that shall contain four times as much as that cube whose side is 12?

$$\begin{array}{r}
 12 \times 12 \times 12 = 1728 \\
 4 \\
 \hline
 & 6912(19.04 \\
 & 1 \\
 \hline
 651)5912 \\
 & 5859 \\
 \hline
 10852816)53000000 \\
 & 43511264 \\
 \hline
 & 9488736
 \end{array}$$

*Prob. 3.* To determine the dimensions of a similar solid, by having the dimensions or capacity of a similar solid given.

*Ex.* If a ship of 100 tuns be 44 feet long at the keel, of what length shall the keel of that ship be, whose burthen is 220 tuns?

$$\sqrt[3]{\frac{44 \times 44 \times 44 \times 220}{100}} = 57.225. \text{ Answer.}$$

2. Let the length of a ship's keel be 125 feet, and the breadth of the midship beam 25 feet, and the depth of the hold 15 feet; I demand the dimensions of another ship of the same figure, that shall carry three times the burthen of the given ship? The cube roots of the cubes of the respective given dimensions, multiplied by 3, will produce the several dimensions required; *viz.*

The length of the keel - - 215.9 feet.  
 Breadth of the midship beam, 36 feet.  
 Depth in the hold - - - 21.6 feet. } Answer.

3. Suppose the length of a ship's keel be 76.95 feet, the breadth of the midship beam 29.58 feet, and the depth in her hold 14.877 feet; required the dimensions of another ship of the same mould, that will carry only 72 tuns burthen?

The ratio of the given dimensions is 5, for  $\sqrt[3]{\frac{76.95}{100}} = 5$ ; wherefore, if the given dimensions are severally divided by the cube root, the ratio, the quotients will be the dimensions required; *viz.*

Length	45
Breadth	17.3
Depth	8.7

4. Required the side of a cube that will be equal in capacity to a vessel which contains 1728 solid inches?

$$\sqrt[3]{1728} = 12$$

5. Suppose a cannon-ball, 3 inches diameter, to weigh 54 ounces Avoirdupois; what must the diameter of that mould be, in which another ball weighing 27 pounds may be cast?

$54 \cdot 3 \times 3 \times 3 = 432$   
Abridged  $2 : 1 \therefore 432 : \sqrt[3]{216} = 6$  inches, Answer.

6. Suppose a carpenter is employed to make a cubical vessel, of equal length, breadth, and depth, which shall just contain 100 Scots gallons of wort; required the dimensions thereof?

100 gallons.

872 solid inches in a Scots gallon.

$\sqrt[3]{87200} = 44.34$  inches, the length, breadth, and depth required.

7. Required to make a gurnal, which shall contain 1500 bushels of malt, whose length shall be double the breadth, and the breadth and depth equal?

First, 2150 solid inches in a bushel.

1500 bushels.

3225000 the solid content in inches.

2dly, Put  $a$  for the required length, the breadth and depth will be each  $\frac{1}{2}a$ ; therefore by multiplying length, breadth, and depth together, we have  $a \times \frac{1}{2}a \times \frac{1}{2}a = \frac{1}{4}aaa = S$  the solid content, and  $aaa = 4S$ . If therefore we multiply the solid content by 4, and extract the cube root of the product, we have the required length,  $\frac{1}{2}$  whereof gives the breadth, to which the depth is equal.

Hence 3225000

$\sqrt[3]{12900000} = 234.54$  inches = the length.  
 $\frac{1}{2} = 117.27$  inches = the breadth.

8. Suppose a ship 150 tuns burden, and the dimensions as follow: 65 feet by the keel, 18.5 by the beam, and as much depth in the hold, by which a carpenter is to proportion the dimensions of another ship of 1200 tuns burden; required the length of her keel, and other dimensions in proportion?

First,

*First,*  $150:65 \times 65 \times 65 :: 1200:\sqrt[3]{2197000}=130$  the length.

*2dly,* For the breadth and depth,

$65:18.5 :: 130$

Abridged  $13:3.7 :: 130:37$  the breadth.

*RECAPITULATION of the principles of Arithmetic  
and Algebra.*

### A X I O M I.

Since whole numbers increase, and decimals decrease in a decuple proportion, 10 is the universal ratio of any series of numbers whatever; and the reason for carrying at 10 in addition and multiplication is self-evident, since 10 in any place to the right is equal to 1 in the next place to the left. Hence also the reason for carrying according to the subdivisions of any integer, when several denominations are to be added.

### A X I O M II.

If two numbers are equally increased, their difference is always the same. Hence the reason of borrowing 10 in one place to the right, and paying it back by carrying 1 to the next place. Hence likewise the reason will be evident, for placing the first figure to the right of the product of every particular multiplier directly below its own multiplier.

### A X I O M III.

The multiplicand will be increased or diminished, in proportion to the multiplier, when the same multiplicand is used. Hence the reason why the multiplicand is increased, when it is multiplied by any thing greater than unity, and decreased when it is multiplied by a fraction.

### A X I O M IV.

The dividend will be increased or diminished in proportion to the divisor, when the same dividend is used. Hence to divide by any thing greater than unity, will quote a number less than the dividend; and, on the contrary, to divide by any thing less than unity, will quote a number greater than the dividend.

### A X I O M

## A X I O M V.

The whole is equal to all its parts taken together. Hence one sum may be made equal to several by addition, and subtraction may be proved by adding the difference to the least given sum.

## A X I O M VI.

If equal quantities are added to equal quantities, the sums will be equal; and if equal quantities are taken from equal quantities, the remainders will be equal. Hence the reason of the reduction of equations by addition and subtraction.

## A X I O M VII.

If equal quantities are either multiplied or divided by other equal quantities, the products and quotients will respectively be equal. Hence the reason of reducing equations by multiplication and division, and of abridging commensurable terms, and cancelling equal quantities and numbers.

## A X I O M VIII.

To multiply any quantity or number by other quantities or numbers continually, is the same as to multiply by the product of these other numbers. Hence the reason of multiplying by component parts, &c.

## A X I O M IX.

To divide one number or quantity by other numbers or quantities continually, is the same as to divide that one quantity or number by the product of the rest. Hence the reason of dividing by component parts, &c.

## A X I O M X.

If four numbers or quantities are proportional, the rectangle or product of the extremes will be equal to the product of the means; and, *vice versa*, if the product of the extremes be equal to that of the means, the numbers or quantities are proportional.

## A X I O M XI.

The quotient of any two succeeding powers, when the next higher is divided by the next lower, exhibits the root of these

these powers. On the contrary, if any power is multiplied by the root of that power, the product will be the next higher power of the root; and if a higher power is divided by the root, the quotient will exhibit the next lower power. Again, if a proportional part of a higher power is divided by a proportional part of the next lower power, the quotient will exhibit a proportional part of the root. Hence the first figure or figures in the root of any power being raised to the power next lower than that whose root is wanted, and that power multiplied by a number expressing the proportion which the given power bears to its root, produces a proportional divisor, whose ratio, compared with the dividend, is a proportional part of the root, which being annexed to the former part of the root, and raised to the full power of the given number, will be either the whole, or a proportional part of the given power, discoverable by subtraction, &c. Hence we have a general rule for extracting the root of any power whatever.

T H E

T H E  
UNIVERSAL ACCOUNTANT.

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P A R T IV.

The Application of Arithmetic to the  
Business of the Merchant, the Banker,  
Custom-house, Insurance-office, &c.

C H A P. I.

Simple PROPORTION, or RULE of THREE.

**P**Roportion may be defined in general, The identity, similitude, or equality of ratios, as ratio is the relation, or habitude of two numbers, which determines the value of the one from the value of the other; for instance, the ratio of 4 and 8 is 2, and the ratio of 8 and 16 is likewise 2; hence, the relation betwixt 4 and 8, and 8 and 16 being the same, these four numbers are said to be in proportion.

The rule of proportion, or rule of three, finds a fourth proportional to three numbers given, one of which shall have the same ratio to that fourth, which exists betwixt the remaining two, as was demonstrated in the algebraic part. But to speak in applicate terms:

Proportion is that rule, by which the value, quantity, or number of one species of things is proportioned to the value, quantity, or number of another species of things, according to some fixed stipulation, or known conclusion. For instance, if I purchase 4 yards of cloth for 10 shillings, and then agree to take the piece of 16 yards at the same price *per* yard; it is plain that the thing required here, is to proportion the price of 16 yards to the price stipulated for 4 yards, by still preserving the same ratio betwixt 16 yards and the price thereof,

of, as betwixt 4 yards and the stipulated price of 10 shillings.

yds. s. yds.

Thus,  $4 : 10 :: 16 : 40$ ; in which 16 hath the same ratio to 40 that 4 hath to 10, and 10 the same ratio to 40 that 4 hath to 16.

All questions in this rule are either in a direct or reciprocal proportion.

1. Direct, when the first bears the same ratio to the second, as the third doth to the fourth; in which case, the greater the second term is in respect to the first, the greater will the fourth term be in respect to the third, and the contrary.  
 $4 : 10 :: 16 : 40$ . Here, because 10 is greater than 4, 40 is proportionally greater than 16; and  $40 : 16 :: 10 : 4$ . Here, because 16 is less than 40, 4 is of consequence less than 10 in the same proportion. Hence we have this corollary for proving all operations in direct proportion, that the product of the extremes will always be equal to that of the means: for it is plain, that  $4 \times 40 = 10 \times 16$

and that  $40 \times 4 = 16 \times 10$

2. Reciprocal, when the third bears the same ratio to the first, as the second doth to the fourth; in which case the less the third term is in respect to the first, the greater will the fourth term be in respect to the second, and *vice versa*. For instance: Suppose 8 men could do a certain piece of work in 4 days, and it were required to know in what time 16 men could do it; upon the least consideration it would occur, that 16 hands would do more work than 8, and consequently require less time, wherefore, as  $8 : 4 :: 16 : 2$ . In which 16 bears the same proportion to 8 that 4 doth to 2; and by shifting the supposition,  $16 : 2 :: 8 : 4$ . Hence, when the terms are in reciprocal proportion, the product of the two first terms will always be equal to the product of the two last.

For  $8 \times 4 = 16 \times 2$ , and  $16 \times 2 = 8 \times 4$ .

From the foregoing considerations are deduced the following rules.

1. For stating, or ranking the numbers in a proportional order. Make that number the third term upon which the demand lies; that number the first term which is of the same kind, or signifies the same thing, with that term which was made the third; then will the remaining one, which is to possess the second place, be of the same kind, or signify the same thing, with the fourth, or number required.

2. For finding a fourth proportional. If the terms are in direct

direct proportion, that is, if more require more, or less require less, the product of the two last divided by the first will quote the answer, or 4th proportional. — But if the terms are in reciprocal proportion, that is, if more require less, or less require more, the product of the two first divided by the last will quote the answer.

## EXAMPLES.

1. Bought 1420 yards of osnaburghs, at 12 s. the score, or 20 yards; what will be the charge of the whole?

$$\begin{array}{rcccl} \text{yds.} & & \text{s.} & \text{yds.} & \text{L. L. s.} \\ \text{First } 20 : .6 = 12 & :: & 1420 : 42.6 = 42-12 \\ & & \text{For } 1420 & & \\ & & & .6 & \\ & & & \hline & \\ 2 | 0) & 85.20 & & & \\ & \hline & 42.6 & & \end{array}$$

And  $20 \times 42.6 = .6 \times 1420$

2. It is computed that 6 men would build a wall in 40 days, but the proprietor would have it finished in 10 days; how many men, according to that computation, must be hired for building the wall?

$$\begin{array}{rcccl} \text{days.} & \text{m.} & \text{days.} & \text{m.} \\ \text{First, } 40 : 6 & :: & 10 : 24 \\ & & \text{For } 40 \times 6 & & \\ & & & \hline & \\ & & & 24 \text{ men.} & \\ & & 10 & & \end{array}$$

and  $40 \times 6 = 10 \times 24$ .

*Illustration of the last two Examples.*

In the first example, the number upon which the demand lies is 1420 yards, and therefore by the rule it stands in the third place; the correspondent number to 1420 yards must be that one of the other two which implies yards, or some denomination of that integer, which in this case is found to be 20 yards, and therefore by the rule adopted for the first term: but to prove that we are so far right, we have still another check, namely, that the remaining term for the second place must be money, because by the question the answer must be money:

money : here we find it is 12 shillings, and therefore we may conclude that the terms are properly stated. Next, we consider that the 3d term, 1420 yards, contains a greater quantity than the first term, 20 yards, and consequently requires a greater price ; therefore we conclude the terms to be in direct proportion, and find the answer, by dividing the product of the two last terms by the first.

In the second example the demand lies upon 10 days for the 3d term, to which 40 days correspond for the first ; and 6 men must be the second, because it corresponds with what is required. It will likewise be obvious, that to finish any work in 10 days, will require more men than it would do to finish it in 40 days, and therefore we conclude that the terms are in reciprocal proportion.

#### O B S E R V A T I O N.

When the terms are mixed numbers, or of different denominations, they may be made homogeneal by reduction, vulgar fractions, or decimals ; and the operations thereafter abbreviated, when possible, in any of those methods proposed in the first part, which may seem most adapted to the purpose ; or by other methods which judgment and experience may dictate, equally, and perhaps still better calculated for dispatch.

To give a more particular idea of my meaning, I shall vary the work of the next question, by different methods of operation, and afterwards give the solution of others in that method which bids fairest for dispatch, which, next to accuracy, ought to be the principal object attended to by an accountant ; and though some figures that are not necessary to the operation, such as the stating of the questions, for the sake of illustration, may be introduced, no figure shall be omitted, for the sake of an affected brevity, which I myself have occasion to use in the operation.

*Quest. 3.* If for  $5\frac{1}{2}$  yards of velvet I get L. 4, 12 s. what may I reckon  $8\frac{1}{2}$  yards worth, which is all that I have of the kind ?

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1. By reduction.

$$\begin{array}{rcl}
 \text{Yds.} & L. s. & \text{Yds.} & L. s. \\
 \frac{5\frac{1}{4}}{20} & : & \frac{4-12}{20} & :: \frac{84\frac{2}{4}}{92} & : & \frac{67-12}{92} \\
 23 \text{ qrs.} & \hline & 338 \text{ qrs.} & \\
 & 92 s. & 92 & \\
 & & \hline & 676 \\
 & & 3042 & \\
 & & \hline & 23) 31096 \\
 & & 210 & \\
 & & \hline & 13512 s. \\
 & & \hline & L. 67-12
 \end{array}$$

2. By vulgar fractions.

$$\begin{array}{rcl}
 \text{Yds.} & L. & \text{Yds.} & L. \\
 \frac{23}{4} : \frac{92}{20} \text{ or } \frac{23}{5} & :: & \frac{169}{2} & : \frac{15548}{230}
 \end{array}$$

$$\begin{array}{rcl}
 \text{For} & \frac{23}{5} \times \frac{169}{2} & \\
 & \hline & = \frac{15548}{230} = L. 67-12 \\
 & \frac{23}{5} &
 \end{array}$$

3. By decimals.

Abridge the dividing and multiplying terms equally by 5, and they will stand

$$\begin{array}{rcl}
 \text{Again} & 1.15 : 4.6 & :: 16.9 \\
 \text{by } 23 & .23 : 4.6 & :: 3.38 \\
 & .01 : .2 & .2 \\
 & \hline & .01) .676 \\
 & & \hline & 67.6
 \end{array}$$

4. By

4. By component parts.

$$\begin{array}{rcl} Yds. & L. s. & Yds. \\ 5\frac{1}{4} & : & 4\cdot 12 :: 84\frac{1}{2} & : & 67\cdot 12 \\ & & \underline{7} \\ & 32 & 4 \\ & & \underline{12} \end{array}$$

$$\begin{array}{rcl} 386 & 8 & \text{into } 84 \\ 2 & 6 & \text{into } \frac{1}{2} \\ \hline 23) & 388 & 14 \\ & 16 & 18 \\ & & \underline{4} \end{array}$$

L. 67 12 Answer.

Many of these figures might have been omitted but for illustration.

5. By multiplying the number of yards  $84\frac{1}{2}$  into the price of 1 yard,  $\frac{1}{4}$  of which is found by dividing the given price 92 s. by the number of quarters in the first term 23, and the quotient will be 4 s. which multiplied by 4, produces 16 s.; then  $16 \times 84\frac{1}{2} = 1352$ , as formerly.

6. Or take  $\frac{1}{4}$  for 4 s. of 338 quarters, and you will have L. 67, 12 s. as before.

7. Or multiply 84.5 by .8, the decimal of 16 s. and it will produce L. 67, 12 s.

*Quest.* 4. Bought 36 pipes of wine for L. 1221 : 19 : 10, how may I sell it per pipe to save one for my own use, and lose nothing on the purchase?

*Note.* When one pipe is deduced, there remains 35, to be valued at the given price. Therefore,

$$\begin{array}{rcl} \text{Pipes.} & L. & s. & d. \\ 35 & : & 1221 & - 19 - 10 & : & 1 & : & 34 - 18 - 3\frac{1}{4} \end{array}$$

$$\begin{array}{rcl} \text{For} & 5) & 1221 & 19 & 10 \\ & & \underline{5} & & \\ & 7) & 244 & 7 & 11\frac{3}{4} \\ & & \underline{35} & & \\ & & 34 & 18 & 3\frac{1}{4} \end{array}$$

*Quest.*

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*Ques<sup>t</sup>. 5.* The perpendicular height of my staff from the ground is 4 feet 3 inches, and it casts a shadow of 5 feet 6 inches; what is the height of a steeple which casts a shadow of 573 feet 9 inches at the same time of the day?

By reducing the inches to decimals, which is done mentally, and dividing the two first terms by 5, the terms will stand thus:

Sha.	H.	Sha.	H.
1.1 :	.85	573.75	$443\frac{4}{5}$
For	573.75		
	.85		
		286875	
		459000	
		487.6875	
			Feet. Inch.
			$443.352 = 443 - 4\frac{2}{5}$ Answer.

*Ques<sup>t</sup>. 6.* What will 245 days salary amount to at 85 guineas per annum?

Because the third term is days, the first term will be 365 days = 1 year, and that term and the third being each divided by 5, the terms will stand abridged thus:

Days.	L.	Days.	L. s. d.
73 :	89.25	49 :	59 18 $1\frac{1}{4}$
For	89.25		
	7		
		624.75	
		7	
		73) 4373.25	
			$59.907 = L. 59-18-1\frac{1}{4}$

*Ques<sup>t</sup>. 7.* How much printed paper  $\frac{3}{4}$  of a yard broad will line a room 70 yards in circumference, and 6 yards high?

Reciprocally,

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*Br. Length. Br. Length.*  
Reciprocally, as 6 : 70 :: 75 : 560

$$\text{For } \frac{70}{6} \quad \text{or} \quad \frac{70}{24}$$

$$5) \underline{420.00} \quad 3) \underline{1680}$$

$$5) \underline{8400} \quad \underline{560}$$

$$3) \underline{1680}$$

560 *Answer.*

Or abridged.

Thus,

$$\begin{array}{r} 14 \\ 6 \\ \hline .15) 84 \\ \hline 560 \end{array}$$

Thus,

$$\begin{array}{r} 14 \\ 2 \\ \hline .05) 28 \\ \hline 560 \end{array}$$

Or thus,

$$\begin{array}{r} 70 \\ 8=6\times 4\div 3 \\ \hline 560 \end{array}$$

Quest. 8. What may one save at the year's end, who hath L. 456, 15 s. per annum, and spends only L. 4, 13 s. 4 d. per week?

First find what he spends a-year.

$$\begin{array}{lll} \text{Week. L. s. d. Weeks. L. s. d.} \\ 1 : 4 - 13 - 4 :: 52 : 242 - 13 - 4 \end{array}$$

$$\begin{array}{lll} \text{For } 4 13 4 \text{ or } 3|52 & \text{He bath L. } 456 15 0 \\ 10 & 4\frac{2}{3} & \text{He spends } 242 13 4 \\ \hline 5 & & \\ \hline 233 6 8 & 208 & \text{He saves } 214 1 8 \\ 9 6 8 & 17 6 8 & \\ \hline & 242 13 4 & \end{array}$$

$$\begin{array}{l} \text{He spends} \\ \text{annually} \end{array} \begin{array}{r} \\ 242 13 4 \end{array}$$

Quest. 9. A can ditch 40 roods in 20 days, B can do it in 30; in what time will they do it together?

This

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This requires two operations.

*Days. R. Days. R.*  
First, for what *A* can do in *B*'s time,  $20 : 40 :: 30 : 60$

*R. R. D. R. D.*

For what time they will take together,  $60+40:30::40:12$  *Ans.*

*Quest. 10.* If 10 men or 14 women can drink a butt of beer in 87 days, in what time would 4 men and 6 women drink it?

*M. W. M. W.*

First,  $10 : 14 : 4 : 5.6$

*W. Days. W. W.*

Then,  $14 : 87 :: 5.6+6$  reciprocally : 105 days.

$$\begin{array}{r} \text{For} & 87 \\ & 14 \\ \hline 11.6) & 1218.0 \end{array}$$

105 days. *Answer.*

*Quest. 11.* When the bushel of wheat sold at 10*s.* the fourpence loaf weighed  $4\frac{1}{2}$  lb. what should the sixpence loaf weigh, when the bushel of wheat sells at 15*s.*?

*s. lb. s.*

First,  $10 : 4.5 :: 15$  reciprocally : 3

*d. lb. d. lb.*

And,  $4 : 3 :: 6 : 4\frac{1}{2}$

These numbers are so simple, that the operation may be entirely mental.

*Quest. 12.* There is a case 6 feet 9 inches long, 5 feet 3 inches broad, and 7 feet 6 inches thick, what freight will it pay at *L. 6, 10s. per tun*?

$$\begin{array}{r}
 6.75 \\
 5\frac{1}{4} \\
 \hline
 33.75 \\
 1.6875 \\
 \hline
 35.4375 \\
 7\frac{1}{4} \\
 \hline
 248.0625 \\
 17.71875 \\
 \hline
 \text{Feet. } L, \quad \overline{265.78125} : 43.1895 \\
 40 : 6\frac{1}{4} :: 265.78125 : 43.1895 \\
 \overline{6\frac{1}{4}} \\
 \hline
 1594.68750 \\
 132.895625 \\
 \hline
 40) 1727.583125 \\
 \hline
 43.1895 = L. 43 3 9
 \end{array}$$

*Quest. 13.* A merchant, in balancing his books, finds he is due L. 575, 17 s. and that his whole subject taken together goes no higher than L. 487, 18 s. 6 d.; how much may he offer his creditors of the pound?

Debt.      Subject.      D. Sub.

$$575.85 : 487.925 :: 1 : 16 \frac{1}{8}$$

s.      d.  
For 575.85)487.925(.847 fere = 16  $1\frac{3}{8}$ . Answer  
460680

$$\begin{array}{r}
 272450 \\
 230340 \\
 \hline
 421100 \\
 423095 \\
 \hline
 1990 \text{ too much.}
 \end{array}$$

*Quest. 14.* A merchant wants a piece of ground before his land or tenement paved with stones, 3 feet by 2; the ground is 30 yards by 4; required the number of stones?

$$\begin{array}{r}
 & 30 \\
 & 4 \\
 \text{Feet} & \hline \\
 3 & 120 \\
 2 & 9 \\
 \hline
 6 : 1 :: 1080 : 180 \text{ Stones.} \\
 \text{For } 6) \ 1080 \\
 \hline
 180 \text{ Answer.}
 \end{array}$$

*Quest.* 15. If I lend a friend L. 200 for 6 months, how long ought I to retain L. 500 of his at another time to indemnify myself?

$$L. \ m. \quad L. \ m. \\ 200 : 6 :: 500 : 2\frac{2}{3} \text{ Answer.}$$

*Quest.* 16. When L. 36 valued rent pays 4 s. 10 d. of cess; what will L. 70 pay?

$$\begin{array}{r}
 L. \ s. \ d. \quad L. \ s. \ d. \\
 36 : 4 \ 10 :: 70 : 9 \ 4\frac{2}{3} \\
 \hline
 7 \\
 \hline
 33 \ 10 \\
 5 \\
 \hline
 6) 169 \ 2 \\
 \hline
 3) 28 \ 2\frac{2}{3} \\
 \hline
 5. \ 9 \ 4\frac{2}{3} \text{ Answer.}
 \end{array}$$

The following questions are proposed and answered, but the young practitioner is referred, for the method of solution, to the application of the foregoing.

*Quest.* 17. Suppose a purchase is made of 5 pieces Dutch holland, each measuring 56 Flemish ells, at 3 s. 2 d. per ell Flemish, what will be gained upon the whole, if it is sold at 5 s. 8 d. per ell English? *Answer* L. 3, 5 s. 4 d.

*Quest.* 18. When wheat was at 12 s. the bushel, the 6 d. loaf weighed 1 lb. 4 oz. what ought it to weigh when the wheat falls to 9 s. 6 d.? *Answer* 1 lb. 8 oz. 4 pwt. 3 gr.

*Quest.*

*Ques.* 19. There is an island 134 miles in circumference, in which at the same instant, and from the same point, *A* and *B* set out back to back, to travel round it. *A* travels 11 miles every 2 days, and *B* 17 every three days. After what time, and how many miles travel to each, will they meet? *Answer*, *A* will travel 66 miles, *B* 68, and they will meet in 12 days.

*Ques.* 20. If the longest end of the beam of a balance be 36 inches, and the shortest 27; how much suspended on the shortest end will equiponderate 84 lb. on the longest end? *Answer* 112 lb.

## C H A P. II. COMPOUND PROPORTION.

**I**N this rule five numbers are given to find a sixth proportional, which may be answered by two successive operations in the last rule, but much more expeditiously as follows.

Of the given numbers three are conditional, or supposed, and the other two move the question; therefore, of the three conditional terms, let that which is the principal cause of gain or loss, increase or decrease, action or passion, be put for the first term; that number which denotes distance of time or place be put for the second term, and the remaining number which will denote action, passion, gain or loss, be put for the third term, then place the other two terms which move the question in the same order with the preceding.

*Rule 1.* If the term sought be of the same name with the first or second, multiply the 1st, 2d, and last terms continually for a dividend, and the other two for a divisor, the quotient arising therefrom will be the 6th proportional.

*Rule 2.* But if the term sought, be of the same name with the 3d, the continued product of the three last terms, divided by the product of the first two, will quote the sixth proportional.

*Ques.* 1. If 8 men receive L. 4, 16 s. for 6 days work; how many men may be paid with L. 19, 4 s. for 16 days work?

$$\begin{array}{lll} \text{men. days.} & L. & \text{days..} \quad L. \\ 8 : 6 :: 4.8 : 16 : 19.2. \end{array}$$

## I L L U S T R A T I O N.

If 8 men for 6 days work receive L. 4, 16 s. These are the conditional or supposed terms in the question, and therefore possess the three first places; 8 men, as being the cause of action, or gain, make the first term; 6 days, as being the space of time, make the second; and the money which is gained in that time becomes the third term: then, because days are put before money in the conditional terms, 16 days stand before L. 19.2 in the terms which move the question. When the terms are compared, it occurs at once that a number of men is demanded, and therefore the question is wrought by the first rule, and may be previously abridged as follows.

$$\begin{array}{rccccc} \text{men.} & d. & & L. & d. \\ \text{State resumed} & 8 : 6 :: 4.8 : 16 : 19.2 \\ & 1 & 1 & .8 & 2) 24 \\ & & & & \underline{1} \\ & & & & 12 \end{array}$$

12 Answer.

Here it is necessary only to divide 24 by 2 after the terms are abridged, whereas otherwise the process would have been,

$$\begin{array}{r} 8 \times 6 \times 19.2 \\ \hline 6 \times 4.8 \end{array} = 12$$

*Quest. 2.* If the interest of L. 100 for one year is L. 5, what will be the interest of L. 5780 in 120 days?

$$\begin{array}{rccccc} L. & \text{days.} & \text{int.} & L. & \text{days.} \\ 100 : 365 : L. 5 :: 5780 : 120 \\ 73. & 1 & & 120 \\ \hline & & & & \\ & 73) 6936.00 & & & \end{array}$$

L. 95 0 3 $\frac{1}{4}$

Here the term sought was of the same kind with the third, and the answer found by the second rule.

## O B S E R V A T I O N.

If we put  $N$ = the given number of men in the first question,  $t$ =6 days, their time,  $S=L. 4, 16 s.$  their wages,  $T=16$  days, the time proposed in the question,  $P=L. 19, 4 s.$  the sum proposed, and  $a$ = the number of men required.

If we state it twice, it will be

1.  $S:N::P:\frac{PN}{S}$  the number of men in equal times.
2.  $t:\frac{PN}{S}::T:a$ .

By

By multiplying extremes and means, it will be

$$ta = \frac{PNT}{S}, \text{ and } Sta = PNT$$

and by division

$$a = \frac{PNT}{St} \text{ according to the rule, and so of any other.}$$

*Ques.* 3. An undertaker contracted to finish 500 yards of turnpike in 30 days, and for that purpose hired 60 men; but, at the expiration of 20 days, he found he had only got the length of 260 yards; how many men must be added to finish the work in the stipulated time?

m.	d.	yds.	d.	yds.		500
60	: 20	:: 260	: 10	: 240		260
						—
						240

By abridging the terms, and cancelling an equal number of ciphers,

$$\begin{array}{ccccccc} m. & d. & y. & d. & yds. \\ \text{It will be } 6 & : 1 & :: 13 & : 1 & : 240 \end{array}$$

Then  $6 \times 1 \times 240$  men.

$$\begin{array}{r} \underline{13} \\ 60 \end{array} = 103 \text{ from which deduce the men he had, viz.}$$

43 to be added.

All questions in this rule may be readily proved by multiplying the divisor into the quotient, the product of which must always be equal to the dividend; or by shifting the conditions of the question thus: If 103 men in 10 days can finish 240 yards, how many yards may 60 men finish in 20 days? &c.

### C H A P. III.

#### R U L E O F C O N J U N C T I O N.

THE rule of conjunction joins together several statings in the rule of proportion into one, and by the relation that several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

To

To dispose this rule aright, the antecedents must be ranged in the left-hand column, and the consequents in the right-hand one.

The first antecedent and the last consequent, whose antecedent is sought, must be of the like species; so must the second consequent and the third antecedent; and this order must be continued throughout the whole.

The terms being thus disposed, the divisor is found by multiplying all the antecedents into one another continually, and the dividend by multiplying all the consequents in the same manner, the quotient arising from these two factors gives the antecedent required.

This rule may be so abridged by cancelling equal quantities and abbreviating commensurables, that the whole operation may be performed with very little trouble.

### E X A M P L E S.

I. Suppose 100 lb. of Amsterdam = 100 lb. of Paris, 100 lb. of Paris = 150 lb. of Genoa, 100 lb. of Genoa = 70 lb. of Leipsic, 100 lb. of Leipsic = 160 lb. of Milan; how many lbs. of Milan will equiponderate 548 lb. of Amsterdam?

Antecedents.	Consequents.	Abridged.
lb.	lb.	Ant. Con.
100 of Amsterdam = 100 of Paris.		5 3
100 of Paris = 150 of Genoa.		5 2
100 of Genoa = 70 of Leipsic.		7
100 of Leipsic = 160 of Milan.		548
How many of Milan = 548 of Amsterdam?		
Then $3 \times 2 \times 7 \times 548$		
	$= 920\frac{1}{2}$ lb. of Milan = 548 lb. of Amsterd.	
	$5 \times 5$	

### I L L U S T R A T I O N.

The two 100's on both sides cancel each other, and let the last cipher of the three remaining antecedents be cancelled, 100 lb. of Paris, 100 lb. of Genoa, and 100 lb. of Leipsic, which is dividing them by 10; and to preserve the equality on the side of the consequents, cancel also the last ciphers in 150, 70, and 160; after which divide one of the remaining 10's on the antecedent-side by 5, and the 15 on the consequent-side by 5, and the quotients will be 2 on the side of the antecedents, and 3 on that of the consequents: then 2 will measure 2 on the antecedent-side, and 16 on the consequent-side;

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side; as it will do 10 and 8, and the quotients thereafter will be 5 and 4; which being again repeated for the remaining 10 and 4 on both sides, leaves another 5 on the antecedent-side, and 2 on the consequent-side. And as there is no further room for abridging, by reason of the odd numbers 5 and 5 on the one side, and 7 and 3 on the other, the operation is made, and the answer found as above.

The use of this rule may be extended to all questions in proportion whatever, whether simple or compound, integral or fractional.

*Exam. 2.* If 12 yards of cloth cost L. 10, 10 s. what will 20 yards cost?

Ant.	Con.	Abridged.
•12 yds.	L. 10.5	3 10.5
What will	20 yds cost.	5

Then  $\frac{10.5 \times 5}{3} = L. 17\ 10.$

*Exam. 3.* If L. 100 in 12 months gain L. 5 interest, what will L. 500 gain in 6 months?

Ant.	Con.	Abridged.
L. 100 principal in 12 months.	L. 5 Interest. 6 months.	2 5
What interest will 500 principal gain?		5

Then  $\frac{5 \times 5}{2} = L. 12\ 10$

*Exam. 4.* If 18 rods of ditching be done by 3 men in 16 days, when the day is 15 hours long, how much may be done by 8 men in 4 days, when the day is 9 hours long?

Ant.	Con.
3 men.	18 rods
16 days.	4 days.
15 hours long.	9 hours long.

How many rods may 8 men do?

Each side being properly abridged, it will be

$\frac{6 \times 3 \times 2}{5} = 7\frac{1}{5}$

*Exam.*

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*Exam. 5.* If 12 cwt. be carried 100 miles for 6 guineas, how many cwt. may be carried 150 miles for 12 guineas?

Ant.	Con.
First, 12 cwt.	6 guineas.
100 miles.	150 miles.
For 12 guineas how many cwt.	

Abridge both sides, and it will be  $\frac{2 \times 2 \times 12}{3} = 16$  cwt.

*Exam. 6.* If 12 men build a wall 30 feet long, 6 feet high and 5 feet thick, when the day is 10 hours long, in how many days will 60 men build a wall 300 feet long, 8 feet high, and 6 feet thick, when the day is but 8 hours long?

The terms being properly arranged and abridged, it will be  $2 \times 30 = 60$ .

*Exam. 7.* If 24 men dig a ditch 724 yards long, 7 feet deep, 3 feet broad at the bottom, and 5 feet broad at the top, in 236 days, when they work 13 hours and 24 minutes each day; in how many days will 16 men dig a ditch 975 yards long, 9 feet deep, 7 feet at the top, and 5 at the bottom, when they work 12 hours and 36 minutes every day?

The terms being properly arranged and abridged, it will be

$$\frac{3 \times 59 \times 201 \times 975}{2 \times 62 \times 181 \times 7} = 1520 \text{ days.}$$

*Exam. 8.* If 18 men build a wall 40 feet long, 3 feet thick, and 16 feet high, in 12 days, how many men must be employed to build a wall 360 feet long, 8 feet thick, and 16 feet high, in 60 days?

The terms being properly arranged and abridged, it will be  $2 \times 9 \times 3 = 54$  men. *Answer.*

All questions performed in this manner may be quickly proved by reversing the question: thus, If 54 men build a wall 360 feet long, 8 feet thick, and 16 feet high, in 60 days, how many men, &c.? The answer will be found to be 18.

## C H A P. IV.

## R U L E S O F P R A C T I C E.

These compendiums in proportion, which are distinguished by the name of the *rules of practice*, because they were invented occasionally by merchants in expediting practice, comprehend a great part of the calculations used in counting-houses, particularly when an unit is the first term in the proportion; and it is certain, when any process is short and unperplexed, one is less liable to error than when he hath to do with heavy multiplications and divisions. In order to assist the young practitioner, I have inserted a table of aliquot parts, and given the method of inventing it.

Table of the aliquot parts of a pound.

s.	d.	L.	s.	d.	L.
10	0	$\frac{1}{2}$	1	8	$\frac{1}{12}$
6	8	$\frac{1}{3}$	1	0	$\frac{1}{20}$
5	0	$\frac{1}{4}$	0	8	$\frac{1}{30}$
4	0	$\frac{1}{5}$	0	6	$\frac{1}{45}$
3	4	$\frac{1}{6}$	0	4	$\frac{1}{60}$
2	6	$\frac{1}{7}$	0	3	$\frac{1}{80}$
2	0	$\frac{1}{9}$	0	2	$\frac{1}{120}$

This table, and any other of the same kind, may be effected by reducing the given parts or denominations to the lowest name mentioned in them for a divisor, and an unit of the integer to the same name for a dividend, the quotient is the fractional part in its lowest terms; for instance, 6s. 8d.=80 pence, and 1l.=240 pence; but  $\frac{80}{240} = \frac{1}{3}$ , consequently 6s. 8d.= $\frac{1}{3}$  of 1l. and so of any other.

*Case 1.* When the price of the integer is any aliquot part of a pound contained in the table, the answer is found by one single division.

*Exam. 1.* At 10s. what cost 375 yards?

2) 375

L. 187 10; for  $\frac{1}{2}$  of 1l. remaining = 10s.

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*Exam. 2.* At 6 s. 8 d. what cost 545 lb.?

$$3) \underline{545}$$

L. 181 13 4; here the remainder is  $2 \times 6$  s. 8 d.

*Exam. 3.* At 5 s. what cost 475 yards?

$$4) \underline{475}$$

L. 118 15; for  $\frac{3}{4} = 15$  s.

*Exam. 4.* At 4 s. what cost 274 yards?

$$5) \underline{274}$$

L. 54 16; for  $\frac{4}{3} = 16$  s.

*Exam. 5.* At 2 s. 6 d. what cost 875 lb.?

$$8) \underline{875}$$

L. 109 7 6; for  $3 \times 2$  s. 6 d. = 7 s. 6 d.

*Exam. 6.* At 2 s. what cost 7425 yards?

$$10) \underline{7425}$$

L. 742 10; for  $5 \times 2$  s. = 10 s.

*Exam. 7.* At 1 s. 8 d. what cost 4167 yards?

$$12) \underline{4167}$$

L. 347 5; for  $3 \times 1$  s. 8 d. = 5 s.

*Exam. 8.* At 6 d. what cost 478 yards?

$$4|0) \underline{457|8}$$

L. 114 9; for 18 sixpences = 9 s.

*Exam. 9.* At 4 d. what cost 975 yards?

$$6|0) \underline{975}$$

L. 16 5; for 15 fourpences = 5 s.

*Exam. 10.* At 3 d. what cost 6875 yards?

$$8|0) \underline{6875}$$

L. 85 18 9; for  $\frac{15}{4} = 18$  s. and the remaining  $3 \times 3 = 9$  d.

*Exam.*

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*Exam. 11.* At 2 d. what cost 3745 ounces?

$$120) \underline{374|5}$$

$$L. 31\ 4\ 2; \text{ for } \frac{4}{6} = 4 s. 2 d.$$

*Case 2.* When the given price is any even part of a pound less than 2 d. it will be an even part of some of those mentioned in the table, and the answer is found by dividing and subdividing accordingly. The remainders may be valued as above, or carried on decimals.

*Exam. 1.* At 1 farthing what cost 3600 yards?

$$12) \underline{3600}$$

$$80) \underline{300} \text{ threepences; for } 12 \text{ qrs} = 3 d.$$

$$L. 3\ 15 \text{ by the table.}$$

*Exam. 2.* At  $1\frac{1}{4}$  d. what 504 yards?

$$1\frac{1}{4} d. = \frac{1}{4} \text{ of } 5 d. \text{ therefore } 4) \underline{504}$$

$$6) \underline{126} \text{ five-pences.}$$

$$8) \underline{21} \text{ half-crowns.}$$

$$L. 2\ 12\ 6 \text{ by the table.}$$

*Case 3.* If the given price is no aliquot part of a pound, it will be composed of aliquot parts, which either may be divided for severally, as before, or, when the remaining part is any even part of the foregoing, divide the quotient for it, and the sum of the quotients will give the answer. Sometimes we can, with great propriety, divide for the nearest aliquot part above the given price, and when the value of the difference is subtracted from that quotient, the remainder will be the answer. A few examples will be the best illustration.

*Exam. 1.* At  $2\frac{1}{2}$  d. what cost 8754 lbs.?

$$120) \underline{8754}$$

$$\frac{1}{2} d. \text{ is } \frac{1}{4} \text{ of } 2 d. \quad 4) \underline{72\ 19} \text{ at } 2 d. \\ 18\ 4\ 9 \text{ at } \frac{1}{2} d.$$

$$L. 91\ 3\ 9$$

Y 2

By

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By dividing for the next even part, *viz.* 3 d.

$$80) \overline{8754}$$

$$6) \overline{109} \quad 8 \quad 6 \text{ at } 3 \text{ d. whereof } \frac{1}{2} = \frac{1}{6}.$$

*Exerc. 1.* At  $1\frac{1}{4}$  d. what cost 574 ounces?

$$120) \overline{574}$$

$$8) \overline{415} \quad 8 \text{ at } 2 \text{ d. of which } \frac{1}{4} = \frac{1}{8}.$$

$11 \ 11\frac{1}{2}$  at 1 farthing to be deduced.

$$\underline{\underline{L. \ 4 \ 3 \ 8\frac{1}{2}}} \text{ Answer.}$$

*Exam. 2.* At  $3\frac{1}{4}$  d. what cost 3525 lb.?

$$80) \overline{3525}$$

$$\frac{3}{4} \text{ d. is } \frac{1}{4} \text{ of } 3 \text{ d. } 4) \overline{44} \ 1 \ 3 \text{ at } 3 \text{ d.}$$

$$\underline{\underline{11 \ 0 \ 3\frac{3}{4}}} \text{ at } \frac{3}{4}$$

$$\underline{\underline{L. \ 55 \ 1 \ 6\frac{1}{4}}} \text{ value at } 3\frac{3}{4} \text{ d.}$$

*Exam. 3.* At  $5\frac{5}{8}$  d. what cost 4738 lb.?

$$5 \text{ d. is } \frac{5}{8} \text{ of } 2 \text{ s. } 6 \text{ d. } 6) \overline{4738}$$

$$8) \overline{789.6666} \text{ at } 5 \text{ d. red. to half-crowns.}$$

$$\frac{5}{8} \text{ of } 5 \text{ d. is } 98.7083 \text{ at } \frac{5}{8} \text{ in half-crowns.}$$

$$8) \overline{888.3749}$$

$$\underline{\underline{111.0468 = L. \ 111 \ 0 \ 11\frac{1}{8}}}$$

*Exam. 4.* At 7 s. 6 d. what cost 5478 lb.?

$$4) \overline{5478}$$

$$2) \overline{1369 \ 10 \text{ for } 5 \text{ s.}}$$

$$684 \ 15 \text{ for } 2 \text{ s. } 6 \text{ d.}$$

$$\underline{\underline{L. \ 2054 \ 5 \text{ at } 7 \text{ s. } 6 \text{ d.}}}$$

*Exam.*

*Exam. 6.* At 8 s. 9 d. what cost 4785 yards?

$$5 \text{ s.} = \frac{1}{4} \text{ and } 4 \text{ s.} = \frac{1}{2}) \underline{4785}$$

$$\begin{array}{r} 957 = \frac{1}{2} \\ 3 \text{ d. is } \frac{1}{20} \text{ of } 5 \text{ s. } 20) \underline{1196} \quad 5 = \frac{1}{4} \\ 2153 \quad 5 \quad \text{at } 9 \text{ s.} \\ 59 \quad 16 \quad 3 \quad \text{at } 3 \text{ d. to be deduced.} \\ \hline \end{array}$$

$$\text{Remains L. } 2093 \text{ 8.9 at } 8 \text{ s. 9 d.}$$

*Exam. 7.* At 13 s. 8 $\frac{7}{8}$  d. what cost 475 yards?

$$10 \text{ s.} = 2) \underline{475}$$

$$\begin{array}{r} 2 \text{ s. } 6 \text{ d.} = 4) \underline{237 \text{ 10}} \\ 1 \text{ s. } 3 \text{ d.} = 2) \underline{59 \text{ 7} \quad 6} \\ \text{Deduce } \frac{1}{16} \text{ of } \underline{29 \text{ 13} \quad 9} \\ \hline \end{array}$$

$$\begin{array}{r} 326 \text{ 11} \quad 3 \\ 4 \quad \underline{11\frac{3}{4}} \\ \hline \end{array}$$

$$\text{L. } 326 \text{ 6 } 3\frac{3}{4} \text{ Answer.}$$

*Exam. 8.* At 15 s. 6 d. what cost 4834 lb.?

$$2) \underline{4834}$$

$$\begin{array}{r} 2) \underline{2417} \quad \text{at } 10 \text{ s.} \\ 10) \underline{1208 \text{ 10}} \quad \text{at } 5 \text{ s.} \\ \quad 120 \quad 17 \quad \text{at } 6 \text{ d.} \\ \hline \end{array}$$

$$\text{L. } 3746 \text{ 7 at } 15 \text{ s. 6 d.}$$

*Note.* At any price which is any even number of shillings, the value may be found by multiplying by the decimal of the price, and doubling the unit's place in the product for shillings.

*Exam. 9.* At 14 s. what cost 475?

$$475$$

$$\cdot 7$$

$$\text{L. } 332 \text{ 10 Answer.}$$

*Exam.*

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*Exam. 10.* At 6 s. 6 $\frac{1}{4}$  d. what cost 494?

$$\begin{array}{r}
 494 \\
 -3 \\
 \hline
 12 \Big| 148\ 4 & \text{at } 6\text{s.} \\
 8 \Big| 12\ 7 & \text{at } 6\text{d.} \\
 \hline
 1\ 10\ 10\frac{1}{2} & \text{at } \frac{1}{4}\text{d.} \\
 \hline
 L. 162\ 1\ 10\frac{1}{2} & \text{at } 6\text{s. } 6\frac{1}{4}\text{d.}
 \end{array}$$

*Exam. 11.* At 18 s. 9 d. what cost 478?

$$\begin{array}{r}
 40) 478 \\
 \cdot 9 \\
 \hline
 430\ 4 & \text{at } 18\text{s.} \\
 2 \Big| 11\ 19 & \text{at } 9\text{d.} \\
 \hline
 5\ 19\ 6 \\
 \hline
 L. 448\ 2\ 6 & \text{at } 18\text{s. } 9\text{d.}
 \end{array}$$

*Case 4.* If pounds are mentioned in the price, multiply for them; and if there are parts included in the quantity, reduce them to decimals, or take parts of the given price for them.

*Exam. 1.* At 2 l. 2 $\frac{1}{4}$  d. what cost 325 $\frac{1}{4}$  cwt.?

$$\begin{array}{r}
 120) 325.25 \\
 \quad \quad \quad 2 \\
 \hline
 650.50 & \text{at } 2\text{l.} \\
 8 \Big| 2.710416 & \text{at } 2\text{d.} \\
 \quad \quad \quad .338802 & \text{at } \frac{1}{4}\text{d.} \\
 \hline
 L. 653.549218
 \end{array}$$

Valued 653 10 11 $\frac{1}{4}$  Answer.

*Exam. 2.* If a dividend of 1 l. 13 s. 3 d. is made upon 1 l. of the stocks of a separating company, what will 526 l. 10 s. amount to?

$$\begin{array}{r}
 2 \Big| 526.5 & = 1 \\
 4 \Big| 263.25 & = 0\ 10\ 0 \\
 5 \Big| 65.8125 & = 0\ 2\ 6 \\
 2 \Big| 13.1625 & = 0\ 0\ 6 \\
 \quad \quad \quad 6.58125 & = 0\ 0\ 3 \\
 \hline
 L. 875.30625 & L. 1\ 13\ 3
 \end{array}$$

*Exam.*

*Exam. 3.* 96 tuns, 18 C. 2 qrs. 24 lb. at L. 6, 15 s. per tun?

$$\begin{array}{r} 96.93571 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 8) 581.61426 = L.6 \\ 72.70178 = 15 s. = \frac{1}{8} \text{ of } L.6 \end{array}$$

$$\underline{654.31604} = L. 654 6 \frac{3}{4}$$

*Exam. 4.* 47 oz. 14 pwt. 18 gr. at 3 l. 15 s. 5 d.?

$$\begin{array}{r} 2) 47.7375 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 143.2125 = L.3 \quad 0 \quad 0 \\ 2 \quad 23.8687 = \quad 0 \quad 10 \quad 0 \\ 12 \quad 11.9343 = \quad 0 \quad 5 \quad 0 \\ .9945 = \quad 0 \quad 0 \quad 5 \end{array}$$

$$\underline{180.0100} = L. 3 \ 15 \ 5$$

*Exam. 5.* 54 oz. 9 pwt. 14 gr. at 3 l. 18 s. 7½ d.?

$$\begin{array}{r} 54.47916 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 5 \quad 163.43748 \text{ for } L.3 \quad 0 \quad 0 \\ 2 \quad 32.68749 \text{ for } \quad 0 \quad 12 \quad 0 \\ 2 \quad 16.34374 \text{ for } \quad 0 \quad 6 \quad 0 \\ 4 \quad 1.36197 \text{ for } \quad 0 \quad 0 \quad 6 \\ .34049 \text{ for } \quad 0 \quad 0 \quad 1\frac{1}{2} \end{array}$$

$$\underline{214.17117} \text{ for } L.3 \ 18 \ 7\frac{1}{2}$$

*Exam. 6.* 16 tun, 3 hhds, 53 gal. at 28 l. 7 s. 2 d.

$$\begin{array}{r} 3) 16.960317 \\ \hline 28 \end{array}$$

$$\begin{array}{r} 135.682536 \\ 339.20634 \\ 5.6534 \text{ for } 6 s. 8 d. \\ .424 \text{ for } 6 d. \end{array}$$

$$\underline{480.9663} \text{ Answer.}$$

*Case 5.* When there is a fraction in the given price, it may be entirely avoided in the computation, by multiplying the given

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given price, and dividing the given quantity by the denominator of the fraction.

*Exam. 1.* Required the cost of 1000 yards sailcloth at  $9\frac{7}{8}$  d. per yard?

$$\begin{array}{r} 8) \underline{1000} \\ \underline{72} \\ 125 \end{array} \qquad \begin{array}{r} 9\frac{7}{8} \\ \underline{79} \\ 79d. = 6s. 7d. \end{array}$$

Therefore 125 at 6s. 7d. For 1000 at  $9\frac{7}{8}$

$$\begin{array}{r} \underline{-3} \\ 37 \ 10 \quad \text{at } 6s. \quad \underline{\quad} \quad \begin{array}{r} 25 \\ \underline{12} \ 10 \quad \text{at } 3d. \end{array} \\ 3 \ 2 \ 6 \text{ at } 6d. \quad \begin{array}{r} 3 \ 2 \ 6 \text{ at } \frac{3}{4} \text{ or } \frac{6}{8}. \\ 10 \ 5 \text{ at } 1d \quad \begin{array}{r} 10 \ 5 \text{ at } \frac{1}{4} \text{ or } \frac{5}{8} \text{ of } \frac{6}{8}. \\ \hline 41 \ 2 \ 11 \text{ at } 6s. 7d. \quad 41 \ 2 \ 11 \end{array} \end{array} \end{array}$$

Hence it is obvious that 125 yards at 6s. 7d. will cost just as much money as 1000 yards at  $9\frac{7}{8}$  d. and the same kind of reason will hold good in every other instance.

*Exam. 2.* Required the value of 1260 yards of incle at  $1\frac{5}{12}$  d. per yard?

$$\begin{array}{r} 12) \underline{1260} \\ \underline{12} \\ 105 \end{array} \qquad \begin{array}{r} 1\frac{5}{12} \\ \underline{15} \\ 1s. 5d. \end{array}$$

$$\begin{array}{r} 5 \ 5 \text{ for } 1d. \\ 1 \ 15 \text{ for } 4d. \\ \hline 7 \ 8 \ 9 \end{array}$$

*Exam. 3.* At 5s.  $11\frac{9}{10}$  d. what cost 4578 yards?

$$\begin{array}{r} 5 \ 11\frac{9}{10} \\ \underline{5} \\ L. 4 \ 15 \ 5 \end{array} \qquad \begin{array}{r} 16) \underline{4578} \\ \underline{4} \\ 286.125 \end{array}$$

$$\begin{array}{r} 1164.500 \text{ at } L. 4 \ 0 \ 0 \\ 143.0625 \text{ at } 0 \ 10 \ 0 \\ 71.53125 \text{ at } 0 \ 5 \ 0 \\ 5.9601 \text{ at } 0 \ 0 \ 5 \\ \hline 1385.05385 \end{array}$$

In the preceding cases there will be found a sufficient number of examples for showing the method of solving any question

tion in the rule of proportion, when unity is one of the first terms, with the greatest expedition possible, that can be communicated in this way. At the same time I would recommend to the young arithmetician, to go over all these cases and examples again and again, till he not only be well acquainted with the method, but be able to do any of the examples quickly: nor content with that, he ought, and will find it to his account, to invent many more to himself, try them in different ways, to prove one another, and thereby be enabled to calculate invoices, bills of parcels, &c. for which these rules are particularly adapted, and which make up a great part of the business of computation in the counting-room, as hath been formerly observed.

*Specimen of a bill of parcels.*

Mr Thomas Johnston, bought of Glen and Peters,

	L.	s.	d.	L.	s.	d.
5 pieces of Scotch linen, 125 yards, 3 s. 3 $\frac{1}{2}$ d.	20	11	5 $\frac{1}{2}$			
25 Irish ditto, 625 yds, 1 s. 9 $\frac{3}{8}$	55	13	3 $\frac{3}{8}$			
14 dozen hats, 7 l. 15 s. 6 d.	108	17	0			
37 pieces broad cloth, 555 yds, 14 s. 3 $\frac{1}{2}$ d.	396	11	10 $\frac{1}{2}$			
48 pieces of lawns, 480 yds, 2 s. 7 $\frac{5}{8}$ d.	63	5	0			
37 pieces of cheques, 740 yds, 9 $\frac{7}{8}$ d.	30	8	11 $\frac{1}{2}$			
				675	07	6 $\frac{7}{8}$

*Specimen of an invoice.*

Invoice of sundries, shipped on board the Sailsure, Capt. Paton master, bound for St Kitts, consigned to David Macfarlane merchant in Basseterre, to sell for my account, as under, *viz.*

	L.	s.	d.	L.	s.	d.
D. M. 160 pieces cheques, 4000 yds, 1 s. 2 $\frac{3}{4}$ d.	245	16	8			
Bales 9 ditto Fife linen, 2751 yds, 1 s. 7 $\frac{5}{8}$ d.	224	19	0 $\frac{1}{4}$			
80 Irish ditto, 2000 yds, 1 s. 6 $\frac{7}{8}$ d.	157	5	10			
1. 100 ditto gauzes, 1000 yds, 3 s. 1 $\frac{3}{4}$ d.	157	5	10	785	7	4 $\frac{1}{4}$
350 razors, 1 s. 3 $\frac{3}{4}$ d.	22	19	4 $\frac{1}{2}$			
420 penknives, 9 $\frac{1}{2}$ d.—90 ditto, 5 $\frac{3}{4}$ d.—40 ditto, 10 $\frac{5}{8}$ d.	21	1	0 $\frac{1}{2}$			
2. 1000 pair of scissars, 2 $\frac{3}{8}$ d.—500 ditto, 3 $\frac{3}{4}$ d.—230 ditto, 4 $\frac{3}{4}$ d.	22	13	1 $\frac{1}{2}$			
150 sets of buckles, 1 s. 2 $\frac{1}{8}$ d.— 100 d°, 10 $\frac{3}{4}$ d.—50 d°, 8 $\frac{1}{2}$ d.	15	1	6 $\frac{1}{2}$	81	15	1
500 pair mens shooes, 2 s. 9 $\frac{3}{8}$ d.	70	6	3			
500 ditto pumps, 3 s. 1 $\frac{1}{4}$ d.	77	12	1			
500 ditto womens, 3 s. 7 $\frac{1}{4}$ d.	90	2	1			
500 ditto boxes, 2 s. 7 $\frac{1}{8}$ d.	64	16	10 $\frac{1}{2}$	302	17	5 $\frac{1}{2}$
	Z	1169	19	8 $\frac{3}{4}$	=	1169 19 8 $\frac{3}{4}$

## VARIETIES in PROPORTION.

## INTRODUCTION.

**S**O extensive and various is the business of the counting-house, that it would be impossible, with any degree of propriety, to crowd it altogether into one general head: and therefore, to render the computations relative to the merchant, the banker, the insurer, the stockjobber, the customhouse, &c. as practical and intelligible as possible, to each species of computation I have assigned a particular variety, in which the method of calculation is illustrated with proper examples; the nature of the transactions, which give rise to these computations, explained; the laws and regulations which regard the more critical parts of the mercantile business, pointed out; with every thing else that may serve to elucidate the different subjects that compose these varieties.

## Var. I. TARE and TRETT.

**B**Y Tare and Trett, may be understood, any defect, waste, or diminution in the weight, quantity, or quality of goods, by reason of certain circumstances, for which a certain abatement is to be made by the seller to the buyer, and is different, in different merchandises, and in different countries. To have a more particular idea of the use and design of this variety, it will be proper to consider, that,

In weighing several commodities, the weight of the package is included in the invoice-weight of the goods, and the whole, upon that account, called *gross weight*; the allowance for which is regulated either by custom, or some express stipulation betwixt the buyer and seller, and goes under the following denominations.

1. *Tare*, which is an allowance for the weight of the cask, chest, box, &c. in which the goods are packed, and allowed to be either so much *per bag*, *barrel*, *chest*, &c. at so much *per cwt*, or at so much of the gross weight, called *invoice-tare*.

2. *Trett*, which is an allowance of 4 lb. *per 104 lb.* for dust contracted by keeping, waste, by freight, carriage, &c.

When a deduction is made for the allowance of tare from the gross weight, the remainder is called *nett*, unless trett is

is likewise allowed, when it is called *suttle*; and in that case the nett does not appear till  $\frac{4}{10\frac{4}{5}}$  or  $\frac{1}{2}$  be subtracted from the suttle, and then the remainder gives the nett.

These allowances may be taken off the gross by several methods; but, as it is no part of our design to dwell upon tedious ones, we shall illustrate the rule only by such as seem most convenient for dispatch.

*Case 1.* When the allowance is at so much *per bag*, barrel, chest, &c. the answer or nett is found by multiplication and subtraction, as in the following examples.

*Quest. 1.* What is the nett weight of 40 hhd's of tobacco, weighing gross 210 cwt, 3 qrs, tare 70 lb. *per hhd*?

$$\begin{array}{r} \text{lb. qrs. lb. hhd's. } 210 \ 3 \text{ gross.} \\ 70 = 2 \ 14 \times 40 = 25 \text{ tare to be deducted.} \\ \hline 185 \ 3 \text{ nett weight.} \end{array}$$

*Quest. 2.* What is the nett weight of 4 hhd's tobacco, weighing gross 16 cwt, 8 lb. tare 85 lb. *per hhd*?

$$\begin{array}{r} \text{cwt. lb.} \\ \text{lb. } 16 \ 8 \text{ gross.} \\ 85 \times 4 = 34 \text{ tare to be deducted.} \\ \hline 13 \ 4 \text{ nett.} \end{array}$$

*Case 2.* When the allowance is at so much *per cwt*, the best way of finding the tare, is in general by aliquot parts; though sometimes the nett may readily be found by making 112 lb. the first term in a direct proportion, the difference betwixt the given tare and 112 the second, the given gross the third; the fourth proportional will be the nett, without any subtraction.

#### T A B L E of ALIQUOT PARTS.

lb.	cwt.	lb.	half cwt.	lb.	qr.	qrs.	cwt.
$16 = \frac{1}{2}$	$\frac{1}{2}$	$8 = \frac{1}{2}$	$\frac{1}{2}$	$7 = \frac{1}{2}$	$\frac{1}{2}$	$2 = \frac{1}{2}$	
$14 = \frac{1}{3}$	$\frac{1}{3}$	$7 = \frac{1}{3}$	$\frac{1}{3}$	$4 = \frac{1}{3}$	$\frac{1}{3}$	$3 = \frac{1}{2} + \frac{1}{2} \text{ of } \frac{1}{2}$	$\frac{1}{2}$

*&c.*

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*Quest. 1.* What is the nett weight of 256 cwt, 2 qrs, 19 lb. tare 14 lb. per cwt?

$$\begin{array}{r} \text{lb.} & 256.669 \text{ gross.} \\ 14 = \frac{1}{8} = & 32.083 \text{ tare to be deducted.} \\ \hline \end{array}$$

224.586 nett 224 cwt, 2 qrs, 9 lb. *Answer.*

Or thus,

$$\begin{array}{r} \text{lb.} \quad \text{lb.} \quad \text{lb.} \\ 112 : 112 - 14 :: 256.669 : 224.586 \text{ as before.} \end{array}$$

*Quest. 2.* What is the nett of 410 cwt, 2 qrs, 12 lb. at 20 lb. per cwt?

$$\begin{array}{r} 410 \ 2 \ 12 \text{ gross.} \\ \text{lb.} \\ 16 = \frac{1}{7} = \frac{58}{2} \ 17\frac{3}{4} \\ 4 = \frac{1}{4} \text{ of } \frac{1}{7} = 14 \ 2 \ 18\frac{1}{4} \\ \hline \\ 20 = \frac{73}{1} \ 8 \text{ tare to be deducted.} \\ \hline \\ 337 \ 1 \ 4 \text{ nett.} \end{array}$$

$$\begin{array}{r} \text{lb.} \quad \text{lb.} \quad \text{lb.} \quad \text{cwt gross} \quad \text{cwt nett.} \\ \text{or } 112 : 112 - 20 :: 410 \ 2 \ 12 : 337 \ 1 \ 4 \end{array}$$

*Quest. 3.* What is the nett weight of 384 cwt, 3 qrs, 24 lb. tare 17 per cwt?

$$\begin{array}{r} \text{cwt. qrs. lb.} \\ 384 \ 3 \ 24 \\ \text{lb.} \\ 14 = \frac{1}{8} = \frac{48}{0} \ 13\frac{1}{2} \\ 2 = \frac{1}{4} \text{ of } \frac{1}{8} = \frac{6}{3} \ 14 \} \text{ added.} \\ 1 = \frac{1}{2} \text{ of } \frac{1}{4} \text{ of } \frac{1}{8} = 3 \ 1 \ 21 \\ \hline \\ 17 = \frac{58}{1} \ 20\frac{1}{2} \text{ tare to be deducted.} \\ \hline \\ 326 \ 2 \ 3\frac{1}{2} \text{ nett.} \end{array}$$

*Note.* In cases of this kind the most minute exactness is not required; for merchants, instead of subdividing the lbs for the tare, take the nearest quarter of a lb. as in the last example, for sufficient exactness.

*Quest. 4.* At 10 lb. per cwt tare, what is the nett of 410 cwt, 2 qrs, 12 lb. gross?

cwt. qrs. lb.

2) 410 2 12 gross

205 1 6 reduced to double cwts, of which

lb.

8 =  $\frac{1}{7}$  = 29 1 92 =  $\frac{1}{4}$  of  $\frac{1}{7}$  = 7 1 936 2 18 tare to be deducted.373 3 22 nett.

*Cafe 3.* When tare and trett are both to be allowed, find and divide the tare as in the last case, the remainder will be the suttle,  $\frac{1}{20}$  of which is always the trett; and when that is taken from the suttle, the remainder is the nett weight.

*Quest. 1.* What is the nett of 375 cwt, 1 qr, 15 lb. tare 13 lb. per cwt, trett 4 per 104?

2) 375 1 15 gross.

187 2 21 $\frac{1}{2}$  reduced to half-cwts, of which,

lb.

8 =  $\frac{1}{7}$  = 26 3 7 }  
4 =  $\frac{1}{2}$  of  $\frac{1}{7}$  = 13 1 18 } add.  
1 =  $\frac{1}{4}$  of  $\frac{1}{2}$  of  $\frac{1}{7}$  = 3 1 11 }43 2 8 tare to be deducted.26) 331 3 7 suttle.  
      12 3 1 trett.319 0 6 nett.

*Quest. 2.* In 732 cwt, 1 qr, 21 lb. gross packed in 57 butts, tare 19 lb. per butt, and trett 4 per 104 lb. how many cwt nett?

Butts lb. 732.4375 gross.

57 × 19 = 9.67 tare to be deducted.

722.7675 suttle.27.798 trett to be deducted.694.9695 = 694 cwt, 3 qrs, 24 lb. nett.

The

The operations in this rule are so extremely easy, that more examples would be unnecessary.

### Variety 2. COMPUTATION of CUSTOMS, BOUNTIES, &c.

CUSTOMS are certain duties or tolls, imposed by the sovereign, or legislative power of the nation, on certain imports and exports, for the maintenance and support of government, receivable at the customhouse, and regulated by tariffs, or books of rates; the principal whereof may be reckoned tonnage, poundage, old subsidy, new subsidy,  $\frac{1}{2}$ ,  $\frac{2}{3}$  subsidy, &c.

BOUNTIES are certain premiums allowed for the exportation of certain British manufactures.

Drawbacks are certain duties, either of the customs, or of the excise, for British manufactures that have paid duty to the excise, or certain foreign merchandises, that have paid duty at importation.

In so small a space as can be allotted here for the article of customs, it is not to be supposed, that I can exhibit a book of rates, or instance the several acts of parliament, by which the several rates have been imposed, renewed, or confirmed. To be well acquainted with these, we must have recourse to the acts themselves, and in place thereof to Crouch or Saxby. My business shall be to show the method of computation, and the use to be made of a book of rates.

### E X A M P L E S.

I. Suppose it was required to compute the duty on 1200 lb. of British plantation tobacco from the book of rates by Saxby.

Under the letter T, in the table (page 244.), and opposite to Tobacco of St Christopher's, Barbadoes, or any of the Caribbee islands, Virginia, &c. we find it rated at 1 s. 8 d. per lb. and upon this rate the computation of the duties is made. The marginal or referring letters to this species are B a, M b, which by the explanatory table (page 74.) we find point to the N° 1. III. 2. 3. 5. 8. Hence the computation will stand as follows.

Rate

Rate of 1200 lb. at 1 s. 8 d. = L. 100

5 per cent. =  $\frac{5}{100}$  or  $\frac{1}{20}$  of the rate = 5 o gross old subsidy.  
 Disc<sup>t</sup> at 25 per cent. =  $\frac{1}{4}$  to be deducted, 1 5 if paid down.

N <sup>o</sup> 1. page 10.	3 15 nett sub. to be paid.
II. page 15. rate and disc <sup>t</sup> as before,	3 15 nett addit. duty.
2. page 20. rate and disc <sup>t</sup> as before,	3 15 nett new subsidy.
3. page 22. $\frac{1}{3}$ rate and disc <sup>t</sup> as before,	1 5 nett $\frac{1}{3}$ subsidy.
5. page 23. rate and disc <sup>t</sup> as before,	3 15 nett subsidy.
8. page 29. rate 15 per cent. disc <sup>t</sup> 25 11 5 nett impost.	

Total duty, 27 10 to be paid down.

## ILLUSTRATION.

The gross old subsidy is found by taking  $\frac{1}{20}$  of the rate, the discount by taking  $\frac{1}{4}$  of that; additional duty and new subsidy are the same with the old subsidy, and  $\frac{1}{3}$  subsidy is made out by taking  $\frac{1}{3}$  of any of the above; subsidy is the same with any of the three first mentioned articles, and impost is effected by multiplying subsidy by 3. There was another subsidy added anno 1759, which is not mentioned in any edition of Crouch or Saxby that I have seen, and is the same with nett subsidy; therefore to the last found duty L. 27 - 10

add sub. 1759 3 - 15

Total duty 31 - 5

When the old subsidy is only paid ready money, and the other branches bonded, the computation will be as follows.

Lb. tobacco	50000
Rate at 1 s. 8 d.	L. 4166 13 4
$\frac{1}{20}$ for 5 per cent.	208 6 8
25 per cent. for $\frac{1}{4}$ of the gross,	52 1 8
$\frac{1}{3}$ of the gross	156 5 0 nett old subsidy. 20 16 8
Carried forward	177 1 8 additional duty. 177 1 8 new subsidy.
	354 3 4

Brought forward

354	3	4
59	0	6 $\frac{3}{4}$ $\frac{1}{3}$ subsidy.
531	5	0 impost.
177	1	8 subsidy 174 $\frac{7}{8}$
177	1	8 subsidy 1759

---

1298	12	2 $\frac{3}{4}$ duties bonded.
156	5	0 paid on importation.

---

1454 17 2  $\frac{3}{4}$  total duties.

The duties on tobacco, or any other goods imported, may very readily be calculated by analogy thus; for 5000 lb. of tobacco.

$$\text{lbs.} \quad \text{duties secured} \quad \text{lb.}$$

$$5000 : L. 1298 12 2 \frac{3}{4} :: 5000$$

Cancel 3 ciphers on each side, and abridge the remaining figures by 5, and it will be,

$$10) 1298 12 2 \frac{3}{4}$$

$$\underline{\underline{129 \ 17 \ 2 \frac{3}{4}} \text{ secured.}}$$

$$\text{and } 10) 156 \ 5$$

$$\underline{\underline{15 \ 12 \ 6} \text{ paid down.}}$$

$$145 \ 9 8 \frac{11}{40} \text{ total duties.}$$

Or by having the duties on 1 lb. ready computed in a pocket-book, the duties on any number of lbs may be readily found by practice.

Supposing the whole duties paid down.

	s. d.		
Rate of 1 lb.	1 8		
5 per cent. of that	<u>1</u>		
Discount $\frac{1}{4}$	$\frac{1}{4}$		
N° I. - - $\frac{1}{4}$	$\frac{1}{4}$	N° I. - - .75	
III. - - $\frac{3}{4}$	$\frac{3}{4}$	III. - - .75	
2 - - $\frac{3}{4}$	$\frac{3}{4}$	2 - - .75	
3 - - $\frac{3}{4}$	$\frac{3}{4}$	3 - - .25	
5 - - $\frac{3}{4}$	$\frac{3}{4}$	5 - - .75	
8 - - $2 \frac{1}{4}$	$2 \frac{1}{4}$	8 - - 2.25	
Sub. 1759 - - $\frac{3}{4}$	$\frac{3}{4}$	Sub. 1759 - - .75	
Total	$6 \frac{1}{4}$		6.25

Hence

Hence to compute the duties paid down on 1200 lb.

60) 1200

20 for 4 d.

10 for 2 d.

1 5 for  $\frac{1}{4}$ .

31 5 total as above.

Calculations of this kind may be checked as in the following

### E X A M P L E.

436 hhds of tobacco, *per* the Argyle, Alexander Morison master, from Virginia, containing 283.400 lb. tobacco of the British plantation.

	d.
20) 283.400 lb.	at $6\frac{4}{5}$ and $\frac{2}{3}$ of $\frac{1}{50}$
3) 14.170	
2) 4723 6 8	
20) 2361 13 4	
3) 118 1 8	
39 7 2 $\frac{3}{4}$	
7242 8 10 $\frac{3}{4}$	
1003 14 2	
8246 3 0 $\frac{3}{4}$	

Nett old sub. L. 885 12 6 paid down.

Addit. duty,	1003 14 2
New subsidy,	1003 14 2
$\frac{1}{2}$ sub.	334 11 4 $\frac{1}{4}$
Impost on tobacco,	3011 2 6
Subsidy 174 $\frac{7}{8}$	1003 14 2
Ditto 1759	1003 14 2

7360 10 6 $\frac{3}{4}$  secured.

8246 3 0 $\frac{3}{4}$  total duties.

Brown and Muscovado sugars of the British plantations are  
VOL. I. A a rated

rated at L. 1, 10 s. per cwt, the 5 per cent. of which is the subsidy gross, the 5 per cent. of which gross is the old subsidy nett: the new subsidy is the same,  $\frac{5}{3}$  subsidy is  $\frac{2}{3}$  of the new subsidy, and subsidies 174 $\frac{7}{8}$  and 1759 the same with the gross. Hence, the duties on 200 cwt Muscovado sugar of the British plantation, will be computed as under.

	cwt.		
	200 at L. 1-10 s.		
Rate	100		Proof.
	<u>300</u>		<u>4) 200 cwt. at 6 s. 3<math>\frac{1}{2}</math>d.</u>
Sub. gross,	15		5) 50
5 per cent.	0 15		4) 10
	<u>—</u>		4) 2 10
N. f.	14 5		5) 12 6
O. f.	14 5		2 6
$\frac{1}{3}$ f.	4 15		63 5 0 as before.
S. 174 $\frac{7}{8}$	15 0		
D° 1759	15 0		Hence 63 5 s. d.
	<u>—</u>		<u>= 6 3<math>\frac{1}{2}</math>d.</u>
Total duties	63 5		200

Computation of the duties on 4 puncheons, containing 400 gallons rum from Jamaica, of the British plantation.

	L. s. d.
1st, 400 gallons rated at 1 s. 8 d. per gallon, 33	<u>6 8</u>
5 per cent. of the rate,	1 13 4
5 per cent. discount,	<u>0 1 8</u>
Old subsidy nett,	1 11 8
	3
Subsidies on spirits,	4 15 0
Coinage ditto,	1 11 9
Subsidy 174 $\frac{7}{8}$ = the gross	1 13 4
Customs,	<u>8 0 1</u> to be paid down,

To

To the customs formerly found, L. 8 0 1  
 Add the excise at 4 s. 8 d. per gallon, 93 6 8 bonded.

and the total duties will be L. 101 6 9

For a proof, the duties on 400 gallons may be computed as under.

Gross subsidies at 3 d. L. 5 0 0  
 Of discount at 5 per cent. 0 5 0

Subsidies on spirits,	4 15 0
Coinage on ditto,	1 11 9
Sub. $174\frac{7}{8} = \frac{1}{3}$ of the gross,	1 13 4
	8 0 1

Excise at 4 s. 8 d. per gallon, 93 6 8

Total duties, L. 101 6 9 as before.

Hence the duties on rum per gallon will be

$$\frac{101 6 9}{400} = L. 0 5 0 \frac{321}{400}$$

But the rates of rum vary according to the proof.

Bounties are calculated with the greatest ease by practice.  
 For instance, Suppose it was required to calculate the bounty on 5000 yards of fail-cloth exported, at 2 d. per yard.

$$\begin{array}{r} 12|0) 500|0 \\ \hline 41 \ 13 \ 4 \quad \text{Answer.} \end{array}$$

400 barrels of salmon, at 4 s. 6 d. per barrel.

$$\begin{array}{r} 5) 400 \\ \hline 8) 80 \\ \hline 10 \\ \hline L. 90 \quad \text{Answer.} \end{array}$$

7650 yards of Fife linen, at 9 d. per yard, bounty  $1\frac{1}{2}$  d.

$$\begin{array}{r} 2) \ 7650 \\ \hline 8|0) \ 382|5 \end{array}$$

47 16 3. *Answer.*

Drawbacks are computed as the duties, in some goods the whole being allowed, and in others only a part, which is specified in the book of rates.

### Variety III. RULE OF MIXTURES.

**I**N groceries they sometimes mix several sorts of their wares together, for the convenience of sale; and to proportion the price of the mixture to the several prices of the simples, or to find the quantity of each ingredient, which will proportion the mixture to a certain price, will admit of the following cases.

*Case 1.* When the quantity of each ingredient is the same, the several rates per lb. oz. cwt, &c. added together, and their sum divided by the number of quantities, will give the rate of the mixture,

*Example.* A grocer would mix sugar at 50 s. per cwt, at 60 s. per cwt, and at 70 s. per cwt; what is 1 cwt of the mixture worth?

$$\begin{array}{r} 50 \\ 60 \\ 70 \\ \hline 3) \ 180 \end{array}$$

60 s. *Answer.*

*Case 2.* When the quantities as well as the prices are different, find the several values according to their different rates and quantities, and divide the sum of their values by the sum of the quantities; the quotient gives the answer.

*Exam. 1.* A grocer bought 275 lb. of almonds at  $5\frac{5}{8}$  d. per lb. and 295 lb. of raisins at  $3\frac{3}{4}$  d. per lb. which he mixed together; what is 1 lb. of the mixture worth?

275 lb.

$275$  lb. of almonds  $295$  lb. of raisins at  $3\frac{3}{4}$  d.

$$\begin{array}{r} 5 \\ \hline 8) 1375 \text{ at } 5 \text{ d.} & 4) 885 \text{ at } 3 \text{ d.} \\ 171\frac{7}{8} \text{ at } \frac{5}{8} \text{ d.} & 221\frac{1}{4} \text{ at } \frac{3}{4} \text{ d.} \end{array}$$

$$\begin{array}{r} \text{Almonds } 1546\frac{7}{8} \text{ at } 5\frac{5}{8} \text{ d.} \\ \text{Raisins } 1106\frac{1}{4} \text{ at } 3\frac{3}{4} \text{ d.} \end{array}$$

$$275 + 295 = 570) 2653\frac{1}{8} (4\frac{5}{8} \text{ d. nearly.}$$

$\underline{2280}$

$\underline{373}$

$\underline{2685}$

$\underline{2850}$

$165$  too much.

*Exam. 2.* If  $278$  gallons of rum at  $11$  s.  $6$  d. per gallon were mixed with  $174$  gallons at  $9$  s.  $3$  d. per gallon, what would a gallon of the mixture be worth?

$$\begin{array}{r} 278 & 174 \\ 11\frac{1}{2} & 9\frac{3}{4} \\ \hline 3058 & 1566 \\ 139 & 43\frac{1}{2} \\ \hline 3197 & 1609\frac{1}{2} \\ 1609\frac{1}{2} & \end{array}$$

$$278 + 174 = 452) 4806\frac{1}{2} (10 \text{ s. } 7\frac{1}{2} \text{ d.}$$

$$\begin{array}{r} 452 \\ \hline 286 \\ 12 \\ \hline 3438 \\ 3164 \\ \hline 274 \\ 4 \\ \hline 1096 \\ 904 \\ \hline (192) \end{array}$$

*Exam.*

*Exam. 3.* A grocer hath 75 lb. of confections, which he can sell at 1 s.  $6\frac{3}{4}$  d. 96 lb. of another kind, which he can sell at 1 s.  $4\frac{1}{2}$  d. and 68 lb. of another kind, which he can sell at 1 s. 3 d.; what is 1 lb. of the mixture worth?

$$\begin{array}{rcl}
 75 \text{ lb. at } 1 \text{ s.} & = & 75 \text{ o} \\
 & \text{at } 6 \text{ d.} & = 37 \text{ 6} \\
 & \text{at } \frac{3}{4} \text{ d.} & = 4 \text{ } 8\frac{1}{4} \\
 75 \text{ lb. at } 1 \text{ s. } 6\frac{3}{4} \text{ d.} & = & 117 \text{ } 2\frac{1}{4} \\
 96 \text{ lb. at } 1 \text{ s.} & = & 96 \\
 & \text{at } 4 \text{ d.} & = 32 \\
 & \text{at } \frac{1}{2} \text{ d.} & = 4 \\
 96 \text{ lb. at } 1 \text{ s. } 4\frac{1}{4} \text{ d.} & = & 132 \\
 68 \text{ lb. at } 1 \text{ s.} & = & 68 \\
 & \text{at } 3 \text{ d.} & = 17 \\
 68 \text{ lb. at } 1 \text{ s. } 3 \text{ d.} & = & 85 \\
 \hline
 239 & - & - & - & ) 334 \cdot 2\frac{1}{4} \text{ (1-4}\frac{3}{4} \text{ Answer.} \\
 & & & & \hline
 & & & & 239 \\
 & & & & \hline
 & & 95 & & \\
 & & 12 & & \\
 & & \hline
 & & 1142 & & \\
 & & 956 & & \\
 & & \hline
 & & 186 & & \\
 & & 4 & & \\
 & & \hline
 & & 745 & & \\
 & & 717 & & \\
 & & \hline
 & & (28) & &
 \end{array}$$

*Exam. 4.* A goldsmith or refiner hath 12 oz. of gold at 4 l. per oz. 8 oz. at 4 l. 5 s. per oz. 3 oz. at 4 l. 6 s. 8 d. per oz. and 9 oz. at 4 l. 13 s. 4 d. per oz.; if these were all melted down together, what would an ounce of the composition be worth?

## Var. III. RULE of MIXTURES.

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$$\begin{array}{rcl}
 12 \times 4 & = & L. 48 \\
 8 \times 4 & = & 34 \\
 3 \times 4 & = & 13 \\
 9 \times 4 & = & 42 \\
 \hline
 32 & & 8) \overline{137} \\
 & & 4) \overline{17 - 2 - 6} \\
 & & L. 4 - 5 - 7\frac{1}{2} \text{ Answer.}
 \end{array}$$

*Cafe 3.* To increase or diminish a compound proportionally, by knowing the several quantities of the simples in the composition.

## R U L E.

As the sum of the particular quantities of the compound given is to the whole quantity proposed to be increased or diminished, so is each particular quantity in the given compound, to the due proportion required of that specie, fineness, &c.

*Example.* Let the compound in the last question be increased to 48 oz. how much must be taken of each simple ingredient?

$$\begin{array}{rcl}
 32 : 48 & :: & 12 : 18 \text{ at } L. 4 \text{ o o} \\
 32 : 48 & :: & 8 : 12 \text{ at } 4 \text{ } 5 \text{ o} \\
 32 : 48 & :: & 3 : 4\frac{1}{2} \text{ at } 4 \text{ } 6 \text{ 8} \\
 32 : 48 & :: & 9 : 13\frac{1}{2} \text{ at } 4 \text{ } 13 \text{ 4} \\
 \hline
 & & 48
 \end{array}$$

*Cafe 4.* Having the simples of any compound given, to find how much of any simple ingredient is in any part of that composition.

## R U L E.

As the total of the composition is to the quantity of any simple in that composition, so is the total quantity proposed to be proportionally compounded, to the quantity of each simple to be in that proposed quantity.

*Example.* It is required to find out how much of each ingredient is in a pound weight of gold, or 12 ounces, at the prices mentioned in the last example, when there are 32 ounces in the composition?

$$\begin{array}{rcl}
 32 : 12 \text{ or } 8 : 3 & :: & 12 : 4\frac{1}{2} \text{ at } L. 4 \text{ o o} \\
 8 : 3 & :: & 8 : 3 \text{ at } 4 \text{ } 5 \text{ o} \\
 8 : 3 & :: & 3 : 1\frac{1}{8} \text{ at } 4 \text{ } 6 \text{ 8} \\
 8 : 3 & :: & 9 : 3\frac{3}{8} \text{ at } 4 \text{ } 13 \text{ 4} \\
 \hline
 & & 12
 \end{array}$$

*Cafe*

*Case 5.* The total of the compound of two simples, with the total value of that composition, and the value of an unit of each simple being given, to find the quantity of each simple ingredient in the composition.

## R U L E.

Multiply the total quantity of the composition by the lesser price of the unit, deduct the product from the total value of the composition, and divide the remainder by the difference in value of an unit of the two simples given, and the quotient is the quantity of the higher priced simple, whose complement to the total of the compound gives the other quantity.

*Example.* Suppose there are 20 ounces of gold melted into one mass, consisting of gold at 4*l. per oz.* and gold at *L. 4, 5 s. per oz.* the value of the whole being 82*l.*; it is required to find how much of each was taken to make the composition?

$$\begin{array}{r}
 \text{oz.} \\
 20 = L. 82 \\
 4 = 1 \text{ oz. of the lesser price.} \\
 \hline
 80 \\
 82
 \end{array}$$

Diff. of a unit 5*s. = .25*) 2 the difference.

$$\begin{array}{r}
 L. \\
 8 \text{ oz. of the higher price} = 34 \\
 20 \text{ the whole composition} = \left. \begin{array}{l} \\ \end{array} \right\} 82 \\
 \hline
 \text{Diff. } 12 \text{ oz. of the lower price} = 48
 \end{array}$$

## DEMONSTRATION.

Let *T* represent the total quantity of the mixture here, 20 oz. of gold, and let *V* represent the total value of the said mixture = 82*l.*; let *P* = 4*l. 5 s.* the price of an unit of the dearest simple, and *p* = the price of an unit of the cheapest simple; *a* = the quantity of the first, *e* = the quantity of the second. Then *a+e=T* per question, and *e=L-a*; by multiplying each quantity by the respective price of an unit, it will be *aP* = the value of the dearest part of the composition, and *L-p-ap* = the value of the cheapest part; and consequently the sum of both *aP+Lp-ap=V* the total value: and by subtracting

subtracting  $T_p$ , it will be  $aP - ap = V - T_p$ ; and dividing both sides by the coefficients of  $a$ , we find  $\frac{a}{P-p} = \frac{V-T_p}{a}$ . Q. E. D.

*Case 6.* The particular rates of the several simples to be mixed, and the rate of the mixtures given, to find how much of each ingredient must be taken to compose the mixture.

### R U L E.

Write down the different rates, the highest uppermost, the next higher below that, and the rest in the same order; couple the various rates thus ranked, one with another, in such a manner, as the one may be greater and the other less than the mean rate; then take the difference betwixt each of the rates and the mean, and write it down over-against that with which it was linked; lastly, sum up all the differences, and their total will be the sum of the mixtures, at the given rate, and the several differences will ascertain the quantity of each ingredient to be taken in compounding the mixture, according to the method in which the ingredients are alligated.

*Example.* A vintner hath rum which he values as follows: Some at 18 s. some at 16 s. some at 14 s. and some at 13 s. per gallon; but as the highest priced was seldom called for, and the lowest complained of, he proposes to make a mixture out of the whole, which will stand him 15 s. per gallon, how many gallons of each must he take to answer his purpose?

18	--	2	18	--	1
16	--	1	16	--	2
14	--	1	14	--	3
13	--	3	13	--	1
	$\overline{7}$			$\overline{7}$	

By either of these methods he will obtain his purpose.

For 2 gall. at 18 s. = 36 s. and 1 gall. at 18 s. = 18		
1	at 16	= 16
1	at 14	= 14
3	at 13	= 39
—	—	—
7	at 15 = 105	7
		at 15 = 105

The truth of this manner of operation may be accounted for thus:

Suppose a mixture only of two simples, the price of the

one at 4 d. and the price of the other at 8 d. which when mixed may be sold at 6 d. according to our rule we will have

$$6 \Big| 8) \quad 2 \text{ viz. } 8-6=2 \text{ at } 8 \text{ d.} \\ 4) \quad 2 \quad 6-4=2 \text{ at } 4 \text{ d.}$$

Hence there is gained just as much by selling the lower priced simple at 6 d. as there is lost in selling the other at 6 d.; for by selling at 6 d. what was valued at 4 d. there is just 2 d. gained, and by selling at the same price what was valued at 8 d. there is just 2 d. lost. And what is true in this, will likewise be true in every case of this kind, whatever be the number of simples, and with however many others any one is linked: for as it is always a greater with a lesser, there is a balance of gain and loss upon the quantities taken from every linking of two simples, and therefore there must be a balance on the whole.

The quantities found by this case may be multiplied for greater quantities, or divided for lesser, by the same number at pleasure; and therefore, if the answer found is either less or more than suits the purpose, it may be increased or diminished in proportion to the stock of the ingredients on hand for making the mixture.

*Cafe 7.* When the mean price, the particular prices of all the kinds to be mixed, and the quantity of one kind, are given, to find the quantities of the other ingredients.

### R U L E.

First, find what quantities would solve the question, if no particular quantity had been given, by the last case; and since any other numbers in the same proportion will answer the question, it will be, as the difference belonging to that price, whose quantity is given, is to the given quantity, so is each of the other differences to its respective quantity sought.

*Example.* Suppose a grocer with 10 lb. of sugar at 10 d. would mix other sorts at 4 d. 5 d. and 8 d. per lb. and sell the composition at 7 d. per lb. what quantity must he take of each sort?

$$d. \\ 10) \quad - \quad 3 \quad \text{as } 3 : 10 :: 3 : 10 \text{ at } 4 = 40 \\ 4) \quad - \quad 3 \quad \text{Then as } 3 : 10 :: 1 : 3\frac{1}{3} \text{ at } 5 = 16\frac{2}{3} \\ 5) \quad - \quad 1 \quad \text{as } 3 : 10 :: 2 : 6\frac{2}{3} \text{ at } 8 = 53\frac{1}{3} \\ 8) \quad - \quad 2 \quad \underline{\quad 10 \quad \text{at } 10 = 100} \\ \underline{30 \quad \text{at } 7 = 210 \text{ Pence.}} \\ \text{Cafe}$$

*Casē 8.* When the mean price with the particular prices of each simple, and the quantity of the whole mixture, are given, to find the particular quantity of each simple of the mixture.

## R U L E.

First find what quantities would solve the question, if the quantity of the whole mixture was not limited; then it will be, as the sum of all the differences is to the whole quantity given, so is each particular difference to its particular quantity sought.

*Example.* How many bushels of rye at 3 s. 6 d. barley at 2 s. 4 d. wheat at 4 s. 3 d. and oats at 1 s. 4 d. per bushel, must be taken to compound a mixture of 98 bushels at 2 s. 6 d. per bushel?

				Because $49 = \frac{1}{2}$
	42	- - 14	as 1 : 2 :: 14 : 28 rye.	
30	28	- - 21	as 1 : 2 :: 21 : 42 barley.	
	51	- - 2	as 1 : 2 :: 2 : 4 wheat.	
	16	- - 12	as 1 : 2 :: 12 : 24 oats.	

49

Each of the last seven cases is illustrated with but one example, not only because the operations are extremely easy, but likewise very seldom occur.

## Variety IV. Warehouse and Shop Computations of Loss and Gain.

IN buying and selling, it is not only necessary for a merchant to be a perfect judge of the quality of goods, and to be well acquainted with the properest markets for making the best purchases and quickest sales, but he must likewise be able, by comparing both together, on the different articles in which he deals, to make a true estimate of his trade, thereby to judge with certainty, in what articles he ought to launch out, and in what to retrench, as he finds them more or less to his account. To effectuate this, it was found necessary to have some common standard, by which the gain or loss made, or proposed to be made upon any commodity, or article in trade, should be tried and expressed; and this, by universal consent, seems to have been fixed to the *centum* or hundred; so that,

B b 2

when

when we say the gain is at 10 per cent. it is to be understood, that when 100*l.* 100 guineas, 100 crowns, 100 shillings, &c. have been laid out in purchasing goods, 110*l.* 110 guineas, 110 crowns, 110 shillings, &c. have been recovered by the sales; and, in the same manner, if 100*l.* &c. were laid out on the purchase of goods, and but 90*l.* received back, we would say that 10 per cent. was lost by such goods.

This variety will admit of four cases.

*Case 1.* When the buying and selling prices are known, and the rate per cent. gain or loss required.

### R U L E.

As the buying price is to the difference betwixt the buying and the selling price, so is 100 to the gain or loss per cent.

*Exam. 1.* Bought cloth at 15*s.* 6*d.* and sold it again for 18*s.* what did I gain per cent.?

$$6\text{d. } 6\text{d. } 6\text{d. } 6\text{d.}$$

$$31 : 36 - 31 = 5 :: 100 : 16\frac{4}{3}\text{ per cent.}$$

For      31) 500

16\frac{4}{3} Answer.

Hence had 100 sixpences been laid out on cloth at 15*s.* 6*d.* which was sold again for 18*s.* there would have been gained 16 $\frac{4}{3}$  sixpences; if 100 shillings, the gain would have been 16 $\frac{4}{3}$  shillings; and if 100 pounds, the gain would have been 16 $\frac{4}{3}$  pounds, &c.; for the proportion would still have been the same, as will be evident from the last example performed the common way.

$$\begin{array}{r}
 15\cdot6 \quad 2\cdot6 \quad L. 100 \\
 12 \quad 12 \quad 240 \\
 \hline
 186\text{d.} \quad 30\text{d.} :: 24000 \\
 \qquad \qquad \qquad 30 \\
 \hline
 186) 720000 \quad (3870 \\
 \qquad \qquad \qquad 558 \\
 \hline
 \qquad \qquad \qquad 210) 32126 \\
 \qquad \qquad \qquad 1620 \\
 \qquad \qquad \qquad 1488 \\
 \hline
 \qquad \qquad \qquad 1320 \\
 \qquad \qquad \qquad 1302 \\
 \hline
 \qquad \qquad \qquad 180
 \end{array}$$

*Exam.*

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Exam. 2. Bought cloth for 10 s. and sold it again for 9 s. what did I lose per cent.?

$$10) \underline{100}$$

10 per cent. ; for 10 : 1 :: 100 : 10.

Exam. 3. Bought 5 puncheons of rum for 245 l. 19 s. 6 d. and sold them immediately for 295 l. 19 s. 6 d. what did I gain per cent.?

First, 295.975 the selling price.

245.975 the buying price.

$$5) \underline{245.975} : \underline{50} :: \underline{100}$$

$$5) \underline{49.195} \quad \underline{10}$$

$$\underline{9839} \quad \underline{- - -} \quad \underline{200.000} \quad (20\frac{3}{9839} \text{ per cent.})$$

196.78

3.220

The shillings and pence are reduced to decimals mentally, and therefore the operation left out.

Exam. 4. Bought a puncheon of rum for 66 l. 13 s. 4 d. ; it run 150 gallons, which I retailed at 12 s. 6 d per gallon ; whether did I gain or lose, how much, and at what per cent.?

$$2) \underline{150} \text{ gallons.}$$

$$4) \underline{75} \quad 0 \text{ at } 10 \text{ s.}$$

$$\underline{18} \quad \underline{15} \text{ at } 2 \text{ s. } 6 \text{ d.}$$

93 15 0 selling price.

66 13 4 buying price.

$$2 : \underline{27} \quad \underline{1} \quad \underline{8} :: \underline{3}$$

3

$$2) \underline{81} \quad \underline{5} \quad \underline{0}$$

40 12 6 per cent.

Because 66 l. 13 s. 4 d. is just  $\frac{2}{3}$  of 100 l. 2 is made the first term ; and because  $\frac{3}{2} = 100$  l. 3 is made the last term ; so by the answer there is gained 27 l. 1 s. 8 d. which is  $40\frac{5}{8}$  per cent.

Cafe

*Case 2.* When the buying price, and the proposed rate *per cent.* profit are given, to find the advanced price.

## R U L E.

100 in this case will be the first term, the rate added to 100 the second, and the buying price the third, the 4th proportional to which is the advanced price required.

*Exam. 1.* Bought cloth at 15 s. per yard, how may I charge it *per yard*, to gain 25 *per cent.*?

$$\begin{array}{cccc} s. & s. & s. & s. d. \\ 100 : 125 :: 15 : 18.9 \end{array}$$

For abridging the terms, 3

$$\begin{array}{r} 20) 375 \\ \hline \end{array}$$

18 9 Answer.

*Exam. 2.* Bought 400 spindles of yarn for L. 41, carriage and other charges came to 17 s.; how may I retail it *per spindle*, to clear 30 *per cent.*?

$$\begin{array}{cc} L. & s. d. \\ 400 : 13 :: 41.85 : 2-7\frac{1}{2} \\ \hline 13 \\ \hline 4) .54405 \end{array}$$

.13601 = 2 s. 8 $\frac{1}{2}$  d. *per spindle.*

When a cipher is cancelled in the product of 400×100 the first term, and one cancelled likewise in the 2d term 130 to balance it, the question will be properly stated as above; and if from the product of the 2d and 3d terms, 5 places be considered as decimals, for the 2 in the 3d term, and the 3 ciphers in the divisor, the product will be a fraction, and the divisor 4.

*Exam. 3.* Bought 80 puncheons of rum, containing 7548 gallons, at 10 s. 6 d. *per gallon*, lighterage, portage, and other charges came to L. 25 7 s.; there was of leakage 25 gallons; how may I sell the remainder *per gallon*, to clear 12 $\frac{1}{2}$  *per cent.* upon the whole?

7548 gal.  
25 leak.

First, 7548

7523 rem.  
100 X

3774 at 10 s.

188.7 at 6 d.

25.35 of charges,

752300 : 112.5 :: 3988.05 first cost : 115.11d.

For 3988.05

112 $\frac{1}{2}$

446549.6

1993.525

7523) 4485.43125 (.59622 = 11 s. 11 d.

37615

72393

67707

46861

45138

17232

15046

21865

15046

(6819)

Exam. 4. Bought 17 cwt, 3 qrs of sugar, at 60 s. and 14 cwt, 2 qrs, and 14 lb. at 70 s. which I mixed together, and propose to sell at 30 per cent. advance; how may I value 1 cwt of the mixture to effectuate my purpose?

First,

*L.*

$$\text{First, } 17.75 \times 3 = 53.25 \\ 14.625 \times 3 \frac{1}{2} = 51.1875$$


---

$$\text{Quantity } 32.375 \quad 104.4375 \text{ prime cost.}$$


---

$$323.75) \quad 1357.6875 \quad (4.1936 = L. 4, 3 s. 10 \frac{1}{4} d.$$

$$\begin{array}{r} 129500 \\ \hline 62687 \\ 32375 \\ \hline 303125 \\ 291375 \\ \hline 117500 \\ 97125 \\ \hline 203750 \end{array}$$

*Case 3.* If the rate proportional to another upon a new advanced price is required.

## R U L E.

The lowest advanced price will be the first term, the rate added to 100 will be the 2d, the highest advanced price the 3d, and the 4th proportional will be the advanced price required, after 100 is deduced from it.

*Exam. 1.* Sold tobacco at 7 d. per lb. upon which I had 10 per cent. markets rose to 8 d.; what did that advance bring per cent.?

$$7 d. : 110 :: 8 : 125 \frac{5}{7}$$

$$\begin{array}{r} 110 \\ \hline 7) 880 \\ \hline 125 \frac{5}{7} \\ 100 \\ \hline \end{array}$$

$25 \frac{5}{7}$  per cent. Answer.

*Exam.*

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*Exam. 2.* Sold cloth at 18 s. upon which I had 15 per cent.; how much per cent. had my neighbour, who sold his cloth of the same cost, at 19 s. 6 d.? *Answer,* 24 $\frac{1}{2}$  per cent.

*Case 4.* If the prime cost is required, 100 added to the rate per cent. will be the first term, 100 the second, the advanced price the third, and the fourth proportional the prime cost required.

*Exam. 1.* Sold a pipe of wine for L. 43, 15 s. by which I had 20 per cent. what was the prime cost?

L. L. s. d.

$$120 : 100 :: 43\cdot75 : 36\cdot92$$

Abridged  $6 : 5 :: 43\cdot75$

5

$$\underline{6) \ 218\cdot75}$$

$$36\cdot4583 = L. 36\cdot92$$

*Exam. 2.* Sold 50 pieces Scotch lawns, 10 yards each for 135 l. by which I had 15 per cent. what did it cost me per yard?

$$115 \times 500 : 100 :: 135 : 4s. 8\frac{1}{4}d.$$

Abridged  $115 : 1 :: 27$

20

$$\underline{\rule{0.5cm}{0.4pt}} \quad s. d.$$

$$540 (4\ 8\frac{1}{4})$$

$$460$$

$$\underline{\rule{0.5cm}{0.4pt}} \quad 80$$

$$12$$

$$\underline{\rule{0.5cm}{0.4pt}} \quad 960$$

$$920$$

$$\underline{\rule{0.5cm}{0.4pt}}$$

$$40$$

$$4$$

$$\underline{\rule{0.5cm}{0.4pt}} \quad 160$$

$$115$$

$$\underline{\rule{0.5cm}{0.4pt}} \quad (45)$$

VOL. I.

C C

O B S E R .

## O B S E R V A T I O N.

In selling on credit, merchants generally propose a certain profit, which they calculate upon the prime cost of the goods, added to the real and imaginary charges.—The real charges on goods are freight, insurance, lighterage, portage, ware-room rent, &c. and the imaginary charges are risk of bad debts, dilatory payments, short insurance, possible accidents in the carriage, risk of having them long on hand, &c.; and where this calculation is made according to the profit they propose, the goods are marked on the cover with something characteristic of the price, which is known only to those concerned in the shop or ware-room. After all, merchants are frequently obliged to conform themselves to the market, selling under the rate they proposed, when there are few bidders, and as demands rise, taking the best price they can get.

## Variety V. COMPUTATIONS IN BARTER.

**B**Arter is the commutation of one commodity for another, and teacheth so to proportion the quantities to be exchanged, according to the conditions of the barter, that neither party may sustain loss. Questions of this kind may be quickly resolved by one single operation, by the following

## R U L E.

Let the rate of the assigned quantity per yard, lb. cwt, &c. be put for the first term, the given or assigned quantity for the second term, and the rate of the quantity required for the third term, in a reciprocal proportion, then will the fourth proportional be the answer.

*Exam.* 1. How many yards of shalloon may I have in barter for 80 yards of broad cloth, rating the broad cloth at 15s. 6d. and the shalloon at 2s.?

6 d. yds. 6 d. yds.

31 : 80 :: 4 : 620

31

4) 2480

620 yds.

Or  $\frac{31}{4} = 20 \times 31 = 620$  yards of shalloon, as before.

*Exam.*

*Exam. 2.* How many yards of Irish linen at 2*s.* 3*d.* may I have in barter for 80 pieces holland of 20 yards each at 3*s.* 6*d.* per yard?

$$\begin{array}{rcl}
 & \text{threepences} & \text{threepen.} \\
 114 : 80 \times 20 & :: 9 : 2488\frac{8}{9} \\
 \text{For } 80 & & \\
 & 20 & \\
 \hline & 1600 & \\
 & 14 & \\
 \hline & 22400 & \\
 \hline & 2488\frac{8}{9} &
 \end{array}$$

*Exam. 3.* How many pieces of Indian chints may I have in barter for 86 pieces of broad cloth, rating the former at L. 25, 10*s.* and the latter at L. 15, 15*s.*?

$$\begin{array}{rcl}
 15 s. \text{ pieces. } 15 s. \\
 21 : 86 :: 34 : 53\frac{2}{7} \\
 \text{For } 86 \\
 21 \\
 \hline 172 \\
 34) 1806 (53 \\
 170 \\
 \hline 106 \\
 102 \\
 \hline 4 \\
 \hline 34 = \frac{2}{7}
 \end{array}$$

*Exam. 4.* How many lb. of almonds may I have in barter for 379 lb. of raisins, reckoning the almonds at  $7\frac{1}{2}$ *d.* and the raisins at  $3\frac{3}{4}$ *d.*?

$$\begin{array}{rcl}
 3\frac{3}{4} d. & 3\frac{3}{4} d. \\
 1 : 379 & :: 2 : 189\frac{1}{2} \text{ lb. of almonds.}
 \end{array}$$

*Exam. 5.* How many Bibles at 2 s. Salmon's gazetteers at 4 s. and spelling-books at 1 s. of each an equal number, may I have in barter for 525 dictionaries at 6 s. each?

s. books.

$$6 : 525 :: 2+4+1=7 : 450$$

For 7) 525

75  $\frac{2}{7}$  to be deduced.

450 of each.

which amounts to the same as multiplying by 6, and dividing by 7.

There is a second species of barter taken notice of by most authors, wherein it is thought necessary, when one of the parties make a distinction betwixt a bartering and ready money price, to proportion the price of the other party's goods to the same advance with the former; but as this only multiplies figuring to no purpose, since the same answer can be found as above, it was judged proper to take no further notice of it here.

#### Variety VI. COMPUTATIONS in PARTNERSHIP.

When two or more merchants join together to carry on any branch of business, they are said to be in partnership or company; and if their stocks are all equal, the gain, loss, or proceeds that each partner must sustain, or draw from the concern, is found by dividing either by the number of partners; when their different shares of the capital have one common denominator, their respective shares of gain, loss, or proceeds, are found by multiplying the gain, loss, or proceeds by their respective numerators, and dividing the product by the common denominator. When each partner puts in a certain sum as best suits his inclination or abilities, his share of gain, loss, or proceeds must be proportioned to his stock. And, lastly, when the partners not only put into the company trade different stocks, but enter or withdraw these stocks at different periods, and perhaps both, each partner's share of gain, loss, or proceeds must be proportioned to his share of the capital, and the time it was employed. Hence will arise four cases.

*Case 1.* When the capital is divided into equal shares, each partner's dividend of the gain, loss, or nett proceeds, is found, by dividing the whole gain, loss, or nett proceeds by the number of partners.

*Exam.* A, B, and C had each in the common stock L. 500; when their books were balanced, they found there was of nett gain L. 550; how much must be carried to each partner's account in company?

$$3) \underline{550}$$

$$\underline{\underline{L. 183-6-8}} \text{ Answer.}$$

*Case 2.* When the different shares of the capital have one common denominator, divide the gain, loss, or proceeds by that denominator, and multiply the quotient by their respective numerators.

*Ex.* A, B, C, and D, were concerned in a store in Virginia, whereof A had  $\frac{1}{2}$ , and B, C, and D each  $\frac{1}{6}$ ; when the store was sold off, and the debts collected, there was a clear capital of L. 5000; what is the dividend to each?

$$6) \underline{5000}$$

$$\begin{array}{r} 833 \ 6 \ 8 \text{ to B.} \\ 833 \ 6 \ 8 \text{ to C.} \\ 833 \ 6 \ 8 \text{ to D.} \\ \hline 2500 \ 0 \ 0 \text{ to A.} \end{array}$$

$$\underline{\underline{5000 \ 0 \ 0 \text{ as given.}}}$$

*Case 3.* When each partner stocks in a sum at random, as suits his convenience or inclination; let the whole capital be the first term, the proceeds, gain, or loss, the second, and each partner's stock the third; then will the share of gain, loss, or proceeds, due to that partner whose share of the capital was the third term, be the fourth proportional.

*Ex. 1.* A, B, and C freight a ship to Jamaica; A puts in goods to the value of 475 l. 10 s. B to the value of L. 675 : 3 : 4, and C to the value of L. 834 : 6 : 8, including charges: they gained 547 l. 19 s. on the voyage: What is the dividend to each, in proportion to his share of the capital?

	<i>L.</i>	<i>s.</i>	<i>d.</i>
A =	475	10	0
B =	675	3	4
C =	834	6	8

Capital 1985	0	0	: 547.95 gain.
Abridged 397	0	0	: 109.59 :: 475 $\frac{1}{2}$ : 131.259 = A.
	397	0	: 109.59 :: 675 $\frac{2}{3}$ : 186.376 = B.
	397	0	: 109.59 :: 834 $\frac{1}{3}$ : 230.315 = C.

547.95 Proof.

Sometimes the answer can be found more expeditiously, by finding the share of gain, loss, or proceeds, due to 100*l.* and performing the rest of the operation by practice, especially where there are few fractions.

*Ex. 2.* A, B, and C freight a ship to Virginia; A contributed to the adventure 500*l.* B 1200*l.* and C 1300*l.*; they had returns in tobacco, the nett proceeds wherèof amounted to 3800*l.*; what is the dividend to each?

First, 3000 : 3800 :: 100	
Abridged 3 : 38 :: 10	
	10
	—
3 ) 380	
	—
126    13    4 per cent.	
	5
	—
500 = 633    6    8    A draws 633    6    8	
500 = 633    6    8	
200 = 253    6    8    B draws 1520    0    0	
—	
1200 = 1520    0    0	
100 = 126    13    4    C draws 1646    13    4	
	—
Proof 3800    0    0	

It will have the same effect, and sometimes the process will be still shorter, to find the proportional share of gain, loss, or proceeds, to 1*l.* and do the rest by practice.

*Ex. 3.* A, B, and C fit out a ship for Martinico, for which they

they pay their 300 guineas each. A puts into the common stock, for the purchase of ready-money goods, 274*l.* B 389*l.* and C 437*l.* Moreover, they buy upon their joint credit goods at twelve months, to the amount of 2750*l.*: portage, lighterage, packing, and shipping-charges, were paid out of the ready money: — A goes supercargo, and is allowed 2*1*/*2* per cent. proceeds for his trouble of management. Each partner's share of nett proceeds for the ready money advanced was to be proportioned to his inputs, but for the goods on credit they shared equally. — A arrives with a bill of exchange for 1000*l.* and sugar, which, after all charges were deducted, they sold for 4785*l.* A's bill of charges and commission came to 360*l.*; how will the remainder be divided?

Advanced by A in cash	<i>L.</i> 274	Returns in sugar	<i>L.</i> 4785
Ditto by B,	- - 389	In a bill of exchange	1000
Ditto by C,	- - 437		—
Total advances in money	<u>1100</u>	Total returns	<u>5785</u>
Upon the common credit,	<u>2750</u>	Deduct charges, &c.	<u>360</u>
		Nett proceeds,	<i>L.</i> 5425
Total of the capital,	<u>3850</u>		

To find the dividend upon 1*l.* it will be

$$\begin{array}{ccc} \textit{cap.} & \textit{proceeds.} & \textit{cap.} \\ 3850 & : & 5425 :: 1 \end{array}$$

$$\text{Abridged } 154 : 217 :: 1 \text{ } L. 1-8-2\frac{2}{7}$$

Since A put in - - *L.* 274

For 1 <i>l.</i> he draws	<u>274</u>
For 6 <i>s.</i> 8 <i>d.</i>	<u>91 6 8</u>
For 1 <i>s.</i> 4 <i>d.</i>	<u>18 5 4</u>
For 2 <i>d.</i>	<u>2 5 8</u>
For $\frac{2}{7} \frac{1}{7}$	<u>0 4 1\frac{1}{4}</u>

For 154 : 217 : : 274 : As share, *L.* 386 1 9*\frac{1}{4}*

*Note.* These methods prove each other alternately.

*Note.* The ship makes no alteration, unless it had been sold at loss or profit.

Since

*A's share brought forward, L. 386 1 9 $\frac{1}{2}$*

Since *B* put in - - *L. 389 0 0*

$$\begin{array}{rcl} \frac{1}{3} & = & 129\ 13\ 4 \\ \frac{1}{5} & = & 25\ 18\ 8 \\ \frac{1}{8} & = & 3\ 4\ 10 \\ \hline \frac{1}{12} & = & 0\ 5\ 10\frac{1}{4} \end{array}$$

For 154 : 217 :: 389 : *B's share, L. 548 2 8 $\frac{1}{4}$*

And since *C* put in *L. 437 0 0*

$$\begin{array}{rcl} \frac{1}{3} & = & 145\ 13\ 4 \\ \frac{1}{5} & = & 29\ 2\ 8 \\ \frac{1}{8} & = & 3\ 12\ 10 \\ \hline \frac{1}{12} & = & 0\ 6\ 7\frac{1}{4} \end{array}$$

For 154 : 217 :: 437 : *C's share, L. 615 15 5 $\frac{3}{4}$*

Lastly, 154 : 217 :: 1100 : *L. 1550 0 0*  
ready-money proceeds.

From the total proceeds 5425

Deduct - - - 1550

Remains - - - 3875

Deduct goods on credit, 2750

1125

To be divided equally  $\frac{1}{3}$  to each, - - - *L. 375*, which being added to their ready-money shares respectively, gives the particular sum which each partner draws out of the concern.

Quest. 4. *A, B, and C buy a ship for 1200 l. whereof A had  $\frac{1}{2}$ , B  $\frac{1}{3}$ , and C  $\frac{1}{4}$ ; how much money paid each?*

First,

$$\frac{1200}{2} = 600$$

$$\frac{1200}{3} = 400$$

$$\frac{1200}{4} = 300$$

1300

Then 1300 : 1200 :: 100  
Abridged 13 : 12 :: 100 : 92-6-1 $\frac{1}{4}$ .

But

$$\begin{array}{r}
 \text{But } L. 92 \ 6 \ 1\frac{1}{3} \times 6 = 553 \ 16 \ 11\frac{1}{3} \\
 92 \ 6 \ 1\frac{1}{3} \times 4 = 369 \ 4 \ 7\frac{5}{6} \\
 92 \ 6 \ 1\frac{1}{3} \times 3 = 276 \ 18 \ 5\frac{7}{9} \\
 \hline
 & & 13 = 1200 \ 0 \ 0 \quad \text{Proof.}
 \end{array}$$

*Exam. 5.* A, B, C, and D purchase a ship, whereof A had  $\frac{1}{3}$ , B  $\frac{1}{4}$ , C  $\frac{1}{5}$ , and D paid 500*l.*; what was D's share of the ship, how much money paid each of the rest, and what cost the ship all together?

Then 13:500:: 20  
20

$3 \times 4 \times 5 = 60$  denominator.

$1 \times 4 \times 5 = 20$  A's numerator.

$1 \times 3 \times 5 = 15$  B's numerator.

$1 \times 3 \times 4 = 12$  C's numerator.

Complem<sup>t</sup> 13 D's numerator.

60

13) 5000

$$4) \begin{array}{r} 769 \\ 192 \end{array} \quad \begin{array}{r} 4 \\ 6 \end{array} \quad \begin{array}{r} 7\frac{1}{4} \\ 1\frac{3}{4} \end{array} \quad \begin{array}{l} A \text{ paid.} \\ \text{Deduct } \frac{1}{4}. \end{array}$$

5) 576 18  $5\frac{1}{2}$  B paid.  
115 7  $8\frac{3}{8}$  Ded.  $\frac{1}{2}$ .

12) 461 10 9 $\frac{1}{5}$  C paid.  
38 9 2 $\frac{4}{5}$  Add  $\frac{1}{5}$

Proof 500 o o D paid.

Then  $A + B + C + D = 2307 \frac{13}{16}$   
the price of the ship.

In real partnership there are few instances where the shares of the company's capital are so undetermined as in some of the last examples ; if they are not determined by one common denominator, they are generally even hundreds ; or if one advance more than another, the difference is made up by an interest-account at balancing the books. But this case is of singular use in settling compositions in bankruptcy, and average-losses in insurance, to which we refer.

*Cafe 4.* When the partners give in, or withdraw their stocks at different periods, multiply each partner's stock into the time it was employed, and the sum of the products will be the first term; the gain, loss, or proceeds the second; and each particular product the third; then will the fourth proportional be the share of gain, loss, or proceeds due to that partner, the product of whose money and time was the third.

term. Or the proportional share of gain may be found to 1*l.*, as above, and the rest performed by practice or multiplication, as seems most convenient for dispatch.

*Quest.* I. Four merchants, A, B, C, and D enter into partnership, thus :

A put in *L.* 64 10 0 for  $4\frac{1}{2}$  months,  
 B - - 78 15 0 for 6 months,  
 C - - 112 14 0 for  $8\frac{3}{4}$  months,  
 D - - 125 5 0 for  $5\frac{1}{4}$  months,

They gain 108*l.* 18*s.*  $4\frac{1}{2}$  *d.*; what is due to each in proportion to their stocks, and time they were respectively employed?

$$\text{First, } \left\{ \begin{array}{l} \text{A's stock } 64.5 \times 4.5 = 290.25 \\ \text{B's stock } 78.75 \times 6 = 472.5 \\ \text{C's stock } 112.7 \times 8.75 = 986.125 \\ \text{D's stock } 125.25 \times 5.25 = 657.5625 \end{array} \right.$$

The sum of the products 2406.4375

$$\begin{array}{rcl} \text{Then } 2406.4375 : 108.91875 & A. & L. \\ \text{Abridged } 19.2515 : .8719 :: 290.25 & : 13.137 & \text{A's gain} \\ 19.2515 : .8719 :: 472.5 & : 21.3859 & \text{B's do.} \\ 19.2515 : .8719 :: 986.125 & : 44.633 & \text{C's do.} \\ 19.2515 : .8719 :: 657.5625 & : 29.762 & \text{D's do.} \end{array}$$

108.9179 Proof.

Or thus,

$$19.2515 : .8719 :: 1 : .045261 \text{ gain due to 1*l.*}$$

Then	.290.250	472.500
Multiplier inverted	162540.0	162540.0
	<hr/>	<hr/>
	11.6100	18.9
	1.4513	2.3625
	.580	945
	.174	284
	...3	5
	<hr/>	<hr/>

$$\text{A's gain} = 13.1370 \text{ B's gain} = 21.3859$$

986.125

986.125	657.5625
162 540.0	162 540.0
	<hr/>
39.445	26.3025
4.9306	3.2878
.2272	.1315
.592	395
...10	7
	<hr/>

C's gain = 44.633 D's gain = 29.762

The several shares as before.

Questions of this kind seldom or never occur in business; all differences in point of time are in company-affairs settled by an interest-account, and therefore to multiply examples on the head would be entirely useless.

### VARIETY VII. Bankruptcy and Computations relating to Compositions.

Bankruptcy is the failure, absconding, and relinquishment of traffic in a merchant, banker, or any other trader.—In the first ages of banking, the dealers in exchange had benches erected in the public places, upon which they told their money, and wrote their bills, &c. as the itinerant merchants, or chapmen, have at this day.—When a banker failed in his circumstances, his bench was broke, either to denote a fraud, or advertise the public that the person to whom the bench belonged was no longer to be trusted; hence the term *bankruptcy* is of Italian extraction, from two words signifying a *broken bench*.

The French distinguish betwixt a bankruptcy and a failure, by stamping the idea of a designed fraud on the first, whilst the latter is construed as occasioned by misfortunes and necessity.

In stat. 1. Ja. I. c. 15. we have this description of a bankrupt, viz. All and every person who shall use the trade of merchandise, by way of bargaining, exchange, bartering, or otherwise, in grots, or by seeking his or her living by buying and selling, who shall depart his house, or absent himself, or suffer himself to be arrested for any debt, either for money delivered, wares sold, or other good consideration;

D d 2

or

or shall suffer himself to be outlawed, or go to prison, or fraudulently procure himself to be arrested, or his money or goods attached, or make any fraudulent conveyance of his lands, goods, or chattels, whereby his creditors may be defeated in the recovery of their just debts; or, being arrested for debt, shall lie in prison six months, or more, upon such arrest or detention, shall be adjudged a bankrupt.

When it is judged that a merchant hath become a bankrupt, not out of necessity, but with a view to defraud his creditors, and save a capital at their expense; a commission of bankruptcy may be taken out against him under the great seal, directed to five or more commissioners, to inquire into the particulars of the tailant's circumstances, and to act conformably to certain statutes made in that behalf.

*Substance of the laws relating to bankruptcy in England.*

13. *Eliz. c. 7.* Those who depart the realm, conceal themselves, or permit themselves to be falsely arrested, to defraud creditors, if they were in trade, are deemed bankrupts. — The Lord Chancellor, upon a complaint in writing against a bankrupt, appoints commissioners to sell the bankrupt's land, as well copy as free, annuities, goods, chattels, debts, &c. These commissioners have power to convene persons suspected to have any of the bankrupt's effects. — Persons refusing to disclose, or detaining lands, &c. to forfeit double the value. — If the person indebted absent himself, after five proclamations by commissioners, to be out of the King's protection, and persons concealing him to be imprisoned and fined. — The creditor, not satisfied, may afterwards take his course at law.

*Stat. 1. Ja. c. 15.* A creditor shall be received if he come in, in four months. — Grants of land, &c. of a bankrupt in other mens names, except to children upon marriage, or for a valuable consideration, — void. — If, upon warning left at the usual place of residence three times, the bankrupt does not appear, five proclamations to be made, &c. Refusing to be examined, shall be committed. — Committing perjury to 10 l. prejudice of creditors, to stand on the pillory. — Persons suspected to detain any of the estate not appearing, to be arrested; and still refusing, to be committed. — Commissioners may assign debts, &c. to the creditors, and proceed to execution though the bankrupt dies. — Commissioners to render the bankrupt an account, and pay overplus.

*Stat.*

*Stat. 21. Ja. c. 19.* Persons endeavouring to compel creditors to take less than their due, or to gain time beyond six months, and escaping, &c. adjudged bankrupts. — Bankrupt's wife may be examined on oath. — Bankrupt fraudulently concealing to stand on the pillory. — Commissioners may break open a bankrupt's house, chests, &c. Another man's goods in the bankrupt's house to be distributed. — No respect to be had to debts upon judgment, recognisances, or specialties, beyond other debts. *St. 12. and 14. Car. II. c. 14. 9. &c. — 10. W. III. c. 44.* Adventurers in the East-India company, Guinea company, &c. and no member of any society or company, to be adjudged a bankrupt in respect of his stock; and the stock not liable to foreign attachment.

*Stat. 5. G. II. c. 30.* Bankrupts not submitting to be examined thirty days after notice, and not discovering how they have disposed of their goods, and all books, papers, &c. and delivering up all to the commissioners, all such estate, &c. except their wives and childrens necessary wearing apparel, to suffer as felons. — No advantage to bankrupts giving above 100*l.* in marriage with children, unless at the time they could satisfy their debts. — Bankrupts removing, concealing, or imbezzling, to the value of 20*l.* guilty of felony. — No commission of bankruptcy to be issued out, unless the debt of one creditor petitioning amount to 150*l.* or of three to 200*l.* — and bond to be given in 200*l.* penalty, to prove the party a bankrupt.

*Stat. 6. G. I. c. 21. and 7. G. I. c. 31.* A bankrupt in prison, on execution for debt, to be discharged, producing his certificate. — And persons having bills or notes on bankrupts, at a day to come, entitled to dividend, allowing discount at 5 per cent.

*Stat. 5. G. II. c. 30.* Bankrupts not surrendering in forty-two days, and not discovering estates, adjudged guilty of felony; but the Lord Chancellor may give seventeen days further respite.

They are to deliver all books of accounts, writings, &c. to assignees, on oath, and be allowed 5 per cent. not above 200*l.* on paying 10*s.* in the pound, and 7½ per cent. not exceeding 250*l.* on paying 12*s. 6d.* per pound; and the body, not the estate of the bankrupt, to be discharged, unless he pays 15*s.* in the pound.

Four parts in five in number and value of creditors are to sign

sign certificates, &c. Bonds or notes given to consent to any certificates to be void; and if any creditor suing out any commission, has privately more than others, the commission shall be superseded.—Bankrupts to be apprehended upon a commission issued.—Persons who discover their estates to be allowed 5 per cent. and concealing trusts to forfeit L. 100 and double value.—Notice must be given to creditors to meet and chuse assignees, prove debts, &c.; but none to vote where a debt is under L. 10.—New assignees may be chosen by the creditors, and assignees at the end of four months, and within twelve, to account, and dividend to be made; and there may be a second final dividend in eighteen months; &c.—Commissioners to take an oath,—allowed 20 s. a-day, but nothing for expenses, and attorneys bills to be adjusted by a master in chancery.—

Bankers, brokers, or factors, are subject to statutes of bankruptcy; but farmers, graziers, or receivers of taxes, shall not be deemed bankrupts. *Stat. 5. and 16. Geo. II.*

*Substance of the laws relating to bankruptcies in Scotland.*

*Will. & Mary, parl. 1. sess. 2. c. 20.* The Lords shall adjudge the lands, &c. belonging to bankrupts, in favour of the buyer, by the decree of sale, and the buyer shall thereon be infest, so soon as it shall be found that the debtor is utterly insolvent, whether the legal be expired or not. Moreover, if no purchaser is found, a dividend may be made of the lands and estates among the creditors; as also the Lords may determine the value of liferent-escheats, and the lands or price to be divided among the creditors, conform to their several rights and diligences.

*Will. & Mary, parl. 1. sess. 6. c. 5.* He is declared to be bankrupt, who, being under horning and caption, and found by decree of the Lords to be insolvent, is either imprisoned or retired to the abbey, or other privileged place, or fled or absconded, or who defends his person by force; which being found by a declarator of the Lords, all voluntary deeds made by the bankrupt, at or after, or in the space of sixty days before his becoming so, in favour of any of his creditors, either in satisfaction, or for security, shall be null and void; and heritable rights, whereupon infestment may follow, granted by the said bankrupts, are only reckoned, as to this case, to be of the date of the seisin, but prejudice of their validity as to other

other effects.—A fraudulent bankrupt is not only infamous *infamia juris*, but may be farther punished by the Lords of Session, death excepted, and no dispensing with the habit allowed in cession, unless the bankrupt failing through misfortune be libelled and proved.

*Ja. VI. parl. 23. c. 18.* All alienations, dispositions, and assignations made by debtors of their lands, teinds, reversions, and goods whatever, to any conjunct or confidant person, without a true and just cause, after the contracting of their debts, may be declared null at the instance of just creditors. And if any third party purchase lawfully the said lands or goods from the said conjuncts or confidants; his rights shall stand, but the receiver of the price shall be liable to make the same forthcoming to the said creditors.—The fraud may be proved by writ or oath of the party receiver of the said security, that it was without any true or just cause; or that the lands and goods being again sold, the price, or most part thereof, is, or was to be converted to the bankrupt's be-hoof, &c.

The computations relating to this variety, when there are few creditors, may be made as in case 3. of the last variety.

*Exam. 1.* Suppose a bankrupt's effects should amount to L. 1739*l.* 13*s.* 8*d.* what dividend thereof will fall to each of the following creditors in proportion to their respective sums?

He owes to A,	L.	313	7	-3	= 313.3625
to B,	290	4	6	= 290.225	
to C,	700	0	0	= 700.	
to D,	486	13	8	= 486.68333	
to E,	600	0	0	= 600.	
to F,	500	0	0	= 500.	
to G,	381	10	0	= 381.5	
to H,	418	0	0	= 418.	

Total debt, L. 3689 15 5 = 3689.77083

Debt.

<i>Debt.</i>	<i>Subject.</i>	<i>L.</i>	<i>s.</i>	<i>d.</i>	
3689.77083	: 1739.685416 ::	313.3625	14	11 $\frac{1}{4}$	A draws.
		290.225	136	16 9 $\frac{1}{4}$	B draws.
		700.	330	0 10	C draws.
		486.683	229	9 3 $\frac{1}{4}$	D draws.
		600.	282	17 10 $\frac{1}{4}$	E draws.
		500.	235	14 10 $\frac{1}{2}$	F draws.
		381.5	179	17 5 $\frac{1}{2}$	G draws.
		418.	197	1 8	H draws.

L. 1739 13 8 $\frac{1}{4}$  Proof.

By the above method it is plain, there must be an heavy multiplication and division for every creditor, which, when the creditors are numerous, would render the calculation intolerably tedious and troublesome, in which case, find the dividend for 1*l.* and do the rest by practice; or, which will be much shorter, multiply the dividend for 1*l.* into all the nine digits severally, and then into 10; multiply this last product into all the nine digits, and lastly into 10, so shall there be a table constructed, from which the several dividends, were the creditors ever so numerous, can easily be effected.

Let the last example be resumed.

$$3689.77083 : 1739.685416 :: 1$$

$$3689.77083) \quad 1739.685416 (.4714875$$

4714875 X	1 = .4714875	1 = 4.714875
	2 = .9429750	2 = 9.42975
	3 = 1.4144625	3 = 14.144625
	4 = 1.8859500	4 = 18.8595
	5 = 2.3574375	5 = 23.574375
	6 = 2.8289250	6 = 28.28925
	7 = 3.3004125	7 = 33.004125
	8 = 3.7719000	8 = 37.719
	9 = 4.2433875	9 = 42.433875
	10 = 4.7148750	10 = 47.14875

Having thus tabulated the dividend to 1*l.* the use of the table will be discovered from the following instances.

There was due to A 313*l.* 7*s.* 3*d.*

By

By the table the dividend to 100 = 47.14875

For 300 multiply by 3

<i>L.</i>	<i>10</i> =	141.44625
3 =		4.714875
6 s. 8 d. = $\frac{1}{3}$ of 1 <i>l.</i>		1.4144625
6 d. = $\frac{1}{40}$ of 1 <i>l.</i>		.1571625
1 d. = $\frac{1}{8}$ of $\frac{1}{40}$		.01178718
		.00183119
		<i>L. s. d.</i>
		147.74636837 = 147 14 11 $\frac{1}{4}$
		as before.

There was due to C 700

*L.* 100 in the table = 47.14875

For 700 multiply by 7

$$330.04125 = L. 330 \text{ or } 10 \text{ as before.}$$

And so of any other.

When the composition is agreed on, or fixed betwixt the failant and his creditors, the dividend may be at once calculated for any sum practically without a table, especially if the parts can be easily taken.

*Exam. 2.* A. B. breaks, and offers his creditors 15*s.* a pound, which they agreed to accept of; what will C receive, whose debt amounts to 515*l.* 7*s.* 6*d.*?

$$2) \underline{515} \quad 7 \quad 6$$

$$2) \underline{257} \quad 13 \quad 9 \text{ for } 10s.$$

$$\underline{128} \quad 16 \quad 10\frac{1}{2} \text{ for } 5s.$$

$$386 \quad 10 \quad 7\frac{1}{2} \text{ for } 15s. \text{ a pound} \times 386 \quad 10 \quad 7\frac{1}{2}$$

*Exam. 3.* A. B. breaks, and offers his creditors 12*s.* 6*d.* a pound, which they agree to receive; what is C's debt worth, amounting to 756*l.* 19*s.* 6*d.*?

$$2) \underline{756} \quad 19 \quad 6$$

$$\underline{378} \quad 9 \quad 9 \text{ for } 10s.$$

$$\underline{94} \quad 12 \quad 5\frac{1}{4} \text{ for } 2s. 6d.$$

$$\underline{\underline{473}} \quad 2 \quad 2\frac{1}{4} \text{ for } 12s. 6d. \text{ Answer.}$$

*Exam. 4.* A. B. compounds his debt with his creditors at 5*s.* 5*d.* per pound Sterling; what will C receive, to whom he was due 500*l.*?

$$\begin{array}{r} 4) \ 500 \\ \hline 125 \\ 10 \ 8 \ 4 \end{array}$$

at 5*s.* 5*d.*

£35 8 4 at 5*s.* 5*d.* *Answer.*

### Variety VIII. STOCK JOBBING.

TO render this subject more intelligible, it hath been thought proper to divide it into particular sections.

#### SECT. I. *Of stocks, or transferrable funds.*

By the word *stock* was originally meant, a particular sum of money contributed to the establishing a fund, to enable a company to carry on a certain trade, by means of which the person became a partner in that trade, and received a share of the profit made thereby in proportion to the money employed. But this term hath been extended further, though improperly, to signify any sum of money which hath been lent to the government, on condition of receiving a certain interest, till the money is repaid, and which makes a part of the national debt. As the security, both of the government and of the public companies, is reckoned preferable to that of any private person, as the stock is negotiable, and may be sold at any time, and as the interest is always punctually paid when it falls due; so they are thereby enabled to borrow money on a lower interest than what might be obtained from lending it to private persons, where there must be always some danger of losing both principal and interest.—But as every capital stock, or fund of a company, is raised for a particular purpose, and limited by parliament to a certain sum, it necessarily follows, that when that fund is completed, no stock can be bought of the company; though shares already purchased may be transferred from one person to another. This being the case, there is frequently a great disproportion between the original value of the shares and what is given for them when transferred: for if there are more buyers than sellers, a person who is indifferent about selling, will not part with his

his share without a considerable profit to himself; and, on the contrary, if many are disposed to sell, and few purchasers appear, the value of stocks will naturally fall, in proportion to the impatience of those who want to turn their stock into specie.

*The present government-funds are,*

1. Three per cent. reduced bank-annuities.
2. Three per cent. consolidated ditto.
3. Three per cent ditto, 1726
4. ditto, 1751
5. Three and a half per cent. ditto, 1756
6. Ditto, ditto, 1758
7. Four per cent. ditto, 1760
8. Long annuities, 1761
9. Four per cent. subscription, 1762
10. Long annuities, 1762

*Stocks of companies are,*

1. Bank stock.
2. South-sea stock.
3. India stock.
4. South-sea annuities.
5. India annuities.
6. India bonds.

The funding scheme was introduced soon after the revolution, and hath hitherto answered the purpose for which it was calculated, *viz.* the providing for the exigencies of the state in time of war, without the least impeachment or diminution of its credit: whether this will always be the case, is a question which futurity alone can decide. In our statutes since the revolution various funds are specified, such as the East-India, South-sea, aggregate or general fund, and the sinking fund.—Whenever the word *fund* is applied to the South-sea or East-India company, it means their stock or capital, or such duties and articles of the public revenue as are appropriated to pay the interest due on such stocks till redeemed by parliament. The aggregate fund is so called from its being a collection of many things incorporated together, comprehending not only the several rates and duties reserved to pay and satisfy the interest due on the funds at the bank, India, and South-sea house, but all the articles of the public revenue in general, excepting those that compose the King's civil list, which likewise, by an act of the first year of our present Sovereign, make a part of the aggregate fund for £. 800,000, to be paid quarterly out of the said aggregate fund.

SECT. II. *The method of raising the annual supplies granted by parliament, for defraying the public expenses of the state, and the manner of subscribing and negotiating subscription, receipts, &c.*

THE supplies granted by parliament have been raised for some years past by annuities transferable at the bank of England, sometimes with, and sometimes without a lottery. After the parliament have voted these supplies, if the subscriptions have not been already secured by a few persons of fortune, which is sometimes the case, a public subscription is set on foot; in which case, every responsible person is at liberty to apply, by a proper letter, to the first commissioner of the treasury, for leave to be a contributor, specifying in the letter the sum which he would incline to contribute. But when the required sum hath been previously secured by a private subscription, the application must be directed to some of the monopolists. Sometimes indeed the subscription lies open to the public at the bank, or at the exchequer, and in that case every person is allowed to subscribe what he thinks proper; and should there be a surplus, a proportional reduction is made upon each subscription, that the whole may exactly correspond with the sum granted by parliament.

After the subscription is closed, receipts are made out, and delivered to the subscribers; and, for the conveniency of sale, a subscriber for a large sum may have sundry partial receipts for different proportions of his property; and a term of assignment is drawn upon the back of the receipt, which being signed and witnessed, transfers the property to a purchaser. The first deposit is generally of 15 per cent. and is commonly made at the time of subscribing, and every month thereafter a deposit is made of 10 or 15 per cent. till the whole is paid up. Those who chuse to pay in the whole sum before the appointed days, are allowed 3 per cent. from the date of such payment till October; and the receipts for such payments in full, in the language of the alleys, are called *heavy horse*, but the receipts for partial payments *light horse*; because the latter is much easier purchased than the former, and therefore more convenient for jobbing. An instance will be the best illustration. Suppose, in the month of March, just after the second payment, one buys a receipt for 500*l. script.* or light horse, for 25*l.* the method of reckoning would be thus: Suppose the current price is 75 per cent. which is 25 per cent. below par; and as the purchaser buys the script. by itself, without the douceurs that accompanied it, he must be allowed the discount,

discount, *viz.* 25 per cent. out of the payments already made, because he is to continue paying in at par, in the same manner as if he had received the douceurs. The method of making out a bill for 500*l.* script. with two payments made upon it, is as follows:

L.

A sells to B 500 script. at 75 per cent. on which }  
he hath made two payments, at 15 per cent. each, } 150 0 0

B, not having the lottery-tickets, nor the long annuities, which are the douceurs, must be allowed 25 per cent. the difference betwixt the current price of 3 per cent. and par, } 125 0 0

Balance to be paid to A for the receipt, 25 0 0

Hence it appears that B becomes possessed of 500*l.* script. for 25*l.* which he calls *light horse*; and the advantage of script. bought on these easy terms, is, that if a little rise happens, the same profit of 1, 2, or 3 per cent. will present to the holder on this nominal sum, as if the whole had been actually cleared off; and for this reason likewise, there is always a more ready market for light-horse receipts than for heavy-horse, because a purchaser may have 25*l.* who hath not 375*l.*

### SECT. III. *The method of buying into, and selling out of the public funds.*

THE business of stockjobbing is generally carried on by brokers, who, themselves having seldom or never any property in the funds, make their fortunes by bubbling their clients, and have entered into a kind of conspiracy to keep the method of trading in the stocks a mystery to every one but themselves.

Any person who wants to sell or buy stocks, will easily discover his merchant amidst the cries that resound from all quarters.— If one means to be a purchaser, he may, without the assistance of a broker, ask another whom he hath discovered to be a seller concerning the price; who, if he is a common seller, will name the whole price, as 74½, 75¾, or what else it happens to be; but, if he is a doctor, or master of arts in the alley, he will only name you the fractional part of the price, to discover if you are a broker, especially if he finds it to his account to deal with such; he will go about

about in search of his merchant, as you must do in search of yours. When your merchant appears, you then fix the price and the quantity, and give him your name, designation, and place of abode, that he, as the seller, may take care of the transfer, and prepare the receipt. In the mean time, it is your business to have the sum ready in bank-notes, because the hurry in negotiations of this kind is so great, that, in counting money, one is very liable to mistakes; for which reason, those who deal much in stocks, and are well known, make all payments by a draught on a banker, whom they have engaged, for a trifle of commission, to clear all draughts of this kind. The transfer being prepared, and your name called, you then go to the clerk, who hath the transfer-book, to see the form in which the seller hath transferred the sum agreed for, which ought to be to you, your heirs, assignees, &c. You will then be directed to subscribe a form of acceptance of the stock transferred, the seller having first set his name to the transfer. This done, the clerk witnesseth the printed receipt, which the seller gives you signed by him, which you must keep as a voucher of the transfer; then, having paid the sum and taken the receipt, the whole business is transacted with respect to you as a buyer.

If you intend to sell stock, get a quarter of a sheet of paper, and write on it your own name, designation, and place of abode, the sum to be transferred, the fund whence it is to be transferred, and the person's name and description to whom you have agreed to transfer it, as in the form below.

James Dinwiddie, Esq; of Southampton-street,  
in the parish of St Paul, Covent-garden,  
500 l. 3 per cent. consolidated bank-annuities,

T O

Mr James Simple watchmaker on Snow-hill,  
in the parish of St Sepulchre.

This paper must be delivered to the clerk nearest to, or under the letter with which your name begins, who will examine your account, and prepare the transfer. In the mean time, it is your business to be making out the purchaser's bill, which you are to do on a printed receipt, which you will be helped to in the office, a copy whereof is hereto subjoined.

*Consolidated*

*Consolidated 3 l. per cent. annuities, at 74 $\frac{5}{8}$ .*

*Received, the day of May 1762, of Mr James Simple watchmaker on Snow-hill, the sum of seventy-four pounds, twelve shillings, and sixpence, being the consideration for one hundred pounds, interest, or share, in the joint stock of three per cent. annuities, erected by an act of parliament of the twenty-fifth year of the reign of King George II. intitled, An act for converting the several annuities therein mentioned, into several joint stocks of annuities, transferrable at the bank of England, to be charged on the sinking fund, together with the proportional annuity, at 3 l. per cent. per annum, attending the same, by me this day transferred to the said James Simple. Witness my hand,*

*Witness,  
John Gray, Clerk.*

James Dinwiddie.

The receipt being thus made out, and the transfer ready, you sign it in the book, and then deliver your receipt to the clerks, who sign as witnesses; after which deliver the receipt, and receive the money, and the whole affair is transacted.

*Table, shewing the amount of the several stocks, the days and hours for transferring stocks and annuities, and the time of paying the dividends.*

Amount of each capital.

Transfer-days.

Bank stock,	L. 10.780.000	} Tues. Wed. Thu. Fri.
An. 3 per cent. red.	17.701.523 16 4	
Ditto consolidated	21.627.821 5 1 $\frac{1}{2}$	
Ditto 1726,	1.000.000	
Ditto, 3 $\frac{1}{2}$ , 1756,	1.500.000 - - -	Tues. and Thurs.
Ditto 1758,	4.500.000 - - -	Mon. Wed. Frid.
4 per cent. do. 1760,	8.240.000 - - -	Tues. Wed. Thur. Fri.

*Dividends due on bank-stock, and 3 per cent. reduced, on the 5th April and 10th October, on the rest the 5th of January and 5th of July.*

South-sea stock,	L. 3.662.784 8 6	} Mon. Wed. Frid.
Old annuities, - - -	12.404.270	
New annuities, - - -	8.958.255 2 10	
3 per cent. 1751, - - -	2.100.000	Tues. Thur. Satur.

South-

*South-sea stock new annuities, and 3 per cent. dividends, due on the 5th January and 5th July; old annuities, the 5th of April and 10th of October.*

India stock, - - 3.000.000 - - Tues. Thurs. Satur.  
Annuities, - - 3.000.000 - - Mond. Wed. Frid.

*Dividends due on bank-stock, 5th of January and 5th of July.  
Ditto on annuities, 5th of April and 10th of October.*

Hours of transferring.	Of paying dividends.
Bank-stock, from 9 to 12.	9 to 12, and 1 to 3.
South-sea house, from 9 to 1.	9 to 12.
India house, ditto.	9 to 12, and 3 to 5.

The transfer-books are generally shut for a month before the time of paying dividends.

#### SECT. IV. Computations in stock-jobbing.

*Case 1.* When the stock is any number of even hundreds, multiply the rate *per cent.* by the number of hundreds, and the product gives the price.

#### E X A M P L E S.

1. Sold 500 three *per cent.* consolidated annuities, at 75*5*; what does it amount to?

$$\begin{array}{r}
 75 \quad 12 \quad 6 \\
 \times \quad \quad \quad 5 \\
 \hline
 L. 378 \quad 2 \quad 6 \text{ Answer.}
 \end{array}$$

2. Bought 700 India stock, at 135*7* *per cent.*; how much money must I pay?

$$\begin{array}{r}
 135 \quad 15 \\
 \times \quad \quad \quad 7 \\
 \hline
 950 \quad 5 \text{ Answer.}
 \end{array}$$

3. Sold

3. Sold 400 script. at  $92\frac{1}{2}$ , what ought I to receive?

$$\begin{array}{r} 92 \quad 10 \\ \quad \quad 4 \\ \hline 370 \end{array}$$

o Answer.

*Cafe 2.* When the quantity of stock bought or sold is no even number of hundreds, multiply the quantity by the rate *per cent.* and divide the product by 100, the quotient gives the answer.

### E X A M P L E S.

1. What must I pay for 135 annuities  $3\frac{1}{2}$ , at  $87\frac{1}{2}$  *per cent.*?

$$\begin{array}{r} 87.5 \\ \times 135 \\ \hline 4375 \\ 11375 \\ \hline 118.125 = L. 118 \ 2 \ 6 \end{array}$$

Answer?

Or by practice, thus:

$$\begin{array}{r} 87 \quad 10 \quad 0 \text{ for } L. 100 \text{ ann.} \\ 21 \quad 17 \quad 6 \text{ for } 25 \\ 8 \quad 15 \quad 0 \text{ for } 10 \\ \hline \end{array}$$

L. 118 2 6 as before.

2. What must I receive for 29 l. 3 *per cent.* annuities, when the price is 74 *per cent.*?

$$\begin{array}{r} 74 \\ \times 29 \\ \hline 666 \\ 148 \\ \hline 21.46 = L. 21 \ 9 \ 2\frac{1}{4} \end{array}$$

Or by practice, thus :

$$\begin{array}{r}
 74 \\
 \hline
 14 & 16 & 0 \text{ for } 20 \\
 2 & 19 & 2\frac{2}{3} \text{ for } 4 \\
 3 & 14 & 0 \\
 \hline
 21 & 9 & 2\frac{2}{3} \text{ as before.}
 \end{array}$$

3. What is the value of 97*l.* 10*s.* bank-stock, at 120 per cent.?

$$\begin{array}{r}
 97.5 \\
 120 \\
 \hline
 117.000 \text{ Answer.}
 \end{array}$$

In this way are all computations in the stocks, and single blanks and prizes, made ; which is so easy and expeditious, that more examples would be quite unnecessary.

#### SECT. V. *The causes of the rise and fall of stocks.*

THOSE who are concerned in the practice of stockjobbing, frequently make contracts to buy or sell, at a certain distant period, a certain quantity of some particular stock ; against which time they set all their engines to work, to raise or lower the price of such stock, by inventing and spreading false rumours and fictitious stories, in order to induce people, by these coined alarms, to sell out in a hurry, and consequently cheap, if they have contracted to deliver stock ; or, by imagined advantages, fill the proprietors with the hopes of more profit by keeping up stock, and so render it dearer, if they have stock to receive. The persons who thus delude the people, and make those contracts, are not in general possessed of any real stock ; and, when the time comes that they are to receive or deliver the quantity they have contracted for, they only pay such a sum of money as makes the difference between the price the stock was at when they made the contract, and the price it happens to be at when the contract is fulfilled. And it often happens, that persons, not worth 100*l.* make contracts for buying or selling 100,000*l.* stock. In the language of Exchange-alley, the buyer in this case is called the *bull*, and the seller the *bear*. The original

ginal design of these contracts was, no doubt, to render the trafficking in our stocks more easy and convenient for foreigners; who, without this expedient, would neither be able to make the advantage of the rise and fall of stocks, nor create a rise or fall as seems most to their purpose. The four principal times for which bargains or contracts are made, are February, May, August, and November; and these, in the language of 'Change-alley, are called the *rescounter settlements*. Foreigners generally employ merchants to transact business for them in the stocks, who, having little time for that purpose, depute brokers to make these contracts for them; who, going to the alley for the purpose, and finding another broker who hath a commission to sell the same quantity of stock which he hath orders to purchase, the price is agreed on; and the one notes in his book, *Bought of Jeremiah Sly 5000 l. 3 per cent. annuities, for May.* — and the other, *Sold to Jonathan Trap 5000 l. 3 per cent. annuities, for May.* — Thus is the bargain finished, but no principal or proprietor appears in the treaty; and this gives occasion, under the sanction of transacting for foreigners, to brokers and jobbers in the stocks to buy and sell for themselves, without any property in, or out of the funds; and, perhaps, without any design of transferring or accepting the funds they buy or sell, but, at most, only paying or receiving the odds at the time of settling. — Hence, for the different purposes of the jobbers, the bulls and the bears, one thing will be propagated in the city and another at St James's. — The ministry are turned topsy-turvy in whispers, and set by the ears in private letters from the Hague. — Invasions, rebellion, flat-bottomed boats, schemes, secret negotiations, and all the other chimeras that false hopes or fears, or rather an unaccountable ignorance and credulity, render subservient to the purposes of stockjobbing.

This species of deluders, although they are by far the most numerous body, would lose much of their influence, if they were not countenanced and abetted by those very people, whose interest, did they know it, should direct them to a conduct quite the contrary; I mean those gentlemen of rank and fortune, who have very considerable properties in the funds. Thus, for instance, a gentleman, who, perhaps, enjoys a considerable post in the nation, and, for that reason, may be supposed to have the earliest intelligence of all events that have any influence upon the stocks, and whose honour will not admit the supposition of his propagating a falsehood; in order to stile a great account in the alley to his advantage, should he report a piece of news to be true, which had never

existed any where but in his own brain, and retail it in whispers by his emissaries ; should he go further still, and confirm this relation by an actual sale or purchase of L. 1000 stock at a low or high price, as he intends to sell out or purchase, the unthinking proprietors are easily caught by so strong a bait, and he gets his purpose effected before the cheat can be discovered.

Should this man have a principal command in the army, or in the fleet, he hath it in his power to do the public a double prejudice; first, by neglecting his duty, in order to favour his account in the alley, he makes the evil real, which in the first case was only imaginary ; and secondly, this evil hath not only the same effect on the funds, but as it can meet with no real contradiction, it must be consequently more lasting.

The unaccountable artifices, delusions, and fictions made use of to promote the interest of jobbers and gamesters at home, are not a little supported by foreigners, who have properties in our funds. — When they have given orders for making the contracts we have already taken notice of, and have received advice of the purchase, it is then their busines to set every engine to work, to turn the scale in their favour by the time of settling the account. Hence those private letters, which are so often shown upon 'Change, and which the creatures of these foreigners improve to every purpose of hurting their country. — These letters pass from the alley to the news-writer ; and not only are those concerned in the stocks amused to their prejudice, but the whole nation deluded by hopes, or alarmed by fears, concerning events which never did, nor possibly ever will happen.

It is likewise the misfortune of many young merchants and tradesmen decoyed by designing brokers, and elated with the hopes of being rich in an instant, to be drawn in pretty deep in the adventures of the alley, and, ignorant of their true interest, and unacquainted with jobbing-politics, they become dupes to their own servants the brokers, by retailing their fictions in all the coffeehouses they frequent, and never give over acting, till loss of fortune, of credit and sympathy, close the scene. These are the causes of the fluctuations in the funds ; and if we only consider the variety of stocks, and the different events which may affect them, it will be easy to figure the confusion, the uproar, and the ferment that continually prevails in the alley, and how unlikely it is that business can be transacted there with decency or order, while the proprietors in the funds suffer themselves to be duped and bubbled by such gentlemen as have been above described.

A TABLE, exhibiting at one view the intrinsic value per cent. of the several public funds, and the proportion they bear to each other, by which any person may know which it will be most advantageous to purchase, and what proportion such purchase bears to the value of landed estates and life-annuities.

$3 \text{ per cent.}$ at 60.	$\frac{3}{2}$ 70	4 80	$4\frac{1}{2}$ 90	5 100	$5\frac{1}{2}$ 110	6 120	Years purchase 20	Annual in- terest 5 per cent.
61 $\frac{1}{2}$	71 $\frac{3}{4}$	82	92 $\frac{1}{4}$	102 $\frac{1}{2}$	112 $\frac{3}{4}$	123	20 $\frac{1}{2}$	4 17 6
63	73 $\frac{1}{2}$	84	94 $\frac{1}{2}$	105	115 $\frac{1}{2}$	126	21	4 15 2
64 $\frac{1}{2}$	75 $\frac{1}{4}$	86	96 $\frac{3}{4}$	107 $\frac{1}{2}$	118 $\frac{1}{4}$	129	21 $\frac{1}{2}$	4 13 0
66	77	88	99	110	121	132	22	4 10 10
67 $\frac{1}{2}$	78 $\frac{3}{4}$	90	101 $\frac{1}{4}$	112 $\frac{1}{2}$	123 $\frac{3}{4}$	135	22 $\frac{1}{2}$	4 8 10
69	80 $\frac{1}{2}$	92	103 $\frac{1}{2}$	115	126 $\frac{1}{2}$	138	23	4 6 11
70 $\frac{1}{2}$	82 $\frac{1}{4}$	94	104 $\frac{3}{4}$	117 $\frac{1}{2}$	129 $\frac{1}{4}$	141	23 $\frac{1}{2}$	4 5 1
72	84	96	108	120	132	144	24	4 3 4
73 $\frac{1}{2}$	85 $\frac{3}{4}$	98	110 $\frac{1}{4}$	122 $\frac{1}{2}$	134 $\frac{3}{4}$	147	24 $\frac{1}{2}$	4 1 7
75	87 $\frac{1}{2}$	100	112 $\frac{1}{2}$	125	137 $\frac{1}{2}$	150	25	4 0 0
76 $\frac{1}{2}$	89 $\frac{1}{4}$	102	114 $\frac{3}{4}$	127 $\frac{1}{2}$	140 $\frac{1}{4}$	153	25 $\frac{1}{2}$	3 18 5
78	91	104	117	130	143	156	26	3 16 11
79 $\frac{1}{2}$	92 $\frac{3}{4}$	106	119 $\frac{1}{4}$	132 $\frac{1}{2}$	145 $\frac{3}{4}$	159	26 $\frac{1}{2}$	3 15 5
81	94 $\frac{1}{2}$	108	121 $\frac{1}{2}$	135	148 $\frac{1}{2}$	162	27	3 14 0
82 $\frac{1}{2}$	96 $\frac{1}{4}$	110	123 $\frac{3}{4}$	137 $\frac{1}{2}$	151 $\frac{1}{4}$	165	27 $\frac{1}{2}$	3 12 8
84	98	112	126	140	154	168	28	3 11 4
85 $\frac{1}{2}$	99 $\frac{3}{4}$	114	128 $\frac{1}{4}$	142 $\frac{1}{2}$	150 $\frac{3}{4}$	171	28 $\frac{1}{2}$	3 10 2
87	101 $\frac{1}{2}$	116	130 $\frac{1}{2}$	145	159 $\frac{1}{2}$	174	29	3 9 0
88 $\frac{1}{2}$	103 $\frac{1}{4}$	118	132 $\frac{3}{4}$	147 $\frac{1}{2}$	162 $\frac{1}{4}$	177	29 $\frac{1}{2}$	3 7 9
90	105	120	135	150	165	180	30	3 6 8
91 $\frac{1}{2}$	106 $\frac{3}{4}$	122	137 $\frac{1}{4}$	152 $\frac{1}{2}$	167 $\frac{3}{4}$	183	30 $\frac{1}{2}$	3 5 7
93	108 $\frac{1}{2}$	124	139 $\frac{1}{2}$	155	170 $\frac{1}{2}$	186	31	3 4 7
94 $\frac{1}{2}$	110 $\frac{1}{4}$	126	141 $\frac{3}{4}$	157 $\frac{1}{2}$	173 $\frac{1}{4}$	189	31 $\frac{1}{2}$	3 3 5
96	112	128	144	160	176	192	32	3 2 6
97 $\frac{1}{2}$	113 $\frac{3}{4}$	130	146 $\frac{1}{4}$	162 $\frac{1}{2}$	178 $\frac{3}{4}$	195	32 $\frac{1}{2}$	3 1 6
99	115 $\frac{1}{2}$	132	148 $\frac{1}{2}$	165	181 $\frac{1}{2}$	198	33	3 0 7
100 $\frac{1}{2}$	117 $\frac{1}{4}$	134	150 $\frac{3}{4}$	167 $\frac{1}{2}$	184 $\frac{1}{2}$	201	33 $\frac{1}{2}$	2 19 8
102	119	136	153	170	187	204	34	2 18 10

## ILLUSTRATION of the preceding table.

Suppose 3 per cent. annuities may be purchased at 84 per cent. and India stock at 140; — in the column of 3 per cent. find 84, and opposite thereto, in the column of interest, it will be found to produce 3*l.* 11*s.* 4*d.*; — in the column of 6 per cent. the dividend on India stock stands 168, which shews, that when 3 per cent. give 84 per cent. India stock ought to give 168; so that to purchase India stock is in this case by 28 per cent. a better bargain than 3 per cent. annuities, unless some allowance be made for the difference of the securities, as it is not certain that the company will always be able to give 6 per cent. for an annual dividend out of the profits of their trade:

## SECT. VI. Some of the laws relating to stockjobbing.

7. Geo. II. c. 8. All contracts which shall be entered into, upon which any premium shall be given for liberty to deliver or receive, accept or refuse, any public stock or securities, shall be void and null, and all premiums upon such contracts or wagers shall be restored to the person who shall have paid them, who within six months from the making of such contract, &c. may sue for the same with double costs; and it shall be sufficient for the plaintiff to allege, that the defendant is indebted to him, or hath received to his use the money or premium so paid, whereby the action accrued, according to the form of the statute, without setting forth the special matter; and a bill in equity may be preferred, for discovering any contract or wager, and the premium given, which the defendant shall be obliged to answer upon oath.—To enter into such contracts, and neglect such prosecution, incurs a penalty of *L.* 500, to which penalty the writer of such contract, or negotiator, is liable. To make up a difference of that kind without a public action, incurs a penalty of *L.* 100; and no person who shall sell stock to be delivered and paid for at a certain day, if it be refused or neglected to be paid for, shall be obliged to transfer the same; but it shall be lawful for such person to sell such stock to any other, and to receive or recover from the person who contracted for the same, the damage which shall be sustained; and any person who shall buy stock to be accepted and paid for on a future day, and which shall be refused, or neglected to be transferred, may buy the same quantity of such stock of any other person, at the

the current market-price, and recover and receive from the first seller the damage sustained.—All contracts made for the buying or transferring of stock, whereof the persons, in whose behalf the contract shall be made to transfer the same, shall not at the time of making such contract be actually possessed of that stock in their own right, or in the name of trustees, shall be void; and every person in whose behalf, and with his consent any contract shall be so made to sell stock, of which such person is not actually possessed, shall forfeit the sum of *L.* 500, one moiety to the crown, and the other to him who shall sue for the same; and any broker or agent who shall negotiate such contract, and shall know that the person in whose behalf the contract shall be made is not possessed of the stock, shall forfeit *L.* 100, to be divided betwixt the crown and the prosecutor.—Every broker or person who shall act as a broker in the buying or selling of stocks, shall keep a broker's book, in which he shall enter all contracts that he shall make, on the day of the making such contracts, with the names of the principal parties, as well buyers as sellers, and such broker who shall not keep such book, or shall wilfully omit to enter any contract, for every such offence shall forfeit *L.* 50, to be divided betwixt the crown and the prosecutor.

#### Variety IX. F A C T O R A G E.

**F**ACTORS are merchants agents, residing abroad, constituted by letters of attorney to act for their constituents.

Supercargoes are employed by merchants to go voyages, and dispose of cargoes to the best advantage. Storekeepers frequently get the name of supercargoes, who have the chief management of stores abroad, in vending goods, and making remittances.

The premium, or allowance made to a factor for his trouble in purchasing goods, or putting off consignments, &c. is different in different countries, and for different considerations, but always rated, excepting in the case of an annual salary, at so much *per cent.*

In computing the allowance, or commission due to factors, multiply the sum upon which the commission is to be charged by the rate *per cent.* and divide the product by 100, and the quotient gives the answer.

*Ex. 1.* Bought goods for account of A. B. which, with charges of package, portage, lightage, &c. *per invoice*, amount

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amount to L. 5756, 11 s. 6 d.; what commission is to be added at  $2\frac{1}{2}$  per cent.

$$\begin{array}{r}
 5756.575 \\
 \hline
 2\frac{1}{2} \\
 \hline
 11513.150 = 2 \text{ per cent.} \\
 2878.2875 = \frac{1}{2} \text{ per cent.} \\
 \hline
 143.914375 = L. 143 18 3\frac{1}{4}.
 \end{array}$$

*Note 1.* In dividing by 100, there is no occasion at any time to note the divisor, but simply to point off two figures, or places extraordinary for decimals.

*Note 2.* Questions of this nature may frequently be answered more expeditiously by practice, especially when the given rate is any even part of a pound.

*The last example resumed.*

$$\begin{array}{r}
 L. 2 - 10 \text{ per } L. 100 = 6 \text{ d.} \quad 40)5756.575 \\
 \text{Per } L. = 40. \\
 \hline
 143.914375 \text{ as bef.}
 \end{array}$$

*Ex. 2.* Negotiated bills for A. B. to the amount of 4785 l. 19 s. what commission ought I to charge, at  $\frac{1}{2}$  per cent.?

$$\begin{array}{l}
 \text{Here } \frac{1}{2} = \frac{1}{5} \text{ of } 2\frac{1}{2}. \\
 \text{Therefore, } 4|0)478|5.95
 \end{array}$$

$5)119.64875$  at  $2\frac{1}{2}$  per cent.

$23.92975 = L. 23 18 7$  at  $\frac{1}{2}$  per cent.

*Ex. 3.* A. B.'s sales, per the Diamond, amount to L. 4849, 11 s. 6 d. what is the commission at 3 per cent.?

$$2\frac{1}{2} + \frac{1}{2} = 3 \quad \text{Therefore, } 4|0)484|9.575$$

$$\begin{array}{r}
 5)121.239375 \text{ at } 2\frac{1}{2} \\
 24.2478 \text{ at } \frac{1}{2} \\
 \hline
 \end{array}$$

$145.487175$  at 3 per cent.

4. My factor at Jamaica charges me 5 per cent. commission

mision on my account of sales, amounting gross to 5784*l.*  
18*s.*; what will his commission amount to?

$$2|0) \ 578|4\ 9$$

$$\underline{289.245} = L. 289 : 4 : 10\frac{3}{4}.$$

*Ex. 5.* What commision is due on a bill of sales, amounting to 7418*l.* 14*s.* at 4 per cent.?

4 per cent. is  $\frac{1}{25} - \frac{1}{3}$ .

$$\text{Therefore, } 2|0) \ 741|8\ 7$$

$$\underline{370.935 \text{ at 5 per cent.}}$$

$$\underline{74.187 \text{ at 1 per cent.}}$$

$$\underline{296.748 \text{ at 4 per cent.} = L. 296 14 11\frac{1}{2}.}$$

*Ex. 6.* Drawn on my correspondents at London for 5745*l.*; what premium should I have at  $1\frac{3}{8}$  per cent.?

$$1 \text{ per cent.} = \frac{1}{10} ) 5745$$

$$\begin{array}{rcl} \frac{1}{8} = \frac{1}{4} & - & - \\ \frac{1}{8} = \frac{1}{2} \text{ of that} & 4) 57.45 & \text{at 1 per cent.} \\ & 2) 14.3625 & \text{at } \frac{3}{8} \\ & 7.18125 & \text{at } \frac{1}{8} \end{array}$$

$$\underline{78.99375}$$

#### *Abstracts from the laws of England relating to factorage.*

1. If a factor by his commission is empowered to dispose of his employer's goods as if they were his own, the factor's actions will be excused, though to his employer's loss, unless it can be proved, that, in similiar circumstances, he hath done more for himself than he hath done for his employer.  
*Bulstrode, 103.*

2. A factor cannot sell his employer's goods but for ready money, without a particular order, or commission; in which case, though the purchasers should afterwards become bankrupt, provided they were in good credit at the time of the purchase, and the factor done his duty in recovering payment, he is not accountable, excepting by express stipulation, upon the account of an extra-commission. *Hill, 1708. 2 veru. 638.*

3. If a factor should die before the debts arising from the  
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sales of his employer's goods are recovered, however bad the circumstances of the defunct's affairs may be, the money is to be paid to the employer, after deduction of commission, and not retained by the executors or administrators, to be applied for extinction of any other of the defunct's debts. *Hill, &c.*

4. If a factor give time for payment of money, contracted on the sale of his employer's goods, which being elapsed, and no payment made, he, notwithstanding, sell the debtor goods of his own for ready money, who some time after proves insolvent; the factor, for the sake of his own credit, should indemnify the employer, but he is not compellable by the common-law. *Molloy.*

5. If a factor makes sales, and does not advise his employer, the factor, in that case, is answerable for the consequences. *Lex mer.*

6. If a factor, by commission, purchase goods with his employer's money, or credit, and, on account of a sudden advance, dispose of them for his own behoof; the merchant, in this case, hath action against the factor for damages and a fraud. *Lex mer.*

7. If a factor purchase above the price, or not of the quality limited by his commission, his employer may disclaim the goods, as he may likewise do if they are shipped to another port than he had ordered them; but, in the case of a sudden advance, if the factor fraudulently misconstrue the orders, then the employer hath action of damages. *Lex mer.*

8. If a merchant send goods to a factor for a fund of credit, and in consequence, some time after, a month for instance, draws on him for a certain sum, which the factor, having effects in his hands, accepts; if, before the bill becomes payable, the merchant should be bankrupt, and the goods in the factor's hands seized; in this case, it hath been found, the factor must answer the bill, and sue as a common creditor for what he can recover of the contents. *Molloy,*  
442.

#### O B S E R V A T I O N S.

When a factor hath bought or sold, pursuant to orders, he ought immediately to advise his principal, lest his former orders should be contradicted. Frequency and punctuality in writing,

writing, a proper knowledge of the value, fall, and rise of goods, both at home and abroad, diligence in executing orders, and honesty and punctuality in giving a faithful account, are the true and infallible means of raising and securing the reputation of a factor.

2. When a merchant sends out an invoice to his factor, he generally charges it with 5, 10, or 12 *per cent.* according to circumstances; in one case that the factor may not be acquainted with all the profits, but, particularly, with regard to a supercargo, that he may sell at so much profit upon the invoice.

3. There are no particular laws or regulations, that I have seen, in regard to the price of commission. In Jamaica and in most parts of America it runs about 10 *per cent.* for sales and remittances; in Holland, Italy, &c. and other places nearer home, at  $2\frac{1}{2}$ ; in some places on the continent, 2 *per cent.*

### Variety X. I N S U R A N C E.

**I**Nsurance is a security, or assurance, by means of a writ called a *policy*, to indemnify the insured of such losses as shall be specified in the policy subscribed by the insurer, or insurers, and entered in some particular office, as a testimony or voucher of the transaction, by which the underwriters oblige themselves to make good and effectual the property insured, in consideration of a certain premium, at a stipulated rate *per cent.* which varies according to the risk, to be immediately paid down, or otherwise secured, according to the tenor of the agreement. In case of loss, the underwriters can retain a certain discount, generally 2 *per cent.* and only pay what is called the *short recovery*.

*Case 1.* When the premium at a certain rate *per cent.* for insuring a given sum is required, the operation is the same as in factorage.

*Ex. 1.* Insured at Hamilton's office, the value of 574 l. at  $7\frac{1}{2}$  *per cent.*; what is the premium?

$$\begin{array}{r} 2|0)57|4 \\ \hline \end{array}$$

2) 28.7 for 5 per cent.

14.35 for  $2\frac{1}{2}$  per cent.

43.05 for  $7\frac{1}{2}$  = L. 43, 1 s.

*Ex. 2.* Insured with Dunlop, Glen, and Peters, the value of 3768 l. at  $14\frac{1}{2}$  per cent.; what is the premium?

$$\begin{array}{r} 10)3768 \\ \hline \end{array}$$

5) 376.8 for 10 per cent.

75.36 for 2 per cent.

4) 75.36 for 2 per cent.

18.84 for  $\frac{1}{3}$  per cent.

546.36 for  $14\frac{1}{2}$  per cent. = L. 546 : 7 :  $2\frac{2}{3}$ .

*Ex. 3.* Insured at the Sun-fire office, on my tenement in the Gallowgate, the value of 748 l. 19 s.; what sum must I pay yearly, at 2 s. per 100 l. to keep the policy in force?

$$\begin{array}{r} L. 748.95 \\ \cdot 4 \\ \hline \end{array}$$

$$\begin{array}{r} .74895 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 14.97900 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 11.74800 \\ \hline \end{array}$$

*Cafe 2.* When the short recovery is required, find the discount as the premium was found in the last case, subtract it from the given sum, and the remainder is the short recovery.

*Exam. 1.* Discount is allowed at 2 per cent. in recovering L. 1000 insured, what is the short recovery?

$$\begin{array}{r} 50) 1000 \\ 20 \text{ discount.} \\ \hline \end{array}$$

980 short recovery.

*Exam.*

*Exam. 2.* Discount is allowed at  $2\frac{1}{2}$  per cent. in recovering L. 500 insured, what is the short recovery?

40) 500  
12 10 discount.

487 10 short recovery.

*Cafe 3.* To find the sum necessary to be insured, when the adventurer would cover, or make good his outset in case of loss; that is, to recover from the underwriters the whole value at risk; to the rate *per cent.* premium add the ordinary discount, and subtract that sum from L. 100 for the first term, let L. 100 be the second, and the value at risk the third, then will the fourth proportional be the answer.

*Exam. 1.* It is required to cover L. 500, premium 8 per cent. and 2 per cent. discount in case of loss?

$$\begin{array}{r} L. \\ 100 - 8 + 2 = 90 : 100 :: 500 \\ \hline 100 \\ 9 | 0 ) \overline{5000 | 0} \\ \hline \end{array}$$

*Note.* It is plain, that when we want to recover L. 90, we must in this case insure L. 100; therefore to recover L. 500, we must insure L. 555, 11*s.* 1*½**d.*: for when 10 per cent. for discount and premium is deducted, we shall have L. 500 remaining nett.

For 555 II  $1\frac{1}{2}$  insured at 8 per cent.  
44 8  $10\frac{2}{3}$  premium to be deducted.

**511** 2 2 $\frac{2}{3}$  remains.  
**11** 2 2 $\frac{2}{3}$  2 per cent. discount to be deducted.  


---

  
**500** 0 0 first outfit.

*Exam. 2.* It is required to cover L. 575, premium  $14\frac{1}{2}$  per cent. 2 per cent. discount in case of loss?

$$\frac{100}{14\frac{1}{2}+2=16.5}$$

$$83.5 : 100 :: 575$$

100

$$83.5) \underline{57500.0} \quad (681.43 \text{ £c.}$$

5070

$$\begin{array}{r} 6800 \\ - 6680 \\ \hline \end{array}$$

$$\begin{array}{r} 1200 \\ - 835 \\ \hline \end{array}$$

$$\begin{array}{r} 3650 \\ - 3340 \\ \hline \end{array}$$

$$\begin{array}{r} 3100 \\ - 2505 \\ \hline \end{array}$$

595

*Cafe 4.* To find the amount of insurance of a voyage out and home. Deduct the premium from the short recovery; then say, As the remainder is to the premium, so is the amount of the first insurance, together with the outset, to the insurance of the voyage home, which being added to the insurance outwards, makes the total insurance.

In the first example of the last case the premium was

$$= L. 44 \ 8 \ 10\frac{2}{3}$$

500 0 0 outset.

$$90 : 8 :: 544 \ 8 \ 10\frac{2}{3}$$

4

$$5) \underline{2177 \ 15 \ 6\frac{2}{3}}$$

$$9) \underline{435 \ 11 \ 1\frac{2}{3}}$$

$$\begin{array}{r} \text{Insurance home,} \quad 48 \ 7 \ 10\frac{1}{3} \\ \text{Insurance out,} \quad 44 \ 8 \ 10\frac{2}{3} \\ \hline \end{array}$$

$$\text{Total insurance,} \quad 92 \ 16 \ 9$$

To

To find what will cover this sum homewards, it will be as in the last case.

	<i>L.</i>	<i>s.</i>	<i>d.</i>
90 : 100 ::	544	8	$10\frac{2}{3}$
			10
	9)	5444	8 8
			—

*L. 604 18 8 $\frac{2}{3}$*  to be insured home.

After the same manner may the insurance on a triple voyage be computed, by making another operation founded on the foregoing.

	<i>L.</i>	<i>s.</i>	<i>d.</i>
Thus 9 : 8 ::	592	16	9
			4
	5)	2371	7 0
			—
	9)	474	5 4 $\frac{4}{5}$
			—
	52	13	$11\frac{2}{3}$
	92	16	9
			outwards to different ports.
			—
	145	10	$8\frac{2}{3}$
			total insurance.

#### *Cases of insurances, and the method of settling them.*

*Cafe 1.* Sir Alexander Grant *contra* William Innes, settled at London 15th of May 1759.

The ship the Prince of Orange being fitted out and victualled at Waterford, was sent to Jamaica to take in her cargo, which she accordingly did, and on the 2d December 1757 sailed from Montego bay in the said island, without convoy, and on the 2d February 1758 was taken by a French privateer.

Between the 11th and 13th of the same February, the plaintiff caused to be insured by different persons the sum of *L. 1600*, at and from Jamaica to London upon the said ship and freight, beginning the adventure from, and immediately following the ship's arrival at Jamaica, and so to continue until she should arrive at London. About the 12th February 1758 the ship was retaken by a British privateer, called the Britannia, Capt. Dobson, who became intitled to half the nett proceeds; the plaintiff applied to the recaptor's agent, and desired he might have the direction of selling the ship and

and cargo for the benefit of the concerned, but was refused; whereupon two brokers were mutually agreed to by all parties to sell and dispose of the ship and her cargo.

The ship and cargo were accordingly sold by public auction, with the mutual consent of the plaintiff, insurers, and recaptors, and one half of the nett proceeds was paid to the plaintiff, and the other half to the recaptor; and in the account delivered by the broker, the plaintiff was allowed for the ship's freight 1322*l.* 8*s.* 6*d.*; in which event, though he had formerly declared for a total loss, he now only claimed a partial or average loss.—The ship and her cargo being thus sold by public auction, the plaintiff delivered in all his vouchers to the policy-broker, to make up the account, and state the loss for the examination of the insurers; and it was accordingly stated as follows.

	L. s. d.	L. s. d.
The Prince of Orange cost at private sale - - - - -	550 0 0	
Amount of the outset by the ship's books, - - - - -	522 2 2	
Premium of L. 1155 to cover the above from London to Waterford, at 5 guineas per cent. - - - - -	60 17 3	1132 19 5
Provision for the ship's use at ditto, - - - - -	110 9 1	1243 8 6
Deduct freight from London to Waterford, - - - - -	121 0 0	1122 8 6
Deduct return of 2 <i>1</i> / <i>2</i> per cent. commission on L. 1155, - - - - -	28 17 6	1093 11 0
Premium on L. 1190 from Ireland to Jamaica, at 6 guineas per cent. to cover L. 1093, 11 <i>s.</i> - - - - -	75 4 0	1168 15 0
Deduct freight from Ireland to Jamaica, - - - - -	290 14 0	878 1 0
Premium on L. 1320 from Jamaica to London, to cover L. 878, 1 <i>s.</i> at 30 guineas per cent. - - - - -	416 0 6	1294 1 6
Total amount of the freight,	<u>1322 8 6</u>	

	L.	s.	d.	L.	s.	d.
Brought over, - -	1322	8	6			
Deduct the portage-bill and charges in the river, which may be about - - - -	422	8	6			
	900	0	0			
Premium of L. 1350 to cover L. 900 at 30 guineas per cent. - -	425	5	0	1325	5	0
So that there might be insured on the ship, - - - -	1320	0	0			
And on the freight, - - - -	1350	0	0			
In all, - - - -	2670	0	0			
Half the nett proceeds of the ship and freight, - - - -	- - - -	940	11	1		
From which deduct half the crew's wages home, - - - -	- - - -	138	11	3		
Nett, 2670 at 98 per cent. L. 2616 12 0				801	19	10
Received - - - -	801	19	10			
	1814	12	2			

Then 2670 : 1814 :: 100 : L. 67-18-9, being the average-loss claimed by the plaintiff.

#### DEFENDANT'S OBJECTIONS.

It is understood, that the practice among merchants is, to insure no more on ship and freight than the sum it would require to bring her on an equality, whether she arrive or not; but when neither are valued, to ascertain whether a merchant chuses to have a profit in view by her arrival or capture, and for which, when valued, he pays a certain premium, it seems natural, that the insured means only to be secured and reinstated in his real property, into which the insurers have a right to inquire, and to see that the account is properly stated. And it must be owned, that the wear of a ship greatly reduces her value, which the freight again answers, but with heavy charges; therefore ship and freight are the same; or, if they are separated, it depends much on fancy; wherefore, in this case, the owners should only have insured as under.

	L. s. d.
The value of the ship at Jamaica being	L. 878 1 0
Required only insurance on L. 1350 on ship } and freight, at 30 guineas <i>per cent.</i> - - - } 425 5 0	<hr/> <hr/>
	1303 6 0

If a total loss had happened, that L. 1350 secured the owners at 98 *per cent.* in L. 1323.

	L. s. d.	L. s. d.
As she was taken, &c. they received for half ship and freight, - - - - -	801 19 10	<hr/>

501 6 2

Then 1350 : 501 6 2 :: 100 : 37 2, which average-loss was offered to be paid by the defendant.

Some urge it to be the custom to insure nett freight, as well as the value of a ship; and it is too general a custom also to mistake what nett freight actually is, or rather impossible to calculate exactly, what a ship and freight will yield in the middle of a voyage: therefore it is incumbent on an owner to please himself, and declare it by a valuation of both; but if he does not, and leaves all open, it is to be presumed he means only to secure his real property, and insurers can never be liable to pay the loss on an accidental or imaginary profit, in case of a total loss; while the assured, in the event of a safe arrival, and that by heavy charges, such as thirty guineas to sailors for the run home, his supposed nett freight, or profit, does not yield to his expectation, can have recourse upon the insurers for a return of premium, by complaining of his mistake after the ship's arrival. Wherefore all insurances left open, and not valued, are intended only to secure the real original property. But as there are facts in the present case, let it be supposed, the assured or owner meant to make profit, which is granting a contradiction on an unvalued policy; by insuring his freight at a high rate, his claim for return of premiums would be founded on the following state, by proving his interest at the highest, it now turns out to be at the end of the voyage.

Suppose L. 2670 had been insured without valuing the ship and freight, and the ship arrived safe, the assured could have demanded a return of premium, by stating his account thus.

Amount

	L.	s.	d.
Amount of the ship's freight, - - - -	1322	8	6
Ditto of ship sold at - - - -	720	0	0
Total value, - - - -	2042	8	6

Cost at Jamaica, - - - -	L.	878	1	0
Premium on L. 1350 ship and freight, 425	5	0		
Portage bill, - - - -	438	8	9	
				1741 14 9

Nett freight or profit to be insured at L. 286, 14s. to bring it home, - - - - -	300	13	9
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Insurers debtors to premium of L. 2670, at 30 guineas, - - - - -	841	1	0
By insurance on L. 1350 0 0 425 5 0			
By d° on nett freight, 286 14 0 90 6 3			
	515	11	3
Sum over-insured, 1636 14 0 - - - - -	325	9	9
1033 6 0 - - - - -			
L. 2670 0 0 - - - - -	841	1	0

The above nett freight at Jamaica, - - - -	300	13	9
Deduct cost of insurance on L. 286, 14s. - - -	90	6	3

When brought home to London yields nett - - 210 7 6

*Note.* Instead of adding the premium to a supposed profit on ship and freight at Jamaica, the premium thereof must always be deducted, and less insured to make it a real value.

### P R O O F.

If she had arrived safe,	L.	s.	d.
She sold at, or was worth - - - - -	720	0	0
Her freight turned out at - - - - -	1322	8	6
	2042	8	6

Cost at Jamaica, - - - - -	L.	878	1	0
Insurance on L. 1638, 14s. - - -	515	11	3	
Portage-bill, - - - - -	438	8	9	
	1832	1	0	
Nett profit, - - - - -	210	7	6	

H h 2

If

L. s. d.

If there had been a total loss,

L. 1636, 14 <i>s.</i> at 98 <i>per cent.</i>	would pay	-	1603 19 9
Cost at Jamaica, - - -	L. 878	1 0	
Insurance premium, - - -	515 11 3		
			1393 12 3

Nett profit, - - -	210 7 6
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As she was taken and retaken,

L. 1636, 14 <i>s.</i> at 98 <i>per cent.</i>	- - -	1603 19 9
By nett proceeds of half ship and freight, -		801 19 10

The insurers must then have paid	- - -	801 19 11
Nett proceeds of half ship and freight, -		801 19 10

Cost at Jamaica, - - -	L. 878	1 0	1603 19 9
Ditto premium, - - -	515 11 3		1393 12 3

Nett profit, - - -	210 7 6
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*Note.* The underwriter in this calculation does not study the minutest exactness, but slumps small things to bring out his proof.

As certainly every man means his true interest in all his operations in trade, therefore the above state proves proper calculations; as whether a safe arrival, a total loss, or a capture and re-capture had happened, the profit, or nett freight, turns out equally the same. Wherefore, in all insurances of ship and freight, both, or either, left general and not valued, the assured can calculate in the above manner for a return of premium; as also the insurers from paying too much. If the insurers were to pay, respecting the case in question, on the imaginary sum of 2670 *l.* the assured would not only gain L. 896 : 13 : 4, by the capture and recapture, an accident, but saves also L. 115 : 6 : 9, they would have lost had the ship arrived safe, without the least chance of a reciprocal advantage to the insurers by any accident.— If custom is pleaded to a contrary practice respecting freight, it hath no relation to an insurance on ship and freight; because, if a high valuation is made and insured on the one, so much the less

less ought to be done on the other. This cause coming on to be heard, and the plaintiff's witnesses proving, that it was customary in the city of London for an owner of a ship to cover his interest both in ship and freight, by insuring the ship and freight, and the premium paid for insuring the same, the underwriter taking the premium, in case the ship arrives safe, the jury gave a verdict for the plaintiff.

*Cunningham's law of bills, &c.*

*Case 2.* Shewing how a particular average was calculated at Hamburg.

Assurance made for account of Mr C—s C—o de C—o, from Lisbon hither, for the sum of 3000 marks Banco, upon 10 chests sugar, shipped by him on board the Antonetta galley, Capt. Thormay, which, according to the bill of lading and invoice, amount to 826 and 512 mil. and are valued in the policy at 4 marks per milree, without premium,

Is, merks, - - - - -	3306	I
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Premium, at $7\frac{1}{2}$ per cent. - - - - -	239	II
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Premium upon ditto, at $7\frac{1}{2}$ per cent. - - -	17	5
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Marks, 3563	I
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Whereupon hath been insured, marks, 3000

and left uninsured, - - - - -	563	I
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3563	I
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The above ten chests have, as per invoice, weighed in the country neat 375 arrobes, 22 lb. which at 30 lb. per arrobe, make 11,272 lbs. of our weight, and amount, at  $9\frac{1}{2}$  d. with  $8\frac{1}{2}$  per cent. discount, to marks 2998 - 7

But as the above ten chests proved damaged by sea-water, they were sold, by order of the insurers, to the best bidders, and produced marks - - - - - 2514 2

Deduce $8\frac{1}{2}$ per cent. discount, - - - - -	200	8
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2313	10
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Which being deducted from - - - - -	2998	7
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Makes the loss - - - - -	684	13
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To which must be added for extra-charges,

To the captain, for general average of marks

2500, at marks 4: 3: 6, Marks, 105	8
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Carried forward, - - - - -	790	5
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Brought forward,	- - -	790	5
To the notary, for attestations, &c.	- - -	4	8
To brokerage of 5 chests, sold at public sale,	- - -	7	8
Cooperage and porterage,	- - -	2	8
			<hr/>
		Marks, 804	13
Deduct agio of marks 120, at 130 per cent.	- - -	27	11
		<hr/>	

The whole loss - - - - - 777 2  
Which being divided on the value of 300 marks banc,  
affects every 100 marks with marks 25 - 14 - 6.

This account was not objected to, and shows the equity  
of the insurers; who might have stated as under:

5 chests of sugar damaged, weighed at Lisbon, as per in- voice, 186 arr. 22 lb. which, at 30 lb. per arr. make in Hamburg 5602, at 9½ per cent.	Marks, 1621	1
Deduct 8½ per cent. discount	129	4
		<hr/>
		1491 13

*Note.* When tare-allowance for good weight and sweepings  
is deducted, an arroba of sugar from Lisbon will produce  
only 28½ lb. Hamburg.

CHARGES.	Marks.	
Freight, 14 marks per chest,	70	0
Ordinary average	7	0
Custom, at 18½ spetus per chest, with 30 per cent. agio,	7	8
Primage and stader-duty	5	10
Admiralty of marks 700, with 15 per cent. agio,	8	1
Lighterage, porterage, and cooperage,	3	12
Brokerage at 1½	7	8
		<hr/>
Marks 109 - 7, at 130 per cent. agio,	84	4
Commission, at 2 per cent.	29	13
		<hr/>
Had there been no average, they would have produced	114	1
But, as the same were damaged, they yielded but	1377	12
Deduct 8½ per cent.	978	1
	78	0
		<hr/>
	900	1
Carried forward,	1377	12

Brought forward,	- -	1377	12
Ordinary charges, as above,	84	4	
General average,	- - -	105	8
Notary and commission,	-	22	8
	—	212	4
	—	687	13
		689	15

Consequently they yielded less.

Which being proportioned upon the above-mentioned sum of marks 3563-1, according to the valuation in the policy, but which, by an exact computation, comes to marks 3564-8. Hence  $19\frac{7}{10}$  marks must be paid *per cent.*

Wherefore there must be paid by the underwri-

ters on 3000 marks insured,	- - -	580	8
By the proprietors, for what was not insured, <i>viz.</i>	- - -	564	8
		109	4
	—	—	—
		3564	8
		689	12

*Case 3.* Shewing how an account was settled where the ship and cargo was ransomed, having been partly insured at different offices, and partly risked by the proprietors.

A. B. freights a ship out and in for a voyage to the West Indies, and by the charter-party was obliged to make payment to the owner of the ship in a certain sum for every gallon of rum or cwt of sugar she carried inwards, provided she landed safe, and likewise to fill her quite full of one or both these species of goods.

A. B. accordingly freights her outward with goods to the value of	- - -	L. 1963	16	6
Takes goods from C at a certain freight, valued at	102	4	2	
from D at ditto to the value of	-	101	15	1
and the ship was valued at	- -	512	4	6
	—	—	—	—
Total value,	- -	2680	0	3

Of this cargo A. B. covers at London at 20 guineas *per cent.*

And covered and risked together the remainder at 12 guineas *per cent.* at Glasgow and Edinburgh,

The value of the ship with C and D's goods were likewise covered at the same rate,

Total, 2680 0 3  
The

The ship proceeds on her voyage outwards, and was unluckily taken by a French privateer, who returned her with the cargo for a ransom of 1409*l.* 15*s.* 4*d.* When the ransom came to be settled, it was pled by the underwriters, that A. B. being a proprietor for the freight, he ought accordingly to be liable in a proportion of the ransom agreeable thereto; but when the case was advised, it was found, that as A. B. paid no freight for goods shipped outward, there was no real property to insure, nor could it be a charge upon the cargo; that insurances are only intended to procure to the proprietors of ships and cargoes their original cost, and no more can be secured, unless by a special agreement with the underwriters specified in the policy, in which case only profits may be insured. For had the freight been payable in the West Indies, there was something in this case risked; but as there was no freight payable but in the event of the ship's arrival at Clyde, there was surely in this case no insurance to be made by A. B.; wherefore the loss was calculated and charged as under, which was admitted to be right by all parties.

To cover *L.* 231 at London, premium 20 guineas, 2 per cent. discount,

77 : 100 :: 231 : 300 covered property.

To cover the remainder at Glasgow, 12 guineas premium, discount 2 per cent.

<i>L.</i> <i>s.</i>	<i>L.</i> <i>s.</i> <i>d.</i>	<i>L.</i> <i>s.</i> <i>d.</i>
85 8	100	1732 16 6
		: 2029 1 4 $\frac{3}{4}$ covered property at Glasgow.

Then to cover the ship and the remainder of the cargo.

<i>L.</i> <i>s.</i>	<i>L.</i> <i>s.</i> <i>d.</i>	<i>L.</i> <i>s.</i> <i>a.</i>
85 8	100	102 4 2
		: 119 13 7 $\frac{1}{3}$ C's cov'd property.
85 8	100	101 15 1
		: 119 2 11 $\frac{3}{4}$ D's cov'd property.
85 8	100	512 4 6
		: 599 15 10 $\frac{3}{4}$ value of ship cov'd.

<i>L.</i> <i>s.</i>	<i>L.</i> <i>s.</i> <i>d.</i>
A. B. ought to have insured at Glasgow and Edinburgh, - - -	2029 1 4 $\frac{3}{4}$

And he insured at London - - -	300 0 0
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In all, - - -	2329 1 4 $\frac{3}{4}$
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Of this sum he only insured, - - -	2100 0 0
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So that he stands underwriter for - - -	229 1 4 $\frac{3}{4}$
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Hence

	L.	s.	d.	L.	s.	d.
Hence A's underwriters at London are accountable for -	300	0	0			
At Glasgow and Edinburgh for	1800	0	0			
He is himself accountable for -	229	1	4 $\frac{1}{4}$			
	2329	1	4 $\frac{3}{4}$			
C, or his underwriters, are accountable for -	119	13	7 $\frac{1}{2}$			
D, or his underwriters, for -	119	2	1 $\frac{3}{4}$			
The proprietors of the ship, or underwriters, for	599	15	10 $\frac{3}{4}$			
Total covered property, -	3167	13	10 $\frac{3}{4}$			

To proportion the ransom according to the above state,

covered property,	loss.	cov. p.	loss.
First, 3167 13 10 $\frac{3}{4}$ : 1409 15 4 :: 100 : 44 10 1 per cent.			
Then, L. 300 insured at London must pay, L. 133 10 3			
1800 insured at Edin. and Glasgow d°, 801 1 6			
229 1 4 $\frac{1}{4}$ risked by A. B. ditto, 101 18 10 $\frac{3}{4}$			
119 13 7 $\frac{1}{2}$ C's covered property d°, 53 5 2 $\frac{1}{2}$			
119 2 1 $\frac{3}{4}$ D's ditto ditto, - - 53 1 3 $\frac{1}{4}$			
599 15 10 $\frac{3}{4}$ ship ditto ditto, - - 266 18 2 $\frac{1}{2}$			
3167 13 10 $\frac{3}{4}$	Proof - -	1409 15 4	

*Case 4.* Shewing in what manner the owners of goods contributed to the charges of salvage on the amount of their value, at the place where they were landed, more than the sum insured on them.

Insurance was made in London with the London-assurance company for the account of a merchant in Bourdeaux of L. 1000 *Sterling*, viz.

L. 600 upon  $\frac{1}{2}$  of the ship L'Orphée,  
400 upon  $\frac{1}{4}$  of the cargo from Bourdeaux to C. Françoise.

Not far distant from Cape Françoise the ship was chased ashore by an English privateer, and several of the goods plundered; the first cost whereof, according to the account delivered, amounted to 12477 livres, 16 sols, 11 deniers, and was included in the whole cargo, which the assured's agent at London valued at 31675 liv. 12 sols, 9 den. whereof  $\frac{1}{4}$  included in the assurance is 7918 liv. 18 sols, 3 den. exchange at 33 d. per ecu,  
Premium at 15 per cent. - - - - - L. 362 19 1  
54 9 0

	L. s. d.
Brought forward,	417 8 1
L. s. d.	
And $\frac{1}{4}$ of what was plundered	3119 9 3, at $\frac{3}{4} 32\frac{1}{2} d.$
Prémium at 15 per cent.	21 2 4
Commission of the insurance $\frac{1}{2}$ per cent.	161 18 6
Lofs,	162 14 8
The ship's value at L. 2400 Sterl. at $32\frac{1}{2} d.$ per ecu, makes	$\left\{ \begin{array}{l} 53169 4 6 \\ - - - \end{array} \right.$
Being damaged was sold at Cape Françoise for	$\left\{ \begin{array}{l} Liv. \\ 16510 0 \end{array} \right.$
Deduct 25 per cent. for differ. in the money,	$\left\{ \begin{array}{l} 4127 10 \\ - - - \\ 12382 10 0 \end{array} \right.$
Whereof the $\frac{1}{4}$ at $32\frac{1}{2} d.$ per ecu makes the losf	$\left\{ \begin{array}{l} - - - \\ 460 3 9 \end{array} \right.$
Paid in all for salvage upon the ship and cargo per account,	$\left\{ \begin{array}{l} 17302 17 4 \\ - - - \end{array} \right.$
Whereof was deducted $\frac{1}{8}$ ,	$\left\{ \begin{array}{l} 2162 17 4 \\ - - - \end{array} \right.$
And for difference in the money,	$\left\{ \begin{array}{l} 1540 0 0 \\ 3785 0 0 \\ - - - \end{array} \right.$
$\frac{1}{4}$ whereof at $32\frac{1}{2} d.$ per ecu makes of losf,	$\left\{ \begin{array}{l} - - - \\ 128 2 9 \end{array} \right.$
Total losf,	751 1 2

Which was demanded from the said company by the assured's agent. But it was objected, that granting the amount of the first cost of the goods to be right as stated, at 12477 *liv.* 16 *s.* 11 *d.* yet it appeared that they really produced 55620 *liv.* from a certificate of the goods saved, and account of sales at Cape Françoise; and though they did not pretend to any of the profit, yet they were of opinion that the proprietors of those goods ought to contribute towards the charges of salvage, as they had rendered more than the sum insured on them. The company therefore made up the account in the following manner.

Per

*Per account of sales, and invoice,*

	Sold for	Cost
49 casks of bacon, <i>Liv.</i> 2017		<i>Liv.</i> 781 1 6
131 red wine	25798 13 8	6130 11 7
5 filled up,		
28 white wine,	1948 2 9½	480 3 0
1½ filled up,		
85 pickled meat,	4745	3407 16 0
13 pots butter,	441	180 14 4
2 casks nails,	445	171 0 0
25 bundles pipe-staves,	480	137 8 0
3 casks tallow,	230	82 15 0
2 ditto pitch,	600	50 0 0
3 ditto vinegar,	49	15 6 6
321 ditto flour,	17271	6855 6 3
6 ditto oat-meal,	971	417 15 6
3 ditto brandy,	60	27 5 8
24 deals,	100	107 5 0
7 casks lead in bullets,	465	243 7 0
	<hr/> 55620	<hr/> 19089 2 4
The ship,	<hr/> 16510	
	<hr/> 72130	

To cover 19089 *liv.* 2 *sols*, 4 *den.* original cost at 15½ per cent. to pay 98 per cent. in case of loss, it will be

82 10 : 100 :: 19089 2 4 : 23138 to be insured.

Premium of 23138 at 15½ per cent. - *Liv.* 3586 0 0  
 Prime cost as above, - - - 19089 2 4

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22675 2 4

For difference in the money add 25 per cent. 

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7525 0 0

Cost at Cape Françoise, - - - 30200 2 4

Upon the above-mentioned produce at the sale of 72130 *liv.* are to be proportioned the charges of salvage, being *Liv.* 17302  
 And to deduct  $\frac{1}{4}$  instead of  $\frac{1}{3}$ , by the former account, - - - 4325

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12977

I i 2

Which

Which chargeable on the following particulars make,

For 30200 the cost of the goods with the premium, <i>Liv.</i> 5433				
16510 the proceeds of the ship, - - - -	2971			
25420 profit on the goods, - - - -	4573			
<hr/>				
72130	12977			

And the loss wherewith the insurers were chargeable amounts to as follows :

The whole cost, - - - -	<i>Liv.</i> 31675 12 9			
Premium of 38394 <i>liv.</i> at 15½ per cent. 5951 0 3				
<hr/>				

37626 13 0

The whole loss, - - - -	<i>Liv.</i> 12477 17			
Premium of 15124 <i>liv.</i> to cover the above at 15½ per cent. - - - -	{ 2344 3			
Charges of salvage, - - - -	5433 {			
Deduct, - - - -	1358 } - 4075 0			
<hr/>				

So that the loss on the cargo will be about 50½ per c. 18897 0

Upon the ship valued at L. 2400 <i>Sterling</i> , exchange at 32½ d. per ecu, - - - -	<i>Liv.</i> 53169 4			
<hr/>				

Which was sold for - - - -	16510 0			
Deduct for charges of salvage, - - - -	2971 0			
<hr/>				

And for difference of money 25 per cent. - -	13539 0			
<hr/>				
3385 0				

Loss to be paid on the ship 80½ per cent. - -	10154 0			
<hr/>				
43015 4				

53169 4

Upon this state the account was settled, and the London-assurance company did accordingly pay,

For 600 <i>l.</i> <i>Ster.</i> insured on the ship at 80½ per C. L. 485 8 9				
For 400 ditto insured on the cargo at 50½ per cent. 200 16 0				

1000	686 4 9			
Which comes short of the former state, - -	61 16 5			

Amount by the agent's state, - - - -	748 1 2			
<hr/>				

Cafe

*Cafe 5.* Showing in what manner three different premiums, at three different offices, where there was a general average, a return for convoy, and an over-insurance, were settled.

A. B. merchant in Glasgow gives orders for shipping £. 2027, 5s. Sterl. value of goods on board the Caledonia, C—— master, bound from Liverpool for Cork and Antigua; in consequence whereof he makes insurance on the interest he expected to have on board, in the following manner: He insures at Glasgow 550 £. at 15 £. to return 8 per cent. if the Caledonia departed from Cork with West-India convoy: he insures on the same interest at Edinburgh 950 £. at 20 guineas per cent. to return 12 £. per cent. for convoy from Cork; and at London he insures on the same interest 1150 £. at 25 guineas per cent. to return 15 £. for convoy from Cork: which several insurances fully covered the interest he expected was shipped on board. But after these insurances were finished, he is informed, that, through an unforeseen event, such as the vessel being full, and not able to take on board the whole, that there was only shipped on board 1463 £. 15s. 4d. Sterl. of value, exclusive of premium of insurance, by which means there is an over-insurance. The vessel proceeds on her voyage, arrives at Cork, the master takes sailing-orders from the captain of the man of war, and, in consequence of signals from thence, weighs anchor, is underway; but unfortunately runs aground on a sand-bank, by which the vessel is hurt, loses the benefit of the convoy, returns to Cork, where the cargo is unloaded in order to repair the ship's damage, and afterwards the cargo is reshipped, and the vessel proceeds on her voyage, and arrives safe; but there is a general average-loss to be paid by the ship and cargo, and A. B.'s proportion thereof is 21 £. 13s. 4d. Irish, to be paid by his underwriters.

Two questions arise from this cafe.

1<sup>mo</sup>, How much is A. B. owing to each of the insurance-brokers at Glasgow, at Edinburgh, and London, for the premium, allowing  $\frac{1}{2}$  per cent. for the over-insurance?

2<sup>do</sup>, What proportion of the 21 £. 13s. 4d. Irish of general average is to be paid by the underwriters at the above three offices in British money?

	L.	s.	d.	L.	s.	d.
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Insured at Glasgow at 15 per cent. - - - 550 0 0  
Deduct premium and 2 per cent. discount, - 93 10 0  
Remains the uncovered supposed property, - - - 456 10 0 Glasg.

Insured at Edinburgh at 20 guineas per cent. - 950 0 0  
Deduct 23 per cent. - 218 10 0  
Remains the uncovered supposed property, - - - 731 10 0 Edinb.

Insured at London at 25 guineas per cent. - 1150 0 0  
Deduct 28 $\frac{1}{4}$  per cent. - 324 17 6  
Remains the uncovered supposed property, - - - 825 2 6 Lond.

Total supposed property uncovered, 2013 2 6

To proportion the supposed property to the real, it will be

*supposed.*      *real.*      *supposed.*      *real.*

2013 2 6 : 1463 15 4 :: 456 10 0 : 331 18 7 at Glasg.  
2013 2 6 : 1463 15 4 :: 731 10 0 : 531 17 7 at Edin.  
2013 2 6 : 1463 15 4 :: 825 2 6 : 599 19 2 at Lond.

To cover these proportions at the different offices.

1st, 83 : 100 :: 331 18 7 : 399 18 4 at Glasgow.  
2d, 77 : 100 :: 531 17 7 : 690 15 1 at Edinburgh.  
3d, 71 $\frac{3}{4}$  : 100 :: 599 19 2 : 836 3 6 at London.

To find the over-insurance.

	L.	s.	d.		L.	s.	d.
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There was insured at Glasgow, 550 0 0  
There ought to have been insured but 399 18 4

Over-insured at Glasgow, - - - 150 1 8

There was insured at Edinburgh, 950 0 0  
There ought to have been insured but 690 15 1

Over-insured at Edinburgh, - - - 259 4 11

There

	L.	s.	d.	L.	s.	d.
There was insured at London,	1150	0	0			
There ought to have been insured but	836	3	6			
Over-insured at London,	-	-	-	313	16	6

L. 21 13 4 Irish is at par L. 20.

To proportion which,

53 : 20 :: 11 : 4	3 0	chargeable at Glasgow.
53 : 20 :: 19 : 7	3 4 $\frac{3}{4}$	at Edinburgh.
53 : 20 :: 23 : 8	13 7 $\frac{1}{4}$	at London.

L. 20 0 0 Proof.

Hence the account must be stated as under.

	L.	s.	d.
Premium of 550 l. insured at Glasg. at 15 per cent.	82	10	0
$\frac{1}{2}$ per cent. on 150 l. 1 s. 8 d. over-insured,	-	0	15
Gross debit, - - -	83	5	0
Ded. 8 per cent. returns for convoy, L. 44 0 0			
Return of premium 7 per cent. for			
150 l. 1 s. 8 d. - - - - -	10	10	1 $\frac{1}{2}$
Proportion of average, - - -	4	3	0
	58	13	1 $\frac{1}{2}$
Nett debit at Glasgow, - - -	24	11	10 $\frac{1}{2}$
Premium of L. 950 at 21 per cent. insured at			
Edinburgh, - - - - -	199	10	0
$\frac{1}{2}$ per cent. on 259 l. 4 s. 11 d. over-insured,	1	5	11
	200	15	11
Ded. 12 per cent. returns for convoy, L. 114 0 0			
9 per cent. for L. 259 4 11 over-insured, 23 6 7 $\frac{1}{2}$			
Proportion of average, - - -	7	3	4 $\frac{3}{4}$
	144	10	0 $\frac{1}{4}$
Nett debit at Edinburgh, - - -	56	5	10 $\frac{3}{4}$

Premium

	L. s. d.
Premium of L. 1150 at $26\frac{1}{4}$ per cent. insured at London,	301 17 6
$\frac{1}{2}$ per cent. on L. 313 16 6 over-insured,	<u>1 11 4\frac{1}{2}</u>
	303 8 10\frac{1}{2}
Deduct 15 per cent. of L. 1150 for convoy,	L. 172 10 0
$11\frac{1}{4}$ per cent. of L. 313 16 6 over-insured,	35 6 1
Proportion of average-loss,	<u>8 13 7\frac{1}{2}</u>
	216 9 8\frac{1}{2}
Nett debit at London,	- - - 86 19 2

*Case 6.* Average-loss on 474 hhds tobacco insured on the Peggy, from Virginia, on account of A. B. valued at L. 8 per hhd wet tobacco, 7 per cent. premium.

When the hhds were examined, and the contents proved, it was found, that 599 cwt and 69 lb. or 67157 lb. were defaced, and that 22392 lb. were entirely lost, being distinguished and ascertained in a particular account made out for that effect. Hence for the loss it was stated:

	L. s. d.
If 1200 lb. wet tobacco give L. 8, insured per hhd, 22392 lb. must in the same proportion be charged with	149 5 7
67157 lb. cut or defaced, charged at $\frac{1}{4}$ per lb. for defacing,	69 19 1
	219 4 8
Deduct 2 per cent.	- 4 7 8
	214 17 0

For the average loss per cent.

474 hhds.  
at 8 L.

If L. 3792 is to be charged with L. 214, 17 s. loss, L. 100 must be charged with L. 5, 13 s. 4 d.

## OBSERVATIONS.

*Of policies of insurance, and decisions concerning them.*

What a policy of insurance is, and the purpose for which insurances are intended, hath been already taken notice of. The essentials that are to be expressed in a policy, or contract of insurance, are in general :

1. The person's name for whom the insurance is made, to which this unlimited declaration is commonly added, " for " his own, or any other person's account."
2. The goods, or ships, or their value, or, in short, the thing itself upon which the insurance is made.
3. The name of the place where the goods are shipped, whither bound, the time when the risk commences, and when it terminates.
4. All the different kinds of dangers which the insurer takes upon him.
5. The consideration or premium received, or to be received, together with the name of the broker employed in the transaction.
6. How much shall be paid in case of loss, and when due.
7. The day and year in which the contract or policy is executed.
8. The intelligence or advices relative to the adventure, at the time of making the contract or policy.

*Who may insure.*

1. Any person, who, by the laws of his country, hath a right to dispose of his property, may insure ; but in some places, where ordinances relating to insurances are in force, there are exceptions, particularly concerning those who are employed either in the management or direction of insurances, as insurance-brokers, commissioners, and secretaries of any chambers or tribunals for judging of differences that may arise in this branch of business, since they ought all to be men strictly impartial. In England and Sweden brokers alone are excluded from signing or underwriting policies. By all the ancient ordinances, and some modern ones, relating to insurances, the insured are particularly enjoined to run part of the risk themselves, as the best means of avoiding frauds ; yet we find that such injunctions are commonly evaded, or, at least, very seldom long complied with. The custom of overlooking, or dispensing with the disposition of the law in

those respects, hath crept in every where; and when a judge, in cases of dispute, finds no visible fraud committed, he cannot well act otherwise, than to oblige the merchants to adhere to, and fulfil their contracts, as the best way of promoting credit and trade in general.

2. *What may be insured, and how.*

By modern laws and customs, insurance may be made on divers kinds of merchandises; on ships; on the freight or hire of ships; on the money for fitting out of ships; on bottomry, or money borrowed on the bottom of a ship; on ships and their cargoes jointly; on the profit expected by the goods; on interest and no interest, *i. e.* without further proof of interest than the policy; on the rise or continuance of the current price of merchandises; on houses, warehouses, cellars, and the value of goods laid up therein, against danger from fire; on fisheries; on the lives of men and cattle; on lotteries, &c.; the whole according to the circumstances that may happen among the contracting parties.—The goods to be insured ought to be declared in the policy as circumstantially as possible, if the particulars are known, by their marks, numbers, and package, rather than under the general expression of merchandise; or if it be agreed to insert those particulars when known by the insured, they ought by no means to be omitted; more particularly, if the insurance be made for the account of several persons, since by this specification great trouble is prevented in proving to the insurers the particular goods they insured, which were more or less subject to damage. But on goods expected from abroad, it is more adviseable to let the policy be made out under the general expression of merchandise, than to insert particulars, as orders for the purchase of foreign commodities cannot always be strictly complied with by agents or factors, since many circumstances may vary the orders given them for purchasing of particular goods. Besides, in the case of a dubious debt, goods taken in payment, different from those that were ordered, may very prudently be substituted in their place. Every person making insurance under the general expression of merchandise, ought not to conceal any thing he may know which would require a greater premium than is commonly given. And for any damage happening to goods more liable to it than others insured at a lower premium, the insurer ought not to be answerable any farther than in common with the rest of the cargo not so subject to damage.

According

According to the laws of several countries, such things as in their nature are corruptible or perishable, or contraband goods liable to confiscation, are not to be understood under the general expression of merchandise, since the insured is obliged not to conceal any thing from the insurer relating to the risk he takes on him. It may nevertheless happen, that, through the unwarrantable proceedings of others, the ship may be exposed to greater danger than the insured may know; in which case the insurer, not the insured, is accountable for any loss or damage.

Any person who hath laden different sorts of goods, and intends to run part of the risk himself, or to have them insured at different places, hath it in his option, which of those goods he will have inserted in such or such a policy, and may chuse from amongst them such sorts or parcels for himself as are least exposed to average, unless he makes insurance under the general title of merchandise. But then this choice ought to be made at the time of taking out the policies, and the marks and numbers of the merchandise be therein distinguished: for if an insurance hath at the beginning been made under the general expression of goods, and some time after the insured wants to have it expressed in the policy, as is frequently the case, that the risk is run on such and such bales, the insurer no doubt hath a right to inquire into the cause of this specification, and particularly into the quality and condition of the goods; since, as long as they continued insured under the general expression of merchandise, and there happened to be on board others of a greater value, a joint obligation subsisted among the insurers, and none concerned therein ought to be prejudiced by separating or dividing such goods. An equal obligation may indeed more properly be said to subsist amongst such of the underwriters as had insured the value of such goods as were brought aboard at one time, and who, previous thereto, had underwrote a policy; for it is certain; that if the ship wherein these goods were loaded as to-day should be burnt at night, none would insure them to-morrow. The risk of the insurers who first underwrote ought not therefore to be confounded with that which the insurers of a later date must run, especially at those places where they answer for the risks the goods run whilst they are aboard the lighters in their passage to the ship, and only those insurers who signed the first policy should jointly run the risk of the goods in the first lighter, and so on, according to the rank and value. It is, notwithstanding, ordinary in London to bring all the in-

surers, who have signed at separate times for the account of one and the same person, under the general expression of merchandise, into one common risk or average; but considering the remarkable difference there may be in the beginning and duration of the risk, by goods being sooner or later shipped and insured, it will be obvious, that this custom is not founded in equity. This distinction, however, will only be practicable in an outward voyage; for in a voyage inward it is impossible to be particular enough for ascertaining the difference in point of time. The shares of ships ought also to be particularly denominated, and precisely valued in the policy, especially when it is not their first voyage; because it will be difficult to prove what their worth may be in subsequent voyages, or what is really lost of the original value by tear and wear.

For if the owner of one fourth of a vessel, for instance, values it in the policy at L. 500, and only gets L. 400 insured, it is clear, that he ought to be considered as an insurer for L. 100 himself; and although, in a gross average, the insured's share may be charged but at the rate of L. 400, he must nevertheless bear his proportion for L. 100 of that average, as a joint insurer with the rest.

### *3. The damages to which the insurers are liable.*

These are described in all policies, and may be all or either of the following: the seas, men of war, fire, enemies, pirates, rovers, thieves, jettisons, letters of mart and counter-mart, surprisals, takings at sea, arrests, restraints, and detainments of all kings, princes, and people, of what nation, condition, or quality soever, barratry of the master and mariners, and of all other perils, losses, and misfortunes, that have, or shall come to the hurt, detriment, or damage of the said goods, or any part thereof; and in case of any loss or misfortune, it shall be lawful to the insured, their factors, servants, or assignees, to sue, labour, and travel for, in, and about the fence, safeguard, and recovery of said goods, or any part thereof, &c.

However, a distinction is every where to be made between the damage that may happen to insured goods on board a ship, and that which occurs by an actual shipwreck; for this last damage is, without doubt, to be made good by the insurers, but the former may proceed from their bad stowage, or their being put in a place exposed to wet, from a deficiency in caulking of the decks, or otherwise; which faults the ship ought

ought to be answerable for. But it is almost become an universal custom for masters, as soon as they arrive at any port, to go on shore and protest against any damage that in the voyage may have happened to their cargoes, by reciting any bad weather they met with: but this sort of protests, it is to be presumed, will not free the master from a further examination; especially if the owners of the goods have any reason to believe that their damage proceeded from the ship, or the fault of the master, they may at all times bring him and his crew to declare upon oath, what they knew of the ship's condition, and in what place the damaged goods had been stowed.

The East-India company hire all the ships they employ in their trade from private people; and in the charter-party there is this general condition, that every ship shall make good all damages that may happen to the goods on board her; and further, that the company shall contribute nothing to any damage the ships may receive, by either cutting or carrying away masts, losing cables and anchors, or any other direful effects of tempestuous weather; so that what general custom hath made a gross average to be borne by ship and cargo, falls solely on the owners of those vessels which the company employ.

#### 4. *Of general or gross averages.*

Whatever the master of a ship in distress, with the advice of his officers and sailors, deliberately resolves to do, for the preservation of the whole, in cutting away masts or cables, or in throwing goods overboard to lighten his vessel, which is what is meant by jettison, or jetson, is in all places permitted to be brought into a general or gross average; in which all concerned in ship, freight, and cargo are to bear a proportional part of what was so sacrificed for the common good; and it must be made good by the insurers in such proportions as they have underwrote. To make this action legal, the three following particulars are essentially necessary.

1. That what was so condemned to destruction, was in consequence of a deliberate and voluntary consultation held between the master and men.
2. That the ship was in distress, and that such sacrifice was a procedure absolutely necessary to save the rest.
3. That the saving the ship and cargo was actually owing to the means used, with that sole view.

If

If a deep-laden ship be obliged to take out a part of her cargo, previous to her passing some shoals or flats, which, without such lightening, would hinder her getting to her destined port, and the lighters or boats in which the goods of such cargo are put should perish, the owners of the goods so preserved shall not only remain possessed of those goods, but also shall contribute nothing towards the loss of the ship and what was in her. This difference is founded on equity. The lightening of the ship was in consequence of a deliberate and voluntary determination, and for the good of the whole; whereas the lighters being saved, and the ship lost, was owing to an accident, nowise proceeding from a regard to the whole; but a case similar to the saving goods lying nearest at hand when a ship is run ashore. Suppose farther, that a ship running on a bank, throws overboard all her heavy stores, and part of her cargo, by which jettison she gets clear, and returns to sea, yet proves so leaky, that she is obliged to make for, and take shelter in the next harbour, where, upon examination, she is found in so bad a condition as not to be repaired, and therefore is condemned as unfit to proceed on the voyage; in this case, notwithstanding her loss proceeds from her having been ashore, yet as she got clear by the foreaid jettison, and the remainder of the cargo escaped, and arrived safe in harbour, that which was so saved must contribute to the loss of what was thrown overboard. But yet, whatever damage the ship sustained by her running aground, or the damage or loss any particular goods suffered by this unhappy accident, must be borne by the ship's owners, and by the owners of the goods, without any pretension of indemnification from the cargo saved, their redress being only against the insurers. The sea-laws of trading-countries differ greatly in fixing the prices at which goods thrown overboard shall be made good, and for what value those saved are to contribute. By some of the most ancient statutes in different countries, if the accident which occasioned the general average happened before half the voyage was performed, the jettison is to be estimated at the price it cost; but if after, then at the price which the rest, or such like goods shall be sold for at the place of discharge, freight-duties and ordinary charges deducted. The new ordinance of Amsterdam in 1744 seems to differ something from this custom, saying, that only in cases of detentions, and ransoms, the contributions shall be made in these two different manners of valuation. Several ordinances, *viz.* of Antwerp 1563, Hamburg 1603, of France,

France, N° 753. Lewis XIV. and of Conigsberg, N° 860. all agree, that the goods saved and lost shall be rated at the market-price which those saved sell for, and a contribution be made accordingly, after deducting freight and charges. But by another ordinance of Hamburg in 1731, it is ordained, that, in cases of general average, the goods shall be estimated according to their invoice, with the addition of all charges excepting premium of insurance. Hence arises the following question, Which is the justest way to reckon the goods, according to their value at the place they came from, or according to their value where they are landed? If the ship arrived in safety at the destined port, without doubt, both the goods thrown overboard, and those saved, ought to be valued at the price they might have yielded, or did yield there, whether the jettison was made before they came half way or after; for if the goods saved by this jettison arrived at the place where they were destined for, and there produced double what they cost, it would be unreasonable that one half of such produce should contribute nothing to what was cast away; nor would it, on the contrary, be reasonable to make the goods saved, if they came to a losing market, pay for more than they produced: or suppose they were such as suffered by their own perishable nature, what reason could there be to make them contribute *ad valorem* of their cost? As there is no law in England that positively directs what method is to be observed in these cases, the insurers as well as the insured are bound by the determination of referees. Molloy, in his treatise *de jure maritimo*, remarks, that, in his time, the general custom was, that the goods saved, and those lost, were estimated at the rate which the goods saved sold for, freight and other necessary charges deducted; which indeed, from a comparison of transactions in cases of that kind, seems still to be the prevailing custom in England. Accidents may happen that may occasion a contribution before the ship can reach her destined port. When a vessel hath been obliged to make a jettison, or, by the damages suffered soon after sailing, is obliged to return from whence she came, the necessary charges of her repairs, and the replacing the goods thrown overboard, must be proportioned by a general average. In like manner, when ships in time of war are detained in foreign ports, and exposed to greater expenses in procuring their release, there is no other way of settling a contribution for the payment of those charges but on the first cost of the cargo. With regard to some sort of goods that arrive whither they

were

were bound after a jettison, it may be less inconvenient for all the concerned to settle, and make a repartition of the loss upon the cost of the whole cargo, than to calculate upon a forced sale, or an inaccurate appraisement. But no rule in this case can be made without exception, or which will not, in some occurrences, want amendment: for though a law may ordain, that each owner shall contribute proportionably to the cost of his goods, yet if he can prove that they were spoiled, or had fallen considerably in value after shipping, it doth not seem equitable to compel him to any other contribution than *pro rata* of what his goods are worth at the time of settling the average. Hence the new ordinance of insurance at Amsterdam, wherein there is the following clause: "Whereas accidents from whence general averages arise are so variable in their circumstances, that no ordinance can well provide for them all, the decision of them shall be left to the commissioners of the chamber of insurances, to regulate and determine according to law, reason, and equity."—In a codex of commercial laws enacted at Bilboa, anno 1738, c. 21. treating in what manner gross averages are to be regulated, it is said, "That the major part of the concerned shall have it in their choice to regulate the value of the goods by the price appearing in the invoices, or by what they shall be computed at by an appraisement made conformable to their current price at the designed port, regard being had to the condition they come in, and their quality; but that a repartition of no part of the average shall be made on the freight, without the unanimous consent of all the concerned and the captain."

Sailors wages, passengers cloaths, money, and jewels, and goods belonging to the king, are generally exempt from average-contributions; but in voyages from Cadiz and Lisbon, where the carriage of gold and silver makes a great part of the ship's profit or freight, if a person, under the cloak of going passenger, should conceal either in his trunks, or about his body, any such considerable sum of money or jewels as would not be suffered without paying a freight, he must, when discovered, not only satisfy the freight, but also contribute to any jettison. It is customary in London, and most other countries, for the proprietors of whatever gold, silver, or jewels pay freight in merchant-ships to contribute to a jettison for their full value; and the worth of such precious commodities being known, the care of them will be increased in proportion to their worth, to prevent their being thrown overboard.

overboard promiscuously with other things ; and hence their preservation redounds to the common benefit.

The several particulars that are admitted into general or gross averages are as follow.

1. All damage that a ship suffers in her apparel and cargo, in defending her against an enemy, privateer, or pirate.
2. Extraordinary pilotage and charges which a ship shall be at in springing a leak, or being obliged to take shelter in a harbour, by having received some other damage.
3. The necessary expenses for assistance to get clear, when a ship is run aground, or the charges of lightening her on such occasions.
4. Whatever a master may have agreed to pay for the ransom of his ship and cargo to any privateer or pirate when taken.
5. What may be expended in the care and extraordinary attendance on either officers or sailors wounded in defence of the ship ; and also what rewards may be promised by articles to the widows and children of those who may unfortunately lose their lives in the engagement.
6. The extraordinary gratuity which a master may have promised to his men, to animate them to a stout defence or salvage of the vessel.
7. All masts, cables, and rigging, either slipt, cut, or carried away ; and all sails or other appurtenances torn or spoiled in order to save the ship.
8. All that is thrown overboard, or damaged by a jettison, for the ship's safety.
9. The value of any goods or stores taken away or plundered by privateers, or other commissioned ships, with a promise to pay for them.
10. Charges occasioned by any extraordinary quarantine, and other unavoidable accidents.
11. Whatever holes are cut in a ship, in order to have the water run out, or conveyed to the pumps, when by shipping a sea she is filled.
12. The ship's boat, if cut away from its fastenings, and set adrift.
13. The expense of getting a ship afloat after running ashore, and that of repairing the damaged works under water.
14. Goods lost by being put into any other vessel or lighter, and there perished, when otherwise they must have been thrown overboard.

15. The charges which a master pays for bringing his ship to a place of safety on being chased by an enemy; or if she lay in a dangerous part, waiting for a fair wind, or convoy.
16. The charges of lawyers, attorneys, and procurators, for reclaiming ship and cargo. Travelling-expenses of persons concerned in the solicitation; gratuities given on such occasions; protests; tavern-expenses at meetings about it; postages, commissions to merchants or factors who have the direction and care of it; brokerage.
17. Charges for the hire of anchors and cables, &c.
18. Lighterage for the unloading and demurrage.
19. Horse, cart, and waggon hire when goods are carried over land.
20. Gratuities to lords of manors, and proprietors of lands, for leave to dig a ship off that is drove ashore above high-water mark, either by a storm or tide, and the charges in performing it.
21. Cost of repairs to carpenters, blockmakers, smiths, sailmakers, &c.
22. Charges of laying in fresh ballast; rewards promised to the sailors more than their wages for extraordinary labour in the ship's preservation.
23. Interest, premium of insurance, and commission on monies disbursed.
24. Charges of victualling and guarding the ship during her detention, with cost of inspecting customhouse-registers, to find out who may be concerned or proprietors.

These particulars are collected from the ordinances of several countries, and Mr Langenbeck's annotations.

The sum on which a general average ought to be computed, is fixed by examining what the whole ship, freight, and cargo, if no jettison had been made, would have produced nett, if they had all belonged to one person, and been sold for ready money; and this is the sum whereon the repartition ought to be made, all the particular goods bearing their nett proportion, neither more or less.

#### *5. Of particular averages.*

Ever since the year 1749 it hath been the custom in England for the insurers to pay no particular average under 3 per cent. and in most places of Scotland where there are insurance-offices under 5 per cent. In Hamburg general averages are not paid if they do not amount to 3 per cent. but in London

don general averages are always satisfied, be they ever so small. There is certainly a deficiency in almost all ordinances relative to particular averages, in not fully explaining when, and after what manner the damaged shall be deemed to exceed *3 per cent.* For instance, if in a chest containing 100 pieces of linen, three pieces are deducted for damage, and as such allowed to the buyer, the loss ought to be recoverable from the insurers whether that chest was in a policy by itself, or amongst a parcel of an hundred chests; for why should not a person who hath insured and paid an equal premium for 100 chests, have the same advantage as he that is insured but for one? Suppose a merchant has shipped 101 bales of goods from N° 1. to 101. of which, on arrival, three bales are by sea, or some other accident, so spoiled as to be worth nothing; if the damage be calculated as on the whole value of 101 bales, it will not amount to *3 per cent.* and is by most insurers thought not to be recoverable by the insured, especially if the insurance be made, without expressly declaring in the policy the particular sum insured on each bale. It is more than probable, however, were such a case rightly explained to a jury of merchants, the insurers would be condemned to pay the value of these three bales, in the same manner as they would if it had been expressed how much was insured on each bale, or as if the 101 bales had been insured in five different or distinct policies; in which case, supposing a bale totally lost in each of three of the parcels, the underwriters would certainly have been liable for the loss, as it would have been more than *5 per cent.* The law or custom by which underwriters are not chargeable with any particular loss below *3 per cent.* was no doubt made to free the insurers from the vexation of being called upon to make up every frivolous damage that might happen in a few pieces of a chest or bale; but it certainly would be wrong, notwithstanding an insurance should be made under the general expression of goods or merchandise, to consider each different parcel or kind of goods as a part of the whole, when it may amount to a considerable value: it ought certainly to be considered by itself; and if the damage in that chest or parcel amount to *3 per cent.* of the value of the chest or parcel, the insurers should be accountable. However, as this may admit of dispute for want of established rules, it is best for the insured to put the matter past question, by explications in the policy.

6. *Of barratry.*

In London and Antwerp, according to Stracia, and as it is expressed in an Ancona policy of the year 1567, the insurers were even so early as then obliged to answer for the barratry of the master; and a policy of Florence, of the year 1523, also comprehends the risk *di barrateria di padrone e di contrabanda, di Veneziani, salvo di stiva e dogana.* The new Stockholm ordinance of 1750, declares, “ That the insurers “ also shall be answerable for all the damages that may happen to the ship and goods through negligence or knavery “ of the ship and crew, of whatever kind they be, and that “ they may seek their redress against the master.” But this ordinance makes a distinction: for in case any goods should be missing or lost, for which a bill of loading hath been signed, the insured shall first have recourse against the master’s person and estate, and for what falls short in these against the ship and freight; but if all is not sufficient, he then has a right to sue the insurers for the deficiency. The most ancient record that is found in England concerning this matter is that in Edward III.’s time, where one brought an action of trespass against the master for an embezzlement by his mariners of twenty-two pieces of gold, a bow, a sheaf of arrows, a sword, and some other things, and adjudged he should answer. This record was certified into chancery, in order to have it sent into the King’s-bench, to enable the plaintiff to bring an action upon the same judgment in any place of England where he could meet with the defendant; part of which runs thus: *Et prædictus Jurdanus venit et dicit, quod lex de Oleron talis est, quod si aliqua bona et catalla magistro alicujus navis liberata sunt custodienda, unde idem magister, pro eisdem vel pro aliqua alia re, in eadem navi facta manu capit, illo modo magister navis tenetur respondere, non alio modo; et super hac petit judicium. Et prædictus Hen. dicit, quod unusquisque magister tenetur respondere de quacunque transgressione per servientes suos in navi sua facta; et petit judicium similiter.* — The whole of this is recorded in Molloy *de jure maritimo*, l. i. c. 3. § 16.

Where a ship was insured against the barratry of the master, &c. in an action brought thereupon, the jury found, that the ship was lost by the fraud and negligence of the master; the court held, that if the master run away with the ship, or embezzle the goods, the merchant may have an action against him; for it is reasonable that merchants who hazard

zard their stocks in foreign trade, should secure themselves in what manner they think proper against barratry of the master and all other frauds; and this must be intended fraud in the master, not a bare neglect: and they all agreed that fraud is barratry, though not named in the covenant, but negligence ought not. *I Mod. c. 230. 231.*

Cambridge brought a writ of error upon a judgment given against him in the common pleas, in an action brought by the plaintiff upon a policy of insurance of the ship Riga Merchant, at and from Port Mahon to London; and Serjeant Braithwayte for the plaintiff in error insisted, that the judgment was erroneous, because the breach was ill assigned; because the policy was, that the defendant Cambridge should insure the said ship, among other things, against the barratry of the master, and all other dangers, damages, and misfortunes which should happen to the prejudice and damage of the said ship; and the breach assigned was, that the ship, in the said voyage, *per fraudem et negligenciam magistri navis prædictæ, depressa et submersa fuit, et totaliter perdita et amissa, et nullius valoris devenit.* This he insisted was not within the meaning of the word *barratry*, but the breach should have been express, that the ship was lost by the barratry of the master. Besides, the owner of the goods hath a remedy against the owners of the ship for any prejudice he receives by the fraud or neglect of the master; and therefore there is the less reason the insurer should be liable. Besides, if the word *barratry* should import *fraud*, yet it does not import *neglect*; and the fact here alleged is, that the ship was lost by the *fraud* and *neglect* of the master. But the court was unanimously of opinion, that there was no occasion to aver the fact in the very words of the policy; but if the fact alleged came within the meaning of the words in the policy, it is sufficient. Now, barratry imports *fraud*, (*du Fresne gloss. verbo barratara, fraus, dolus*), and he that commits a *fraud* may properly be said to be guilty of a *neglect*, *viz.* of his duty. Barratry of a master is not to be confined to the master's running away with a ship, and the general words of the policy ought to be construed to extend to losses of the like nature, as those mentioned before. Now, losses arising from *fraud* of the master are of the same nature as if he had run away with the ship, supposing barratry was to be confined to that which it is not, because it imports any *fraud*. The judgment was affirmed, April 27. 1724. *Stran. 581.*

7. *How the damage is to be borne when two ships run foul of one another.*

Most modern marine laws ordain, that when two ships no-wise wilfully but accidentally run foul of each other, they shall equally bear the loss or damage which ensues. But if the moiety of the damage done to one of them should exceed the value of what the other can make good, no claim shall extend farther than the produce of the whole; so that the owners, by giving up what was theirs, are freed from any further demands. The custom of dividing the damage equally, which happens by such accidents on each ship, was not heretofore general; for in some parts an estimation of both ships and cargoes was made, and each bore the damage in proportion to their value. In the 68th article of the laws of Wisby, it is thus expressed: "If it happens that by ships running foul of each other, one of them with her cargo be lost, the goods in both ships shall be appraised, and what is lost be rated at so much in the pound, on the value of what each had in, and the same poundage shall be calculated on the value of the ships."

Molloy, *de jure maritimo*, l. 2. c. 6. § 10. says, "If two ships happen to encounter, and cross each other, and the crew swear their innocence, contribution must be made by a just equality; but if one perish, as there can be no medium for proportioning the loss, so there would be no contribution."

8. *Of premiums.*

The premium which an insurer receives, binds him to answer for all such damages and losses as are expressed in the policy; and if the insured pay more than a common premium, it must certainly imply a reason why more than the current premium at the time was given, and an insurer, by accepting it, tacitly admits it to be a sufficient consideration for any circumstance whatever; so that this extraordinary premium being proved, allegations of concealments, or uncertain rumours of retardments, will be of no great weight against the insured.

The payment or non-payment of a premium before a loss happens, makes no alteration in an insurance, according to the present custom, as the insurers may insist upon their being satisfied at the time of their underwriting, if they do not incline to give credit for it. At London, Edinburgh, and Glasgow,

Glasgow, when insurers sign to brokers, and office-keepers, it is they only who are to be looked on as debtors for the premium.

When an insurer hath an open account directly with the insured for the premium, in case of the insured's failure, it is presumable, that he hath a prior right against the goods he insured, when arrived, if remaining on board. A case which was decided at Hamburg, where a bill of lading of goods for account of a person in France, was pawned by the factor to whom they were consigned, together with a policy of insurance he had made on them, seems pretty much calculated to confirm this opinion. The question in this case was, which should first be satisfied, the premium, the loan, or the proprietor? As it happened that the proprietor had not insured the goods in France, the insurance done at Hamburg in case of a loss, must have been recovered, either for his benefit or that of the lender; it therefore was but reasonable that the insurer should be paid his premium prior to both. Hence premiums of insurance are a kind of hypothec of the goods insured, and bills of lading and policies of insurance are but bad securities to lend money on; because, if the aforementioned goods had been insured elsewhere by the shipper, or right owner of them, the insurance made by him who pledged them would not have been paid in case of a loss.

In case the insurer fails while the insured's property is at risk, by the ordinances of most countries, the insured is at liberty to make insurance *de novo*, and to draw back his premium, at least to become a creditor for the premium if it hath been paid, and accordingly share *pro rata* of whatever the bankrupt's estate produces; but if a total or partial loss hath happened, and been made public previous to the insurer's failure, the insured can only claim to be admitted as a creditor on the insurer's estate. Amst. N° 536. Hamb. N° 965. Stockh. N° 1098.

#### 9. *The documents requisite to recover a loss.*

It is not easy to enumerate the different proofs or documents requisite to ascertain and recover a loss.—We shall condescend on a few.

It is common for the insurers to ask,

1. What is insured elsewhere upon the ship or goods for which the loss or damage is demanded?

2. It

2. It is ordinary to ask for the protest, which is a declaration upon oath usually made by the master and some of his people, before a justice, notary-public, or consul, at any place where they first arrive, setting forth whatever bad weather they met with in the voyage, or any other accident that befel them; as also what precautions they took to guard against the ill consequences to be apprehended from those accidents, with the motives they had for going into any other harbour than that they were bound to; which sort of protests are now become almost a mere matter of form, as a notion is propagated at some places, that if the masters neglect protesting immediately after their arrival, against the damages that have resulted from any bad weather, they make themselves answerable for those damages that shall be found in any of the goods aboard, from which they believe a protest frees them. But this cannot be sufficient to clear them on all occasions. It is true, the French ordinance of 1681, *tit. 10. de congés et rapports, art. 4.* enjoins, “that all masters shall be obliged to “make a report within twenty-four hours after their arrival, “before the lieutenant or judge of the admiralty, of all that “hath happened in their voyage.” But it is to be presumed, that the intent of this order is only to prevent the making any partial declarations, by not allowing time and room for it; which might have some weight where the master was no way concerned, and not when the question is, whether the damage did not proceed from his own carelessness, bad stowage, or defect in caulking of his decks; for, on any appearance of such neglects, the proprietors of the goods have a right, notwithstanding the protest, to insist that the ship be visited, and a particular examination of the crew taken, how and where these goods were laid. At Hamburg it is the custom, soon after the ship's arrival, though not just within twenty-four hours, to send a printed notification to every person who hath goods on board, importing, that the master fears his cargo may have received damage; the intent of this is, that all who have goods to receive, may send to inspect the ship and the places where they were stowed, before they take them ashore. This is certainly a very prudent precaution, and greatly tending to the master's justification. At London masters seldom do any thing more than to have their intentions of making a protest noted before a notary-public, without giving any information to the persons who are to receive the goods, &c.

3. For the bills of sale, and often for the customhouse-registers,

registers, to find out the owners when insurance is made on the body of a ship.

4. For bills of lading signed by the master, which are usually called for to prove an interest in goods; but if there is any apparent reason to distrust their being genuine, all such clearances or registers from the customhouses as are ordinarily given where the ship hath been dispatched, are called for; and upon proofs of such authority a greater dependence in general may be made than upon mere bills of lading; more especially upon certificates from those registers which the Spanish ships, in their West-India trade, carry along with them, and of which duplicates remain behind in the customhouse. For the manner of making such a register is, that every person who hath goods to ship, previous to their embarkation, gives an account of their bales and marks, and pays a duty for them, either by weight or measure, which is explained in a cocket; and when they pass the gates, or go to be shipped, they are searched to see whether they correspond with the entry delivered in, and if they do, each parcel is marked with a custom-house-stamp, and the cocket signed by the searchers: when on board, another set of visitors re-examine them, and put their *camplido* upon each cocket, out of all which the general register is framed, to go by the ship, in order to a re-examination by the King's officers, at the places of unloading in America.

By an ordinance of Hamburg *anno 1603*, it is said, that every master is diligently to inspect the stowage of the goods, and to take care they be not exposed to damage; for if any ensue, or they be diminished through the master's fault, or that of the mariners, the former shall be obliged to make satisfaction to the merchant for the same. And it is remarked in Langenbeck's annotations upon this law, that although the city of Bourdeaux, as well as several others both in France and Italy, where wines are shipped, keep a set of sworn labourers, or stowers, to stow goods well aboard of ships, the master is not thereby freed from supervising, or being answerable for the stowage, as some of them have pretended, because the stowers must put them in such places as the master or his officers direct. And the insufficiency of such a plea was proved in a law-suit carried on there before the admiralty in Hamburg in 1725, by Tessier against Capt. Detloft.

In most policies the time is expressed when insurers shall pay in case of a loss. At Hamburg, where else no abandon of effects is admitted, the ordinance says, N° 939. "that

" when a ship bound to a place in Europe shall not be heard  
 " of in three months beyond the usual time for such a voy-  
 " age, she may be considered as lost, and the insured is per-  
 " mitted to abandon ship and cargo to the insurers, and to  
 " demand payment of the sums they have respectively under-  
 " wrote. The insurers shall be obliged to pay 92 per cent.  
 " within two months from and after the time such abandon-  
 " was notified to them, on behalf of the insured, by the bro-  
 " ker, or their dispatcher." And in N° 940. " That in case  
 " the insurers should make any demur to pay within the be-  
 " fore-limited time, the insured may stay until fourteen  
 " months are elapsed from the time of the ship's departure,  
 " and then the insurers shall be obliged to pay 100 per cent.  
 " without any deduction: and if the insured desires in the  
 " mean while to have security given, they shall be obliged to  
 " give it." But whenever there is certain advice that a  
 ship is lost or taken, and her salvage and recovery desperate,  
 the same ordinance directs, N° 961. " That the insurers shall  
 " be obliged to pay at the rate of 98 per cent. within two  
 " months from and after the time that such loss hath by the  
 " broker been notified to them, provided that such loss hath  
 " before been proved by the proper and necessary docu-  
 " ments."

It is appointed by the ordinance of Amsterdam, N° 573. that a loss is to be paid within three months after its being notified; by that of Copenhagen, N° 1259. within two months after proof; by the *recapilacion des leyes de Indias* in Spain, N° 105. within six months after its being duly proved, and within eighteen months after the ship has sailed, if no advice is received. At London it is customary with the insurance-companies, in a week after the adjustment of the loss, to pay 98 per cent. deducting a half per cent. for prompt payment; the private insurers pay the same sum of 98 per cent. in a month after the loss is settled, and both the one and the others, in cases of salvage and reclaims, are always very ready to advance something to account, for carrying on suits, either upon the reasonableness of the thing, or a tender of proper security. At Glasgow and Edinburgh, upon settling the loss, the underwriters grant bills at three months, which the insured can sell for ready money, allowing discount for the time to run.

#### 10. Of return of premium in case of convoy or over-insurance.

It is ordinary, when the insured expects convoy, to agree  
 for

for a certain premium without convoy, so much thereof to be returned in case of convoy; and it is likewise an universal custom, that in case too much is insured, or insurances made at divers places without fraud, the first shall stand good, and the last insurers shall return the premium, keeping only  $\frac{1}{2}$  per cent. for their trouble of keeping a book or account of it. It often happens that goods are insured to come in certain ships expected from abroad, which do not come by them; and as in this case no risk is run, the insurers are obliged to return the premium as above. Sometimes ships that have begun to load change their voyage, and unload again what they have aboard; under which circumstances the insurers are intitled to something more than half per cent. for the time the risk continued, though, whilst the ship was in port, it could be but very small. The ordinance of Amsterdam, N° 534. allows in such cases to keep  $\frac{1}{2}$  per cent. And the Stockholm ordinance, N° 1072. directs, that it shall be so much as impartial arbitrators, or their judicial court of insurances, shall adjudge. At London it is commonly insisted, that all who have insured on goods expected for one and the same person's account, without particularising them, whether they be comprehended in one or more policies, and underwrote sooner or later, shall contribute equally to any loss, or the return of premium, if over-insured; which would be right had all the several policies been signed before any goods went on board, but not otherwise. For, suppose a person who expected goods from Jamaica in the ship *Sailsure* to the value of L. 5000, insures L. 2000 provisionally at the London insurance-office, under the general expression of merchandise, before he knew of any goods being shipped, on his receiving advice some time after that the value of L. 2000 was gone on board, he got L. 3000 more insured by the royal-insurance company. Now, it would seem plain, that L. 2000 value of the goods first shipped ought to be applied to the London insurance-company, and not mixed with the goods loaded last. For it is possible that the *Sailsure* might have perished in the harbour by some accident, with only the first-shipped goods on board, and before the resolution was taken in London to increase the insurance, consequently the royal-exchange company would not have been comprehended in the said loss. The ordinances of most countries with respect to this particular leave room for law-suits, as not being sufficiently explicit. The laws of France, Holland, Hamburg, and Stockholm ordain, that only the persons who underwrote

one and the same policy shall stand upon an equal footing, and that the first-signed policy shall first stand good, which likewise would admit, upon an appeal to equity, of some distinction.

When an insurance is intended to be done abroad, and at home, an explanation in the policies on both sides, which shall have the preference, or stand good first, is a very material circumstance, because this cannot be decided by the dates alone. For instance, a person residing in Hamburg had the value of 10,000 rix-dollars, or *L. 2400 Sterling*, to ship for Lisbon, and had actually loaded on the 1st of July to the value of 4000 rix-dollars, and thereupon gave an order of that date to his correspondent in London, to get insurance done for *L. 1000 Sterling*, without any limitation either in premium or circumstances; and on the 4th July he got insured at Hamburg the remaining *L. 1400*, or  $58\frac{3}{4}$  rix-dollars, although he had not then shipped more than the first mentioned 4000 rix-dollars; but the 5th of July some circumstances occurred that induced him to alter his design, and to resolve on shipping no more: now, the query is, who ought to make a return of premium, according to the sense of the Hamburg ordinance? To determine this, it ought to be considered, that, notwithstanding the underwriting of the London insurers must be of a later date than that of the Hamburg insurers, who underwrote the 5th of July, because the order sent to London on the 1st of July could not possibly arrive there before the 7th, and therefore the underwriting in London could not be before that or the next day; yet it seems evident, that the insurance in London, though later in date, ought to stand good before that made at Hamburg: for if between the first of July, that the order for insurance was sent to London, and the 4th of the same month when it was made at Hamburg, the ship had been burnt, this last-mentioned insurance could not have taken place, and consequently that done in London, on the order given on the 1st of July, must have borne the entire loss of these first-shipped goods; therefore the return of premium ought to be made by the Hamburg insurers, notwithstanding it is said in the ordinance of that city, N° 919. "That the policy which is first  
"in date shall first stand good."

### II. Of re-insurances.

Re-insurances may be as justifiable, and made with as little scruple as insurances, provided the true motives for doing them

them are not concealed. By an act of parliament passed in England 25th March 1747, No 1297. It is enacted, “ That “ it shall not be lawful to make re-insurance, unless the in-“ surer shall be insolvent, become a bankrupt, or die; in “ either of which cases, such insurer, his executors, admini-“ strators, or assignees, may re-insure to the amount of the “ sum before insured, provided it shall be expressed in the “ policy to be a re-insurance.” The liberty given in this act, for the executors of an insurer who dies solvent and in good circumstances, to re-insure, is very necessary, and will often be done by his heirs to secure the estate he hath left. But to make re-insurances for insolvent insurers, or for those who become bankrupts, will seldom happen. The insured will naturally take care of themselves, by making fresh insurances as soon as possible, and not neglect to put the loss of the principal out of the question, when they can do it by only paying a new premium. It is highly expedient for the insured, in this case, not to wait on what is never likely to be done, but to do the best for himself by making a new insurance; and supposing the bankrupt's administrators were to re-insure, if a loss happened, as the premium of this re-insurance must come out of the common stock, it would seem that the recovery of such loss ought to come to the common benefit.

#### 12. *Of insurance on lives.*

Mens lives with good reason are and may be insured, to secure to a creditor the reimbursement of a sum advanced to his debtor for purchasing a post or place, out of the income of which he may have a sufficiency, besides his maintenance, and expenses, interest, and premium, to pay off yearly a part of the capital: however the lender ought not to insure the life of the borrower without having his consent. In some places insurances are not permitted on the lives of persons at the head of government; as at Genoa N° 155. no insurances may be made, *sine licentia senatus, super vita pontificis, neque super vita imperatoris, neque super vita regum, neque cardinalium, neque ducum, principum, episcoporum, neque aliorum dominorum, aut personarum ecclesiasticarum, seu secularium in dignitate constitutarum.* In London however people may make insurance on any one's life without exception; and the insurers seldom inquire much, if there are good or bad reasons for such an insurance, but only what the person's age is, and whether he be of a good constitution. The common premium on a good life, from twenty to fifty years of age, is 5 per cent. and from fifty to six-

ty years of age, 6 per cent. per annum. But these premiums are higher than any computations founded on observations concerning the probability of human life will warrant.—People ought by all means to be prevented from getting insurances done with finister views, especially that inhuman one of committing murder, to gain the sum insured; an instance of which villainy happened a few years ago in a London apothecary, who having got his wife's life insured, soon after killed her. Underwriters ought therefore to inform themselves of the motives why an insurance is required, and not to be contented with obscure, plausible, or fictitious reasons. It is indeed true, that the insurers are not obliged to pay to a murderer convict, as happened in the case of this apothecary; yet this does not restore the life sacrificed. A great part of the insurance done on lives in London are for people who have certain expectations in reversion, after the death of some friend or relation, whose possessions they have a mind in some measure to anticipate by these means. But such insurances seem not easily to be justified, as they frustrate the intention of the bequeather, and frequently overset a good design: and a fine estate may be soon anticipated by this irregular method.

All ordinances of insurances allow it to be made upon the lives of captives in slavery; but to subsist no longer than the bondage does, or till the person be redeemed. An office for the insurance of lives at Serjeants-Inn was erected the 25th July 1706, by a charter from Queen Anne granted to the then Lord Bishop of Oxford, Sir Thomas Allen, and others, for incorporating them and their successors, by the name of the “amicable society for a perpetual insurance-office,” whereby they might provide for their wives, children, and other relations, &c.; and the chief view of the Bishop of Oxford seems to have been, to induce clergymen to save a premium out of their yearly income, for insuring their lives, so that when they died, their families might recover the sums insured, which hath been a comfort to many persons.—This seems to be a good institution, and carries in it a view, not the least beneficial for those for which insurances on lives were originally designed. The lives of cattle may likewise be insured, provided the insurance is made before any reigning distemper hath got amongst them; in this case the value of the cattle ought to be ascertained in the policy as well as the period when the risk begins and ends.

13. *Insurance on lotteries.*

It hath sometimes been practised, when a lottery was on foot, to insure for a certain premium, that a ticket should not be drawn a blank ; and if it was, that the insurer should pay for that ticket a certain sum agreed on. In order to have a clear idea of the design of insuring tickets, it will be proper to take notice, that not above one half of the tickets in any lottery are fairly sold out to the public before the drawing ; that is, so as not to come to market again : for a great many people buy a quantity of tickets, without any intention of venturing a single shilling in the lottery. These buy at a low price, and when they find an opportunity of selling to advantage, they bring them to market again ; and if this opportunity does not offer before the drawing begins, they generally insure them ; for which purpose there are a set of underwriters who open offices in Jonathan's, and paste up their names over their several stalls nearly in this manner : " Tickets insured from " blanks by A. B. and company :" their principal business is to insure tickets from blanks, or, in other words, to give an undrawn ticket for every one that is drawn a blank, during the time they are insured. This branch of insuring is calculated to serve those who go into the wheel with a number of tickets, designing to sell them at a high price, if they grow scarce, or the great prizes happen to stay long in the wheel ; but whether the insuring tickets in this case, or the risking the chance of the wheel, and buying fresh tickets, in the room of those drawn blanks, with the money that is saved by not insuring, be most eligible, is a matter of doubt.

The price of insurance from blanks runs generally betwixt 5*s.* and 5*l.* as the drawing advances ; and it sets off at first lower or higher, in proportion to the number of blanks there are to a prize in the scheme of the lottery, and as the risk is lesser or greater. This insurance is for real property ; but there is a gaming kind of insurance not less considerable than the former, and full as profitable.—As the gentlemen insurers, previous to the opening their books, have held a committee, in which the nicest calculations have been made, and the price fixed, with the moderate allowance to themselves of 25 per cent. profit upon the whole ; it is a matter of indifference to them, whether the persons who bring a list of numbers to insure are really possessed of those tickets or not, provided they comply with the conditions required of those who are really possessed of tickets ; viz. to deliver up the blanks when drawn in

in exchange for undrawn tickets. I am therefore at liberty to carry any indifferent number, or the number of my ticket, (if I have one); the only difference is this, that in the one case I give up the blank for an undrawn ticket, and in the other I allow the value of the blank, and take either an undrawn ticket, or if I am no adventurer in the lottery, the value of an undrawn in money. Another method of gaming is to insure for prizes; that is, to give in any list of numbers whatever, and if any of them are drawn prizes during the time they are insured, I receive for every prize so drawn the value of *L. 20* prize in money. The price of insuring in this manner is generally from *1 s. 6 d.* the first day to *5 s.* towards the end of the drawing.

#### *14. Insurance from fire.*

Insurances from fire are introduced almost into every country under one denomination or other. At London insurances of this kind are obtainable at such easy rates, that there are few merchants but chuse to be insured for their own quiet. Besides, this premium adds to their credit both at home and abroad, when it is known that the great capitals lying in their houses and warehouses are thus secured from the flames. Here the premium on brick houses, or goods lodged in them, is but *2 s.* on each *100 l.* to the value of *1000 l.* and *2 s. 6 d.* to the value of *2000 l.* excepting where hazardous trades are carried on, such as sugar-refiners, distillers, and chandlers, or on dangerous commodities, such as hemp, flax, &c. as will appear by the proposals and policy of the London insurance-company, N<sup>o</sup> 1332. which hath taken a very good and necessary precaution, by inserting the condition, that whenever any houses are burnt, it shall be optional to the said company either to pay the loss or rebuild them. For this takes away the temptation from all insured leaseholders of destroying their houses, as the landlords, and not they, in such case, would be benefited by it. And it is a similar good disposition, that when goods are burnt, the insurers shall have it in their choice either to make a pecuniary satisfaction, or to repair the damage by a quantity of merchandise, equal in kind, value, and goodness to those damaged or lost, which effectually prevents any iniquitous people, who may have a parcel of goods on hand greatly fallen in price, from setting them on fire, in expectation of recovering from the insurers what they cost.

*Decisions relative to the foregoing observations.*

1. A having got a doubtful account of his ship at sea, *viz.* that a ship described like his was taken, insured her without giving any information to the underwriters of what he had heard, either as to the hazard or circumstances which might induce him to believe his ship was in danger, if not actually lost. — The insurers brought a bill in chancery for an injunction, and to be relieved of this insurance as fraudulent; which being found justly alleged, the Lord Chancellor pronounced the following decree: “The insured hath not dealt fairly with the insurers in this case; he ought to have disclosed to them what intelligence he had of the ship’s being in danger, in which case it is impossible to think that the insurers would have underwritten the ship at so small a premium as they had done, but would either not have insured at all, or would have insisted on a larger premium, so that concealing this intelligence is a fraud; wherefore I decree that the policy be delivered up with costs, but the premium to be paid back, and allowed out of the costs.” 2 *eq. Abr.* 635. *Weaver contra Fowler.* In the following term, the same point was again determined by Lord Macclesfield.

2. Seaman *contra Fonereau*, Easter term 1743, before Lord Chief Justice Lee in the King’s-bench. A fleet of ships came out of port together, and afterwards anchored; it was agreed among all the captains of the fleet to keep company. Mr Seaman’s captain went on board one of the ships at anchor, and there dined. In discoursing at dinner he said, his ship was leaky, and a bad sailer, and that she could not keep company. After dinner the captain went on board his own ship, and at night a signal was given to sail; but Seaman’s captain intending to get ahead of the fleet, slipped away when under sail, and arrived safe on the coast of England, where he was taken by a privateer. — The captain of the vessel in which Seaman’s captain had dined, apprehending the ship was foundered, wrote home, that such a ship sailed with them, but that in the night on a sudden his lights disappeared, and therefore he had reason to think she had foundered. This advice reached Seaman time enough to make insurance, but he concealed it from the underwriters; wherefore when the cause came to be heard, although the accident contained in the advice did not happen, the Lord Chief Justice Lee gave a verdict for the defendant. *Stran.* 1183. 16. *Geo. II.*

3. An action was brought upon a policy of insurance, bearing, *inter alia*, "That the said ship, &c. for so much as concerned the assured's, was and should be valued at interest or no interest, free from average, and without benefit of salvage, without farther account to be given by the assureds for the same, touching the adventures and perils," &c. *communi forma*, because the insurers refused to make satisfaction for the loss.—The defendant pleaded the general issue *Non assumpsit*, and issue was thereon joined.

Upon this case it was clearly taken, at the trial before Lord Chief Justice Lee, without any objection or question made upon it, that the plaintiff was well intitled to maintain this action upon the policy of insurance made in the name of Caleb Smith, who was the policy broker employed to procure the insurance, he having, by indorsement upon the policy, acknowledged and declared, that the policy was made in his name, upon the account, and for the sole risk and benefit of the plaintiff; and Smith was allowed, without any objection, to prove the underwriting by the defendant, and his own indorsement. In this case, the defence insisted on for the defendant underwriting the policy was, that the plaintiff had been informed by a letter wrote from Carolina, by a ship called *the Collet*, to one Mr Crockat, that the Polly, the ship insured, had sailed ten days or a fortnight from Carolina before the ship Collet, and that the ship Collet had arrived in England about seven days before the insurance made, and that the plaintiff had not informed the defendant of this; which was insisted on to be a fraud in the plaintiff, sufficient to discharge the defendant of this insurance; it being, as was insisted, a settled and established rule, that, on making an insurance, all material circumstances relating to the adventure ought to be disclosed to the insurer, for him to judge upon: and the Chief Justice allowed this rule, and declared his opinion, that the concealment insisted on was a sufficient circumstance to discharge the defendant from the policy; for he said, that these contracts are made upon a mutual faith and credit, and that to conceal such circumstances, which may make any difference in the adventure, is fraudulent: for the insurer ought to have the advantage of judgment upon them; and that where there is such concealment, the insurance ought not to bind.—But the defendant not being able to make out this fact to the satisfaction of the jury, the plaintiff had a verdict.

N. B. In this case the insurance was a re-insurance; and it

it was said by several policy-brokers, that when policies are made interest or no interest, it is generally in such cases of re-insurances. *Tr. & Com.* 148. 16th December 1743, at Guildhall, book 5. Thurmond.

4. In the year 1678, one Newnham Perkins and Stokes were owners of a vessel, called the *May-flower ketch*; the vessel coming laden with wines on the account of Firebrace and Stone to the isle of Wight; Perkins being then in the same place, contrives, with one Joy the master, to sell the freighters goods privately, and that being effected, to go out to sea, some small distance from the isle, and there privately sink the vessel, and pretend the struck, and then foundered through stress of weather. The plot being laid, Perkins hastens up to London, and makes a policy of insurance on the vessel; which being done, he remits his orders to Joy, to put the contrivance in execution; who accordingly, having disposed of the best part of the goods, stood out to sea, and then with his own hands, by the force of an iron crow, made a hole in the hold, and then in the longboat conveyed himself and mariners ashore. Joy sends advice of the loss; and Perkins, as if he had known nothing of the matter, demands the monies assured with great confidence, and thereupon brings an action for the same; but before the cause came to be heard, Firebrace and Stone bring trover against Perkins, and thereupon the whole fraud was detected, and judgment given for the plaintiff; with this further intimation, that if the owners proceeded in their action on the insurance, they must expect that their practice and fraud would totally poison it; so they thought it advisable to drop it. *Mollov*, book 2. c. 7. § 5.

5. The ship Charming Peggy was insured in 1739 from North Carolina to London, with a warranty against captures and seizures, and in an action the loss was said to be by sinking at sea. All the evidence given was, that she sailed out of port on her intended voyage, and has never since been heard of. Several witnesses proved, that in such a case the presumption is, that she foundered at sea, all other sorts of losses being generally heard of. The underwriter insisted, that as captures and seizures were excepted, it lay upon the assured to prove the loss in the particular manner declared on. But the Chief Justice said, it would be unreasonable to expect certain evidence of such a loss, as where every body on board is presumed to be dead; and all that can be required is the

best proof the nature of the case admits of, which the plaintiff hath given. He therefore left it to the jury, who found the loss according to the plaintiff's declaration. *Stran. 1199.*  
*17. Geo. II. Green ver. Brown.*

6. The defendant insured goods to London, and until the same shall be safely landed there. The ship arrived in the port of London, and the owner of the goods sent his lighter and received the goods out of the ship; but before they reached land, an accident happened whereby the goods were damaged; for which this action was brought against the insurer. For the defendant it was insisted on, that the accident happening after the owner had taken the goods into his possession, it was a loss after the insurance was ended. To which it was answered, that if this had been an action against the master or owners of the ship, that would have been of weight, for they were certainly discharged; but in this action it could be no answer, for during all the voyage it might as well be said that the goods were in the possession of the assured, who took the ship to freight, and whose servant the master was, to this purpose, as much as the lighterman. And these words are put into policies, to guard against all sorts of losses till there is an actual landing; because, in the case of ships of great burden, that are forced to lie off, there may be a carriage for many miles in boats or lighters, and it was in the course of trade for the owner to send his lighters. But the Chief Justice held the insurer was discharged. He said it would have been otherwise had the goods been sent by the ship's boat, which is considered as a part of the ship and voyage. And the jury, which was of merchants, expressing they thought it turned upon that distinction, brought in a verdict as to this point against the plaintiff. *Strange 1236. 18. Geo. II. Sparrow ver. Carruthers.*

7. The ship Success was insured at and from Leghorn to the port of London, and till there moored twenty-four hours in good safety. She arrived the 8th of July at Fresh wharf, and moored; but was the same day served with an order to go to the Hop to perform a fourteen days quarantine. The men upon this deserted her, and on the 12th the captain applied to be excused going back; which petition was adjourned to the 28th, when the regency ordered her back; and on the 30th she went back, performed the quarantine, and then sent up for orders to air the goods. But before she returned, the ship was burnt on the 23d of August; and now the question was,

was, whether the insurers were liable? For the defendant it was insisted, that the ship arriving and being moored on the 8th of July, and remaining so till the 30th, here was a performance of what he had undertaken, and his risk ought not to be extended to so long a time as from the 8th of July to the date of burning, the 23d of August. But it was ruled, that though the ship was so long at her moorings, yet she could not be said to be there in good safety, which must mean the opportunity of unloading and discharging; whereas here she was arrested within the twenty-four hours, and the hands having deserted, and the regency taking time to consider the petition, there was no default in the master or owners; and it was proved, that till the fourteen days were expired, there could be no application to air the goods; wherefore the jury found for the plaintiff. *Strange 1243. 19. Geo. II. Waples ver. Eames.*

8. Upon the execution of a writ of inquiry, before the Chief Justice, it appeared, that the defendant was an insurer to L. 200 upon corn, the value of which was L. 217; that the corn was so damaged in the voyage that it sold only for L. 67, and the freight came to L. 80; and upon this the question was, whether as the freight, which the plaintiff was obliged to pay, exceeded the salvage, this was not to be considered as a total loss? And for the plaintiff it was insisted, that he ought not to be in a worse condition than if his corn had gone to the bottom of the sea; for then he would have had no freight to pay, and now that the voyage hath been performed, whereby the freight is become due, he hath a right to apply the salvage for discharging that. It was proved to be the usage, where the salvage exceeds the freight, to deduct the freight out of the salvage, and make up the loss upon the difference. For the defendant it was insisted, that as his insurance was upon the corn, and the whole did not perish, he ought, in making up the loss, to deduct the salvage: but no instance could be shown, on either side, of an adjustment where the freight exceeded the salvage. The Chief Justice was of opinion, that since it was usual to deduct the freight when the salvage exceeds it, the plaintiff was in this case, wherein it fell short, intitled to have it considered as a total loss, and the jury found for the plaintiff accordingly. *Strange 1065. 10. Geo. II. Bayfield ver. Brown.*

9. The plaintiff insured on ship and freight at and from Jamaica to Bristol. A cargo was ready to put on board, but  
as

as the ship was careening in order for the voyage, a sudden tempest arose, and she and many others were lost. The rigging and part of her were recovered and sold, and the defendant paid in to court as much as upon an average he was liable to for the loss of the ship. But the plaintiff insisted to be allowed £. 600 for the freight of the ship, which she would have earned in the voyage, if the accident had not happened. But as the goods were not actually on board, so as to make the plaintiff's right to freight commence, the Chief Justice held he could not be allowed it. 19. Geo. II. Strange 1251. *Tongue ver. Watts.*

10. A bill in chancery was brought for relief against a verdict and judgment given in the court of common pleas, upon a policy of insurance, and to have an injunction to stay execution upon the judgment. The case appeared to be, that the ship insured was taken by a Spanish privateer, and that after it had been carried *infra hostium præsidia*, it was retaken by an English privateer. It was argued for the now plaintiff, who was the defendant at law, that although, by the law of nations, the first capture of the ship, and its being *infra hostium præsidia*, had absolutely divested the right of the original proprietors, yet that now, by the statute made in the year 1740, it is otherwise, being thereby provided, that if the ships of our English merchants should be taken by an enemy, and afterwards retaken by any of his Majesty's subjects, that the right of the original proprietors in such ships should be reserved, on their paying one moiety of the value of such ships to the recaptors for resalvage. Upon this it was argued, that the verdict and judgment are unjust, in regard that the whole insurance-money is given in damages, when it appears that the plaintiff at law, upon payment of half of the value of the ship, might recover it back, and therefore that half of the insurance-money ought only to have been given in damage, upon which the injunction prayed by the bill was moved for. On the other side, it was insisted, that this was a right verdict, and that the insured were not to be put to the delay, expense, and trouble of ascertaining the value of the ship, in order to recover it back, upon payment of one moiety of the value to the captors: that for recovery thereof the insurers might stand in the place of the insured, and make use of their names, which had been offered: that they did not pretend to oppose so much of the bill as sought this, but insisted, that this could be no ground for granting the injunction prayed: that this point hath been debated before Lord

Chief

Chief Justice Willes, upon trial of the issue at *Nisi prius*, who had declared his opinion, that this right of salvage ought not to preclude the insured from their recovery upon the insurance till the salvage should be settled : that the defendants the insurers would be intitled to stand in the place of the insured, to make what advantage they could of the salvage. Lord Hardwicke Chancellor, being of the same opinion, refused to grant the injunction, and said, that the damage in recovering the salvage is as much a part of the insurance as the ship itself. *Dicit. trad. & com.* 148. *Mich.* 18. *Geo. II. Prendle ver. Hartly.*

11. A ship insured was in her voyage seized by the government, and converted into a fire-ship ; the question was, whether the insurers were liable ? Holt Chief Justice thought it was within the word *detention*, but the cause was referred. *2 Salk. 444.*

12. In a case on a policy of insurance upon *Non assumpsit* pleaded. The case was, Mr Crisp being at the West Indies, sent a letter to Bates to insure goods on the Mary galley of St Christopher's, Capt. A. Hill commander, at London. Bates carried the letter to Stubbs, who writ policies, and he by mistake made the insurance on the Mary, Capt. Haslewood commander, &c. This policy thus made was subscribed by the defendant. The Mary galley was lost, and then Stubbs applied to the insurers to consent to alter the policy, to which they agreed, and the mistake was rectified. It was objected at the trial, that the Mary was a stouter ship than the Mary galley, and that the insurers ought to have an increase of premium for the alteration : but it was held by Holt Chief Justice, that the action well lay, and that the mistake might be set right, and that Stubbs was a good witness ; and he cited this case which happened when Pemberton was Chief Justice : An insurance was made from Archangel to the Downs, and from the Downs to Leghorn, but there was a parole-agreement at the same time, that the policy should not commence till the ship came to such a place. It was held the parole-agreement should not void the writing. *2. Salk. 444. Dec. 3. 1703. Bates ver. Graham & alios, at Guildhall.*

13. *Assumpsit* upon a policy of insurance, where the defendant insured the plaintiff interest or no interest, against all enemies, pirates, takings at sea, and all other damages whatsoever. And upon trial it appeared, that the ship was taken by

by a pirate of Sweden, and was in his possession for nine days, and then retaken by an English man of war, and after the suit commenced brought into Harwich. And the question was, whether in such case the defendant was responsible? It was reserved by the Chief Justice for the opinion of the court; and after argument by Serjeant Whiteacre for the plaintiff, and by Dr Henchman for the defendant, it was determined for the plaintiff. For although it was objected, that the insurer was only responsible where the plaintiff had a property, and that the term of insuring interest or no interest was introduced since the revolution; yet it was said, that such insurance was good, and the import of it is, that the plaintiff has no occasion to prove his interest, and that the defendant cannot controvert it.

And though the ship was here retaken, yet the plaintiff received a damage, for his voyage was interrupted; and the question is not, whether the plaintiff had his ship, and did not lose his property, but what damage he sustained. *Comyns 360. Mich. 7. Geo. I. Depa ba ver. Ludlow.*

### Variety XI. COMPUTATIONS in EXCHANGE.

#### I N T R O D U C T I O N.

**E**Xchange is the commutation of the money of one country for that of another, by means of a bill, instrument, or writ, commonly called *a bill of exchange*; the nature and form of which, with the laws, regulations, and customs concerning it, shall be particularly insisted on in the second volume.

Exchange may likewise be defined, a fixing of the actual and momentary value of money. Silver, as a metal, hath a value like all other merchandises; but as it is capable of becoming the sign of all other merchandises, or the medium by which they can be estimated, it may receive an additional value; for was it no more than a mere merchandise, its value would perhaps be less fluctuating, and of less consideration than it is.—As money, the prince can fix a value upon silver in some cases, and in others he cannot.—He can fix a proportion betwixt silver as a metal and silver as money; betwixt the several metals made use of to pass as money; he establishes the weight and standard of every piece of money, and assigns to it that ideal value, by which it is current. On the other hand, if we consider the money of one country comparatively

ratively with that of another, it receives a new value, which is fixed by the current course of commerce, and the general opinion of merchants; but never by the laws of any particular nation, because it is liable to incessant variations, and depends on the accidental circumstances of trade, the money-transactions between nations, and the state of their public credit. The several nations, in fixing this relative value, are chiefly guided by that particular nation which hath the greatest quantity of specie. If any one nation hath as much specie as several others together, it will then become necessary for these several nations to be regulated by the standard of that one nation. In the actual state of the universe, Holland, in this respect, seems to be the umpire, since she regulates the exchange for almost all Europe, in a manner most agreeable to her own interest. This scarcity or plenty, from whence results the mutability of the course of exchange, is not real, but relative: for instance, when Glasgow hath greater occasion for funds in London than London of having funds in Glasgow, the price of bills must rise at Glasgow. The specie of both cities is the same, both as to weight and standard; and although there should be money enough to purchase bills at Glasgow upon London, yet when there is not a fund of credit at London equal to the debit, the price of bills, not of money, must rise of course.

To set this subject in a more practical point of view, let us consider,

1. That if our purchases and payments in foreign countries exactly balance their purchases and payments in ours, there will be just enough of bills on the one to clear accounts with the other; so that in this case the exchange on both sides will be at par; that is, one who gives money in one country, will receive as much in the other in weight and standard.

2. If a nation supplies us with more than it takes from us, or if we pay that nation more money than it pays to us, there will be a balance against us, which we must necessarily pay; in order to which, the demand for the money of that nation, or its bills of exchange, becomes greater among us than the quantity to supply that demand, which raises the value of their money or bills and lowers ours, or, in other words, puts the price of their money above par, and ours below it, which constitutes what we call *the course of exchange*. From these two considerations we may naturally infer,

1. That the course of exchange betwixt two nations is a herald, which proclaims publicly the state of commerce and money-negotiations betwixt them, and which of the two is indebted to the other.
2. That the nation which is indebted hath the disadvantage in commerce and money transactions, and that the one which hath the balance in its favour, hath in every respect the advantage.
3. That the balance of trade naturally imports specie, and renders money at home more valuable abroad; whereas, on the other hand, when the balance is against a nation, their specie is exported, and becomes thereby less valued.

That nothing may be wanting to render this treatise as complete as possible, I have thought proper to insert Sir Isaac Newton's table of the assays, weights, and values of most foreign silver and gold coins actually made at the mint, by order of the privy-council, before the year 1717, with notes and explanations, and a calculation of the real or intrinsic par of exchange, as it stood 1719, when first published, and as it was republished in London 1740.

#### T A B L E

T A B L E O F F O R E I G N S I L V E R C O I N S.

	Affay dw.	Weight dw. gr.	Stan. dw. gr.	Wt. gr. mi.	Value. d.
The piaffer of Spain, or Seville piece of 8 reals, now reduced to 10	W. 1	17 12	17 10 2	54	.11
The new Seville piece of eight	W. 1	14	13 21 15	43	.83
The Mexico piece of eight	W. 1	17 10	17 8 14	53	.87
The pillar piece of eight	Sta.	17 9	17 9	53	.87
The Peru piece of eight, coiner, but of uncertain allay	W. 1	17 12	17 10 2	54	
The old ecu of France, or piece of 60 sols Tournois	W. 1½	19 14½	19 11 12	60	.39
The new ecu, or piece of 5 livres, or 100 sols					
N.B. <i>The ecu of France should be 2 ducat wofe by law.</i>					
The crusado of Portugal, or ducat worth 400 reas, now marked and raised to 480 reas	W. 2	11 4	11 1 13	34	.31
The patacks or patagons of Portugal, worth 500 reas, now marked and raised to 600	B. 4½	20 22	21 8 2	66	.15
The ducatoon of Flanders, or piece of 60 sols or patars	W. 12	18 1	17 1 13	52	.91
The patagon of Flanders, or crois dollar, or piece of 48 patars	B. 3	20 21	21 3 15	65	.59
The ducatoon of Holland, or piece of 63 flivers	W. 14	18	16 20 17	52	.28
The patagon leg-dollar, or rix-dollar of Holland, or piece of 50 flivers	W. 2	20 8	20 3 12	62	.46
The three guilder pieces of Holland, or piece of 60 flivers	W. 2	6 18½	6 17 1	20	.08
The guilder, florin, or piece of 20 flivers	W. 2	20 6	20 1 13	62	.21
The ten schelling piece of Zeland, or piece of 60 flivers	W. 44	17 14	14 2 7	43	.07
The lion dollar of Holland, or $\frac{2}{3}$ of the ducatoon	B. 3	20 18	21 — 15	65	.02
The ducatoon of Cologn	W. 13	18	16 22 14	52	.53
The rix-dollar, or patagon of Cologn	W. 12	17 22½	16 22 5	55	.48
The rix-dollar, or patagon of the Bishop of Liege					

### FOREIGN SILVER COINS.

	Affay. dw.	Weight dw. gr. mi.	Stan. Wt. dw. gr. mi.	Value. d.
The rix-dollar of Mentz	W. 6 $\frac{1}{2}$	18 8	17 18	.27
The rix-dollar of Frankfurt	W. 9	18 8	17 14	.53
The rix-dollar of the Elector Palatine of the Rhine and Bavaria before 1620	18 5			
The rix-dollar of Nuremberg	W. 6	18 10	17 22	1 55
The old rix-dollar of Lunenburg	W. 10	18 11	17 15	2 54
The old rix-dollar of Hanover	W. 8	18 12	17 20	2 55
The double gulden of the Elector of Hanover	W. 7	18 18	18 3	16 .03
The gulden of the Elector of Hanover, or piece of $\frac{1}{2}$	W. 17 $\frac{1}{2}$	8 10	9 1	18 .29
The half-gulden of the Electorate of Hanover, or piece of $\frac{1}{3}$	B. 17 $\frac{1}{2}$	4 5	4 12	19 14 .07
The gulden of the Duke of Zell, or piece of 16 gutz grosh	W. 43	11 2	8 22	10 27 .07
The gulden of the Bishop of Hildesheim, or piece of 24 marien grosh, now raised to 26	W. 4C $\frac{1}{2}$	11 22	9 17	17 30 .21
The rix-dollar of Magdeburg	W. 10	18 12	17 16	1 54 .27
The gulden, or guilder of Magdeburg	W. 44	11 14	9 6	28 .67
The old rix-dollar of the Elector of Brandenburg	W. 9	18 13	17 19	1 55 .17
The old gulden of Brandenburg, now raised from 24 to 26 marien grosh	W. 43	12 4	9 19	9 30 .41
The gulden of Brandenburg, or piece of $\frac{1}{2}$	W. 43	11 3	8 23	6 27 .81
The half-gulden of Brandenburg, or piece of $\frac{1}{3}$	W. 43	5 13	4 11	14 13 .09
The gulden of the Elector of Saxony, or piece of $\frac{1}{2}$	W. 41	11 3	9 1	14 28 .12
The old bank dollar of Hamburg	W. 8	18 9	17 17	4 54 .92
The old rix-dollar of Lübeck	W. 8 $\frac{1}{2}$	18 16	17 22	17 55 .54

The four mark piece of Denmark, or coarser alloy — — —

W. 6 $\frac{1}{2}$  | 14 8 | 10 9 10 | 32 .23

The four mark piece of Denmark, or coarser alloy	—	—	W. 6 <sup>1</sup>	14 8	10 9 10	32 .23
The four mark piece of Denmark, or finer alloy	—	—	W. 2 <sup>1</sup>	11 13 <sup>1</sup> <sub>2</sub>	10 11 5	32 .45
The eight mark piece of Sweden	—	—	Stran.	20	20	62
The four mark piece of Sweden	—	—	W. 5 <sup>8</sup>	13 12	9 23 7	30 .92
The two mark piece of Sweden	—	—	W.	6 19		
The old dollar of Dantzick	—	—	W. 10 <sup>1</sup> <sub>2</sub>	18 9 <sup>1</sup> <sub>2</sub>	17 12 14	54 .27
The old rix dollar of Thorn near Dantzick	—	—	W. 12	18 8 <sup>1</sup> <sub>2</sub>	17 8 15	53 .85
The rix-dollars of Sigismund III. and Vladislaus IV. Kings of Poland	—	—	W. 10	18 9	17 13 14	54 .04
The rix-dollar of the late Emperor Leopold	—	—	W. 10 <sup>1</sup> <sub>2</sub>	18 9	17 12 4	54 .27
The rix-dollar of his predecessor Ferdinand III.	—	—	W. 10 <sup>1</sup> <sub>2</sub>	18 9	17 12 4	54 .27
The rix-dollar of Ferdinand Archduke of Austria	—	—	W. 10 <sup>1</sup> <sub>2</sub>	18 9	17 8 7	53 .78
The rix-dollar of Basil	—	—	W. 7 <sup>1</sup> <sub>2</sub>	18 18 <sup>1</sup> <sub>2</sub>	18 3 6	56 .24
The rix-dollar of Zane	—	—	W. 13	18 1	16 23 13	52 .65
The old ducat of Venice, with the words <i>Ducatus Venetus</i> upon it; <sup>a</sup> piece of six old livres, afterwards rated, I think, to 6 livres 4 sols de Picoli	—	—	W. 23 <sup>1</sup> <sub>2</sub>	14 15	13 1 17	40 .50
The half-ducat	—	—	W. 23 <sup>1</sup> <sub>2</sub>	7 7 <sup>1</sup> <sub>2</sub>	6 12 18	20 .25
The new ducat, with the N° 124. upon it, signifying 124 sols, or 6 livres 4 sols de Picoli	—	—		18 2		
The half thereof	—	—			9 1	
The crusado croisat, or St Mark of Venice, with the N° 140 upon it, signifying 140 sols, or 7 livres de Picoli	—	—			20 6	
The half-crusado, of the same form	—	—			10 3	
The quarter crusado, of the same form	—	—			5 1	
Another coin of Venice	—	—	W. 46	17 10	13 19 8	42 .08

## FOREIGN SILVER COINS.

	Affay. dw.	Weight dw. gr.	Stan. Wt. dw. gr. mi.	Value. d.
The piece of two jewels	—	—	3 17 7	.05
The ducat de banco of Nap. or piece of 5 tarins, or 10 carlins, or 100 gr.	—	—	13 1	.43
The half-ducat	—	—	6 12 10	.21
The tarin, or fifth part of the ducat	—	—	2 14 12	.09
The carlin, or tenth part of the ducat	—	—	1 7	.04
The escudi escu, or crown of Rome, or piece of 10 Julios, or 100 bayoches	—	—	—	—
The teston of Rome, or piece of 3 Julios	—	—	—	—
The ducat of Florence and Leghorn, or piece of 7 livres, or 10 <sup>1</sup> Julios	—	—	—	—
The Julio of Rome	—	—	—	—
The piaffer ecu, or crown of Ferdinand II. Duke of Tuscany	W. 1	20 3	20 20 6	.62
The piaffer ecu, or crown of Colimus III. present Duke of Tuscany, whose monies are about 4 <i>per cent.</i> lighter than those of his father; this piece is 8 $\frac{1}{2}$ Julios	—	—	—	—
The croisat of Genoa, or piece of 7 $\frac{1}{2}$ livres	—	—	—	—
The ecu d'argent of Genoa, or piece of 7 livres 12 sols	—	—	—	—
The piaffer ecu, or crown of Milan	—	—	—	—
The Philip of Milan, a piece of 7 livres	—	—	—	—
The livre, or 20 sols piece of Savoy	—	—	—	—
The 10 sols piece of Savoy	—	—	—	—
A roupee	B. 16 $\frac{1}{2}$	1 23	7 23 4	.07
A gout gulden, or Florin d'or, a Dutch coin of 28 stivers	W. 75	12 19	8 11 5	.26
Another gout gulden	W. 48	11 0	8 14 18	.72
Another	W. 48	12 0	9 9 15	.15

## GOLD COINS Unworn.

Affay.	Weight.	Stan. Wt.	Value.
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## GOLD COINS Unworn.

GOLD COINS Unworn.		Weight.	Stan. Wt.	Value.
Affay.	car. gr.	dw. gr.	dw. gr. mi.	s. d.
The old Louis d'or	—	—	—	16 9 .3
The half and quarter in proportion	—	—	—	8 5 .6
The new Louis d'or	—	—	—	20 0 .6
The half and quarter in proportion	—	—	—	10 0 .3
The old Spanish double doublon	—	—	—	67 1 .4
The old Spanish double pistole	—	—	—	33 6 .7
The old Spanish pistole	—	—	—	16 9 .3
The new Seville double pistole	—	—	—	16 9 .3
The new Seville pistole	—	—	—	16 9 .3
The half and quarter in proportion	—	—	—	26 10 .4
The doppia moeda or double moeda of Portugal, new coined	—	—	—	26 9 .9
The doppia moeda as they come into England	—	—	—	13 5 .1
The moeda of Portugal	—	—	—	8 .5
The half-moeda	—	—	—	3 .6
The Hungary ducat	—	—	—	9 3 .2
The ducat of Holland coined <i>ad legem imperii</i>	—	—	—	9 3 .2
The ducat of Campen in Holland	—	—	—	9 3 .2
The ducat of the Bishop of Bamberg	—	—	—	9 3 .2
The double ducat of the Duke of Hanover	—	—	—	18 4 .8
The ducat of the Duke of Hanover	—	—	—	10 9 2 .7
The ducat of Brandenburg	—	—	—	9 3 .2
The ducat of Sweden	—	—	—	9 3 .2
The ducat of Denmark	—	—	—	9 3 .2

## GOLD COINS Unworn.

	Affay. car. gr.	Weight. dw. gr.	Stan. Wt. dw. gr. mi.	Value. s. d.
The ducat of Poland	B. 1 2	2 5	2 8 12	9 2 .1
The ducat of Transylvania	B. 1 1½	2 4½	2 7 6	8 11 .6
The sequin, or zacheen of Venice	B. 1 3½	2 5¾	2 10 7	9 5 .7
The old Italian pistole	W. o 0¼	4 6¾	4 6 11	16 7 .6
The double pistole of Pope Urban, 1634		8 14½		
The half-pistole of Innocent II, 1685		8 10	2 4	
A double pistole of Placentia		8 16		
A double pistole of Genoa, 1621		8 13½		
A double pistole of Milan		4 6¾		
A single pistole of Milan		4 8½		
A pistole of Savoy, 1665			4 11	4 18 8
Double ducats of Castile, Genoa, Portugal, Florence, Hungary, {	B. 1 2½			18 7 .7
and Venice				
Single ducats of the same places	B. 1 2½	2 5½	2 9 9	9 3 .8
Double ducats of the several forms in Germany	B. 1 1	4 11	4 17 1	18 4
Single ducats of the same places	B. 1 1	2 5½	2 8 5½	9 2
Double ducats of Genoa	B. 1 2	4 11	4 18 6	18 6 .5
Single ducats of Genoa, Besançon, and Zurich	B. 1 2	2 5½	2 9 3	9 3 .2
Pistole of Rome, Milan, Venice, Florence, Savoy, Genoa, {	W. o 0¼	6 6	4 5 17	16 6 .7
Orange, Trevon, and Besançon				
A Barbary ducat, with Arabic letters on both sides, in square tablets, without any effigies or escutcheon	W. 2 1¼	2 16¼	2 9 6	9 3 .5

N. B. The gold coin having been valued when guineas were at 21 s. 6 d. they are here reduced to the present standard of 21 s.

## Var. XI. EXCHANGE with IRELAND.

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For understanding the foregoing table, it is to be observed, that the English pound Troy contains 12 ounces; 1 ounce, 20 pennyweights; 1 pennyweight, 24 grains; and 1 grain, 20 mites.

The present English standard for gold coin is 22 carats of fine gold and two carats or  $\frac{1}{12}$  of alloy.

The silver coin contains 11 ounces 2 pennyweight fine silver, and 18 pennyweight of alloy in the pound.

The first column of the table expresseth the fineness of the assayed piece, the letter B. signifying better, and W. worse than the English standard.

The second column the absolute weight of the piece.

The third column its standard weight, or its quantity of standard metal.

The fourth column its value in English money.

## Art. I. EXCHANGE with IRELAND.

In Ireland accounts are kept in pounds, shillings, and pence Irish, divided as in Britain; but having no coins of their own, they are supplied by the different countries with which they have commerce. Britain exchanges with Ireland upon the L. 100 Sterling, the par of which in Irish money is L. 108, 6 s. 8 d.; so that the shilling Sterling, according to that equation, will be worth 13 pence Irish. The course of exchange runs from 5 to 12 per cent. according as the balance of trade is in favour of Ireland, or against it.

*Case 1.* When Irish money is required; let L. 100 be the first term, its equivalent Irish according to the course, the second term, and the given Sterling the third; then will the fourth proportional be the Irish money required.

*Case 2.* If Sterling is required; let the course of exchange equivalent to L. 100 be the first term, L. 100 the second, the given Irish the third; then will the fourth proportional be the answer.

### Examples of Irish exchanges.

*Exam. 1.* Glasgow remits to Ireland L. 357, 18 s. Sterling; for how much Irish ought Glasgow to be credited for this remittance, when the exchange is at  $10\frac{1}{8}$  per cent.?

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Sterl.

$$\text{Sterl. Irish}$$

$$L. 100 : 110\frac{5}{8} :: 357.9$$

$$\underline{110\frac{5}{8}}$$

$$39369.0$$

$$178.95 = \frac{4}{8} \text{ or } \frac{1}{2} \text{ of } 357.9$$

$$44.7375 = \frac{1}{8} \text{ or } \frac{1}{4} \text{ of } \frac{1}{2}$$

$$\text{Irish L. } 395.926875 = L. 395 - 18 - 6\frac{3}{8} \text{ Ans.}$$

The same answer can very readily be effected by practice, thus.

$$357.9$$

$$35.79 = \frac{1}{10} \text{ for } 10 \text{ per cent.}$$

$$1.7895 = \frac{1}{10} \text{ of } \frac{1}{10} \text{ for } \frac{4}{8} \text{ or } \frac{1}{2}$$

$$.447375 = \frac{1}{4} \text{ of } \frac{1}{10} \text{ of } \frac{1}{10} \text{ for } \frac{1}{8}$$

$$395.926875 \text{ as before.}$$

*Exam. 2.* Ireland draws on Glasgow for L. 395-18-6 $\frac{3}{8}$ . Irish, exchange at  $10\frac{5}{8}$  per cent.; how much Sterling will Ireland be debited with at Glasgow when this bill is discharged?

$$110\frac{5}{8} : 100 :: 395.926875$$

$$\underline{8} \quad \underline{160}$$

$$5)885 \quad \underline{\underline{}} \quad \underline{\underline{}} \quad \underline{\underline{}}$$

$$5)800 \quad \underline{\underline{}} \quad 177)63348.300000(357.9 \text{ as given in the}$$

$$\underline{177} \quad \underline{\underline{}} \quad \underline{531} \quad \underline{\underline{}} \quad \text{(first example.)}$$

$$\underline{160} \quad \underline{\underline{}}$$

$$\underline{\underline{}} \quad \underline{1024}$$

$$\underline{\underline{}} \quad \underline{885}$$

$$\underline{\underline{}} \quad \underline{\underline{}} \quad \underline{1398}$$

$$\underline{\underline{}} \quad \underline{\underline{}} \quad \underline{1239}$$

$$\underline{\underline{}} \quad \underline{\underline{}} \quad \underline{\underline{}} \quad \underline{1593}$$

$$\underline{\underline{}} \quad \underline{\underline{}} \quad \underline{\underline{}} \quad \underline{1593}$$

$$\underline{\underline{}} \quad \underline{\underline{}} \quad \underline{\underline{}} \quad (0)$$

## Art. II. Exchange with America and the West Indies.

In America and the West Indies, as in other parts of the British dominions, accounts are kept in pounds, shillings, and pence, divided as in Britain, and their money, for distinction's

tion's sake, is called *currency*. Upon the continent payments are seldom made in specie, as there are few coins circulating among them, but some French and Spanish pieces, the value of which, by a statute in the sixth year of Queen Anne, was ascertained as follows.

	<i>Weight.</i>	<i>true value.</i>	<i>cur. value.</i>			
	<i>dwt.</i>	<i>gr.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>
Dollar old plate of Seville	17	12	4	6	6	0
Ditto of new	-	-	14	—	3	7 $\frac{1}{2}$
Mexico ditto	-	-	17	12	4	6
Pillar ditto	-	-	17	12	4	6 $\frac{1}{4}$
Peru ditto, old plate	-	-	17	12	4	5
Cross dollar	-	-	18	—	4	4 $\frac{1}{2}$
Ducatoon of Flanders	-	-	20	21	5	6
French old crown	-	-	17	12	4	6
Crusado of Portugal	-	-	11	4	2	10 $\frac{1}{4}$
Three guilder piece of Holland	20	7	5	2 $\frac{1}{4}$	6	10 $\frac{1}{4}$
Old rix-dollar of the empire	18	10	4	6	6	0

The scarcity of specie obliges them to substitute a paper currency for carrying on their trade, which being subject to innumerable casualties, suffers generally a very great discount for Sterling in the purchase of bills of exchange, or good silver or gold. This is not the case however with the West Indies; for their intercourse with the Spanish settlements furnishes them with such an abundance of specie, that at an average the exchange may be reckoned at 7 currency to 5 Sterling.

The method of computing the exchanges is in this case the same as with Ireland, and therefore the rule needs not be repeated.

*Examples of the exchanges with America, &c.*

*Exam. 1.* Glasgow receives an account of sales from Philadelphia, the nett proceeds amounting to 575 l. 19 s. 6 d. currency; for how much Sterling may Glasgow debit Philadelphia, the exchange being at 80 per cent.?

$$\begin{array}{r}
 180 : 100 \\
 \text{Cancel the ciphers,} \\
 \text{and divide by 2.} \qquad\qquad 9 : 5 :: 575 \ 19 \ 6 \\
 \hline
 9) 2879 \ 17 \ 6 \\
 \hline
 319 \ 19 \ 8\frac{1}{2} \qquad \text{Answer.}
 \end{array}$$

*Exam. 2.* Glasgow receives a bill of exchange from Philadelphia for 319*l.* 19*s.* 8*d.*; for how much currency is Glasgow debited at Philadelphia, the exchange being at 80 per cent.?

$$\begin{array}{r} 100 : 180 \\ \text{By cancelling and } 5 : 9 :: 319 \ 19 \ 8\frac{2}{3} \\ \text{dividing as above.} \qquad \qquad \qquad 9 \\ \hline 5) 2879 \ 17 \ 6 \\ \hline 575 \ 19 \ 6 \text{ currency. } \textit{Ans}. \end{array}$$

*Exam. 3.* Philadelphia is indebted to Glasgow in 1575*l.* 14*s.* 9*d.* currency; what Sterling may Glasgow reckon on to be remitted when the exchange is at 75 per cent.?

First, 175 : 100 :: 1575 14 9, the two first terms of which (being abbreviated.) By dividing them by 5×5, there will be this new state.

$$\begin{array}{r} \text{Cur.} \quad \text{Ster.} \quad \text{Cur.} \\ 7 : 4 :: 1575 \ 14 \ 9 \\ \hline 7 ) 6302 \ 19 \\ \hline 900 \ 8 \ 5\frac{5}{7} \text{ Sterling. } \textit{Answer}. \end{array}$$

*Exam. 4.* Glasgow receives a bill of exchange from Philadelphia for 900*l.* 8*s.* 5*d.* Sterling; for how much currency was Glasgow debited, the exchange being at 75 per cent.?

The first and second terms being abbrev. as above, it will be

$$\begin{array}{r} \text{Ster.} \quad \text{Cur.} \quad \text{Ster.} \\ 4 : 7 :: 900 \ 8 \ 5\frac{5}{7} \\ \hline 4 ) 6302 \ 19 \\ \hline 1575 \ 14 \ 9 \text{ currency. } \textit{Answer}. \end{array}$$

Or by practice thus, L. 900 8 5*d.* for 100

$$\begin{array}{r} 450 \ 4 \ 2\frac{4}{7} \text{ for } 50 \text{ being } 1\frac{1}{2} \\ 225 \ 2 \ 1\frac{2}{7} \text{ for } 25 \text{ being } \frac{1}{2} \end{array}$$

$$1575 \ 14 \ 9 \text{ cur. } 175$$

*Exam.*

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*Exam. 5.* Glasgow consigns to Virginia goods *per invoice* amounting to 578*l.* 19*s.* 6*d.* which are sold for 847*l.* 15*s.* 6*d.* currency; what Sterling ought the factor to remit, deducting 5 per cent. in full of commission and charges, and what doth Glasgow gain *per cent.* upon the adventure, supposing the exchange at 30 per cent.?  $\frac{1}{3}$

*Exch. Com.*

$$\begin{array}{r} 130 + 5 = 135 : 100 \\ 27 : 20 :: 847.775 \\ \hline 20 \end{array}$$

$$3) \underline{16955.500}$$

$$9) \underline{5651.83}$$

627.981 to be remitted.  
578.975 consigned.

49.006 gained on the whole.

$$\text{Then } 578.975 : 49.006 :: 100 \\ \hline 100$$

$$\begin{array}{r} 578.975 ) 4900.600 ( 8\frac{1}{2} \text{ nearest.} \\ \underline{4631.800} \\ \hline 268.800 \end{array}$$

*Exam. 6.* Virginia is indebted to Glasgow in 575*l.* 19*s.* 6*d.* Sterling; with how much currency will Glasgow be credited at Virginia, when the exchange is at 33 *$\frac{1}{3}$*  per cent.?  $\frac{1}{3}$

$$\text{First } 100 : 133\frac{1}{3}$$

$$\begin{array}{r} 3 \\ \hline 3 \end{array}$$

300 : 400 both terms reduced to thirds.

By cancelling the ciphers, it will be 3 : 4 :: 575 19 6

$$\begin{array}{r} 4 \\ \hline 3) \underline{2303 18 0} \\ \hline 767 19 4 \text{ curcy.} \end{array}$$

Or

Or thus, L. 575 19 6 for 100  
 191 19 10 for  $\frac{33\frac{1}{3}}{100}$  or  $\frac{1}{3}$   
 $\underline{191 \quad 19 \quad 10}$   
 767 19 4 currency. Answer.

Exam. 7. Glasgow is indebted to Carolina in 5759 l. 18 s. 9 d. currency, exchange at 550 per cent.; for how much Sterling may Carolina value on Glasgow?

First 550 : 100. Cancel one cipher on each side and divide by 5, then it will stand thus,

$$\begin{array}{r} 11 : 2 :: 5759 \ 18 \ 9 \\ \underline{2} \\ 11) \ 11519 \ 17 \ 6 \end{array}$$

1047 5 2 $\frac{1}{4}$  Sterling. Answer.

Exam. 8. Jamaica is indebted to Glasgow in L. 1550 currency, for which Glasgow is ordered to draw on London, exchange 40 per cent.; how much money does Glasgow draw for the bill, being allowed  $\frac{1}{2}$  exchange for the draught to London?

$$\begin{array}{r} 7 : 5 :: 1550 \\ \underline{5} \\ 7) \ 7750 \end{array}$$

$$\begin{array}{r} 100) \ 1107 \ 2 \ 10\frac{1}{4} \text{ value of the bill.} \\ 2) \quad 11 \ 1 \ 4\frac{1}{2} \text{ at } 1 \text{ per cent. exchange.} \\ \underline{5 \ 10 \ 8\frac{1}{4}} \text{ at } \frac{1}{2} \text{ per cent.} \end{array}$$

1123 14 11 received at Glasgow.

#### O B S E R V A T I O N .

When Britain exchanges upon the L. 100, as in the above instances, the higher the exchange is, the advantage to Britain in remitting is the greater, and in drawing the less. Suppose, for instance, that the exchange betwixt Britain and Ireland is at 12 per cent. I can in that case purchase a bill for L. 100 Sterling to discharge a debt of L. 112 Irish, whereas had the exchange been at 5 per cent. the same bill would have cost me L. 106 : 13 : 4 Sterling; but what I gain by remitting on this occasion is lost by the drawer of the bill; for his credit

credit in Ireland is lessened by *L. 112* Irish, and he hath only received *L. 100 Sterling*, whereas had the exchange been at *5 per cent.* he would have received *L. 106 : 13 : 4 Sterling* for his *112* Irish. They who deal in exchange with Ireland, ought to be well informed at all times of the state of trade betwixt the two nations, and on whose side the balance lies; for as the rise and fall of exchange is the true barometer of the balance of trade, so likewise by having a proper intelligence of the imports from Ireland, and exports thither, one may in a great measure discern what will be the state of the exchange. Suppose, for example, Ireland had imported from Britain goods to the amount of *L. 20,000*, and had exported thither to the value of but *L. 10,000*, it is plain that Ireland can be at par with Britain for no more than *L. 10,000*; and yet there is a balance of *L. 10,000* still due, which must be remitted to Britain before the account is evened. When there is no money due in Britain to compensate this balance, Sterling money in Ireland will become dear, and Irish money cheap, and as the demand for bills increases, the price will be proportionally raised. The Irish merchant who foresees this, will lay in a fund of credit in Britain for the occasion; and though at this time he can receive *L. 112* Irish for his draught of *L. 100*, he may be able, by the time his bills fall due, to purchase remittances at *L. 105*. The British merchant, on the other hand, will muster up all he can to purchase remittances, to raise a stock of Irish money, which, upon the turn of the balance, he can draw for, with the odds of *5 or 6 per cent.* perhaps, in his favour.

When the exchange with the plantations in America is high, which is generally the case where there is not a sufficiency of produce fit for the British market to answer the imports from Britain, bills of exchange are often a very expensive remittance; for which reason those who have stores abroad, and can afford to lie a little out of their money, chuse rather to purchase such produce as will come to the quickest market in some other colony upon the continent, or in the West Indies, in order to be remitted from thence in produce, or in bills of exchange.

*Examples of exchange applied to drawing and remitting.*

*Exam. 1.* When the exchange was at *12 per cent.* Britain remitted to Ireland *L. 5000*; for how much Sterling ought Britain to draw for a reimbursement when the exchange falls to *6 per cent.*?

First,

First, 100 : 112 :: 5000

112

100) 560000

5600 amount in Ireland at 12 per cent.

2dly, 106 : 100 :: 5600

100

106) 560000 (5283 o 4 $\frac{1}{2}$  Sterling. Answer.

530

300

212

880

848

320

318

2

20

40

12

480

424

56

Or more expeditiously as barter.

*Ir.      Ster.      Ir.*

$112 : 5000 :: 106$

112

106) 560000

*L. 5283 o 4 $\frac{1}{2}$  as before.*

Hence Britain gains by this transaction *L. 283 : o : 4 $\frac{1}{2}$* , which will be found to be above 5 per cent.

*Exam.*

*Exam. 2.* When the exchange with Ireland rose to 12 per cent. Ireland drew on London for L. 5000 Sterling ; how much Irish must be remitted to London to discharge the debt when the exchange falls to 6 per cent. ?

First, 5000 at 100 per cent.

500 at 10 per cent.

100 at 2 per cent.

5600 at 112 Irish received for draughts.

2dly, 5000 at 100 per cent.

250 at 5 per cent.

50 at 1 per cent.

5300 at 106 Irish remitted.

Difference 300 gained by Ireland, which is 6 per cent.

### Art. III. EXCHANGE with HOLLAND.

#### I N T R O D U C T I O N.

In Holland there are two banks, the one of Amsterdam, and the other of Rotterdam. That of Amsterdam is not only the more famous of the two, but the most considerable in Europe. It was established on the 31st of January 1609, by the authority of the States-General, under the direction of the burgomasters of the city, who having constituted themselves the perpetual cashiers of the merchants of Amsterdam, are themselves a security for the bank ; and as business in the bank is negotiated by transfers, millions may be paid in a day without the intervention of any cash, which is of the greatest consequence in expediting trade that can possibly be imagined. Authors differ about the real sum of money that is deposited in this bank, but few have reckoned it under 30 millions Sterling.

It is to this bank the city of Amsterdam owes its splendour and magnificence, which, though it possesses the greatest part of the merchants ready money, rather promotes than interrupts their commerce, by the security and dispatch with which a bank-credit is attended.

By the establishment of this bank it is ordained, that bills of exchange, and the payment of wholesale goods, shall only be in bank, unless the sum be under 300 guilders ; in which

case it cannot be entered into bank without a duty of 6 stivers, excepting by the East and West India companies, who can enter into bank what sums they please without any duty whatever.

To have an account opened in the bank, there is a duty imposed of 10 guilders for once only; but if at any time that account should be overdrawn, the merchant incurs a penalty of  $3 \text{ per cent.}$ .

A merchant may draw his cash out of the bank at any time upon allowing  $\frac{1}{10} \text{ per cent.}$  for bank-security; but if at that time the agio, that is, the difference betwixt the bank and current money, or the discount to which the current money is liable, be below  $5 \text{ per cent.}$  the treasurer is obliged to make good the difference, as  $5 \text{ per cent.}$  which seems to have been the original agio, is always charged when it is received.

No merchant can draw for the money he puts into bank, or any part of it, until the next day, without making an allowance of  $\frac{1}{2} \text{ per cent.}$  unless the bank had been opened only the day before, after balancing the books, or at the feast of Pentecost.

All accounts with the bank must be balanced twice a-year, and that within six weeks after the bank is opened, under the penalty of 15 guilders.

No procuration is good at the bank unless it be registered there, and the noting of such procuration costs 50 stivers.

The time of noting in the bank is from seven to eleven in the forenoon; after which to three in the afternoon every note carried in will cost 6 stivers, and after three none are admitted.

When money is deposited in the bank, the depositor may call for it in specie, when he wants it, upon allowing a premium seldom exceeding  $\frac{1}{2} \text{ per cent.}$  for bank-security.

No money can be given in loan but upon security that is unexceptionable, and in judging of such security the directors must be unanimous.

No director can use the freedom to abstract any money from the bank to supply any private demand, though but for a single day, upon pain of death.

The strictness and fidelity with which this famous bank is kept, is productive of so great a security on the part of the citizens, and contributes so much to dispatch in money-matters, when a large sum can be negotiated with a single debit and a credit, that a bank-payment is reckoned from 3 to 6 per cent. better than a payment in cash, besides a premium allowed

lowed the bank for every deposit. It is no wonder then if this bank be immensely rich, since the institution is so judicious, the regulations so wise, and the direction so proper.

In Holland they keep their accounts in pounds, shillings, and pence Flemish, divided as the British pound, but more generally in guilders or florins, stivers, and phinnings, reckoning 16 phinnings to a stiver, and 20 stivers to a guilder or florin.

They reckon likewise 6 guilders or florins to the pound Flemish.

Britain exchanges with Holland upon the pound Sterling, for which the latter gives an uncertain number of shillings and pence, or grotes Flemish, according to the course of exchange, which runs from 30 to 40 s. Flemish per 20 s. Sterling. The par of a pound Sterling, according to Sir Isaac Newton's table, will be found to be L. 1 : 16 : 6 Flemish; but a guinea passes in Holland for 12 guilders, according to which estimation their coins may be reckoned as follows.

A duke	-	-	-	L. 0 0 0 $\frac{1}{16}$
A stiver	-	-	-	0 0 0 $\frac{1}{16}$
A schilling	-	-	-	0 0 6 $\frac{3}{16}$
A guilder	-	-	-	0 1 9
A Zealand dollar	-	-	-	0 2 7 $\frac{1}{2}$
A rix-dollar	-	-	-	0 4 4 $\frac{1}{2}$
A dry guilder	-	-	-	0 5 3
A ducat	-	-	-	0 9 2 $\frac{1}{2}$

To compute the exchanges with Holland, or any other place, where it is made upon the pound Sterling.

*Case 1.* If Dutch money is required, let L. 1 *Sterling* be the first term, its equivalent Dutch the second, and the Sterling to be exchanged the third; then will the Dutch money required be the fourth proportional. The equivalent Dutch money will likewise be found very readily by practice.

*Case 2.* If Sterling is required, let the Dutch money equivalent to L. 1 *Sterling* be the first term, L. 1 the second, and the given Dutch the third, then will the Sterling required be the fourth proportional.

#### Examples of exchanges with Holland.

*Exam. 1.* London is indebted to Holland in L. 270 : 8 : 2 *Sterling*, with how much Flemish is London debited at Amsterdam, when the exchange is at 35 s. 6 d. *Flem.* per pound *Sterl.*?

case it cannot be entered into bank without a duty of 6 stivers, excepting by the East and West India companies, who can enter into bank what sums they please without any duty whatever.

To have an account opened in the bank, there is a duty imposed of 10 guilders for once only; but if at any time that account should be overdrawn, the merchant incurs a penalty of *3 per cent.*

A merchant may draw his cash out of the bank at any time upon allowing  $\frac{1}{10}$  per cent. for bank-security; but if at that time the agio, that is, the difference betwixt the bank and current money, or the discount to which the current money is liable, be below *5 per cent.* the treasurer is obliged to make good the difference, as *5 per cent.* which seems to have been the original agio, is always charged when it is received.

No merchant can draw for the money he puts into bank, or any part of it, until the next day, without making an allowance of  $\frac{1}{2}$  per cent. unless the bank had been opened only the day before, after balancing the books, or at the feast of Pentecost.

All accounts with the bank must be balanced twice a-year, and that within six weeks after the bank is opened, under the penalty of 15 guilders.

No procuration is good at the bank unless it be registered there, and the noting of such procuration costs 50 stivers.

The time of noting in the bank is from seven to eleven in the forenoon; after which to three in the afternoon every note carried in will cost 6 stivers, and after three none are admitted.

When money is deposited in the bank, the depositor may call for it in specie, when he wants it, upon allowing a premium seldom exceeding  $\frac{1}{2}$  per cent. for bank-security.

No money can be given in loan but upon security that is unexceptionable, and in judging of such security the directors must be unanimous.

No director can use the freedom to abstract any money from the bank to supply any private demand, though but for a single day, upon pain of death.

The strictness and fidelity with which this famous bank is kept, is productive of so great a security on the part of the citizens, and contributes so much to dispatch in money-matters, when a large sum can be negotiated with a single debit and a credit, that a bank-payment is reckoned from *3 to 6 per cent.* better than a payment in cash, besides a premium allowed

lowed the bank for every deposit. It is no wonder then if this bank be immensely rich, since the institution is so judicious, the regulations so wise, and the direction so proper.

In Holland they keep their accounts in pounds, shillings, and pence Flemish, divided as the British pound, but more generally in guilders or florins, stivers, and phinnings, reckoning 16 phinnings to a stiver, and 20 stivers to a guilder or florin.

They reckon likewise 6 guilders or florins to the pound Flemish.

Britain exchanges with Holland upon the pound Sterling, for which the latter gives an uncertain number of shillings and pence, or grotes Flemish, according to the course of exchange, which runs from 30 to 40 s. Flemish per 20 s. Sterling. The par of a pound Sterling, according to Sir Isaac Newton's table, will be found to be  $L. 1 : 16 : 6$  Flemish; but a guinea passes in Holland for 12 guilders, according to which estimation their coins may be reckoned as follows.

A duke - - -	$L. 0 0 0 \frac{1}{16}$
A stiver - - -	$0 0 0 \frac{1}{2}$
A schilling - - -	$0 0 0 6 \frac{3}{16}$
A guilder - - -	$0 0 1 9$
A Zealand dollar - - -	$0 2 7 \frac{1}{2}$
A rix-dollar - - -	$0 4 4 \frac{1}{2}$
A dry guilder - - -	$0 5 3$
A ducat - - -	$0 9 2 \frac{1}{2}$

To compute the exchanges with Holland, or any other place, where it is made upon the pound Sterling.

*Cafe 1.* If Dutch money is required, let  $L. 1$  Sterling be the first term, its equivalent Dutch the second, and the Sterling to be exchanged the third; then will the Dutch money required be the fourth proportional. The equivalent Dutch money will likewise be found very readily by practice.

*Cafe 2.* If Sterling is required, let the Dutch money equivalent to  $L. 1$  Sterling be the first term,  $L. 1$  the second, and the given Dutch the third, then will the Sterling required be the fourth proportional.

#### Examples of exchanges with Holland.

*Exam. 1.* London is indebted to Holland in  $L. 270 : 8 : 2$  Sterling, with how much Flemish is London debited at Amsterdam, when the exchange is at 35 s. 6 d. Flem. per pound Sterl.?

Q q 2

L.

*L. St. L. Fl.*

$$1 : 1.775 :: 270.40833$$

$$\underline{1.775}$$

$$\begin{array}{r} 135204166 \\ 1892858333 \\ 18928583333 \\ \hline 27040833333 \end{array}$$

$$479.97479166 = L. 479\ 19\ 5\frac{1}{2}\text{d. Flemish.}$$

By practice thus :

*L. Flem.*

$$\begin{array}{rcc} 270 & 8 & 2 \text{ for } 1\ 0\ 0 \\ 135 & 4 & 1 \text{ for } 0\ 10\ 0 \\ 67 & 12 & 0\frac{1}{2} \text{ for } 0\ 5\ 0 \\ 6 & 15 & 2\frac{1}{2} \text{ for } 0\ 0\ 6 \\ \hline \end{array}$$

$$479\ 19\ 5\frac{1}{2}\text{d. as before.}$$

*Exam. 2.* Holland is indebted to London in nett proceeds of tobacco, booked there at 479*L.* 19*s.* 6*d.* Flemish; for how much Sterling may London draw, the exchange being at 35*s.* 6*d.* Flemish per pound Sterling?

*L. Flem. L. St. 5) L. Flem.*

$$5) 1.775 : 1 :: 479.975$$

$$5) .355 \quad 5) 95.995$$

$$.071 \quad .071) 19.199 (270-8-2 \text{ Sterling. Answer.}$$

$$\underline{\underline{142}}$$

$$\underline{499}$$

$$\underline{497}$$

$$\underline{\underline{29}}$$

$$\underline{20}$$

$$\underline{\underline{580}}$$

$$\underline{568}$$

$$\underline{\underline{12}}$$

$$\underline{\underline{12}}$$

$$\underline{\underline{144}}$$

$$\underline{\underline{142}}$$

$$(6)$$

*Exam.*

*Exam. 3.* Amsterdam remits to London 5104 guilders, 8 stivers, and 2 pennicks, exchange at 35 s. 6 $\frac{1}{2}$  d. Flem. banc. what will this remittance credit Amsterdam for at London?

First, find the guilders contained in the exchange, by proportioning, as 20 is to 6, thus:

$$\begin{array}{r} 2|20 : 2|6 :: 35.5416 : 10.6625 \\ 10 : \quad 3 \end{array}$$

Hence to multiply by .3, being the same as to multiply by 3 and divide by 10, .3 will be a common multiplier; the product of which into the course of exchange will always give the guilders contained in a pound Sterling: wherefore,

$$\begin{array}{r} 35.5416 \\ \cdot 3 \\ \hline \end{array}$$

10.66250 guilders = L. 1 at 35 s. 6 $\frac{1}{2}$  d. Flem. exch.

Then 10.6625 : 1 :: 5104.40625

$$\begin{array}{r} 2.1325 \quad 1020.88125 \\ .4265 \quad 204.17625 \\ .0853 \quad ) 40.83525 (478.725 = L.478-14-6 \\ \hline 3412 \end{array}$$

$$\begin{array}{r} 6715 \\ 5971 \\ \hline \end{array}$$

$$\begin{array}{r} 7442 \\ 6824 \\ \hline \end{array}$$

$$\begin{array}{r} 6185 \\ 5971 \\ \hline \end{array}$$

$$\begin{array}{r} 2140 \\ 1706 \\ \hline \end{array}$$

$$\begin{array}{r} 4340 \\ 4265 \\ \hline \end{array}$$

(75)

Or

$$\text{Or thus, } 35 \frac{6\frac{1}{2}}{12} \quad 5104 \frac{8}{40}$$

$$5) 426.5 \quad 5) 204176.25$$

$$85.3 \quad 40835.25 \quad (478.725 = L. 478.14.6$$

$$\underline{3412}$$

6715

5971

7442

6824

6185

5971

2140

1706

4340

4265

(75)

*Exam. 4.* London is indebted to Amsterdam in L. 478, 14 s. 6 d. Sterling, for how many guilders may Amsterdam draw, the exchange being at 35 s. 6 $\frac{1}{2}$  d. per pound Sterling?

478.725	Or,
35 schillings.	478.725 for L. 1
$\frac{2393625}{1436175} \}$ at 35	239.3625 for — 10
239362 at - - 6	119.68125 for — 5
19946875 at - - $\frac{1}{2}$	11.968125 for — — 6
	.997344 for — — $\frac{1}{2}$
	850.734219 L. Flemish.
17014.684375 schillings.	6
•3	
5104.4053125 guilders.	

*Exam. 5.* A merchant in Glasgow is credited at Amsterdam for 2154 guilders, 12 stivers, and 4 pennicks, current money; for how much Sterling may Glasgow value on Amsterdam, exchange 35 s. 3 $\frac{1}{2}$  d. Flem. banc. agio 4 $\frac{1}{2}$  per cent.?

First,

C.	B.	C.	B.
First, 104.5	: 100	:: 2154.6125	: 2061.83
2dly, 35 3½	2061.83		
12	40		
<hr/> 423.5	) 82473.20		

194.742 Answer, = L. 194.14.10

Exam. 6. Glasgow draws on Amsterdam for L. 194:14:10 Sterling, exchange at 35 s. 3½ d. Flem. banc. per pound Sterling; how much current money will pay the draught, agio at 4½ per cent.?

First, 194.742	for 20 Then 5°  2061.83
97.371	for 10                  41.236 for 2 per cent.
48.6855	for 5                  41.236 for 2
2.434275	for — 6                  10.309 for ½
.4057125	for — ½ <hr/> 2154.613 cur. guilders.
<hr/> 343.6384875	Flem. pounds.
6	<hr/> 2061.830925 guilders banco.

Exam. 7. When the exchange is at 35 s. what is the price of the guilder?

35		
12		
<hr/> 420	d. Flem.	: 240 :: 40
<hr/> 3) 21		<hr/> 12
7		<hr/> 4
		<hr/> 40
7) 160		<hr/>
		22½ d. Sterl. Answer.

Exam. 8. When the guilder is sold at 22½ d. Sterling, what is the exchange betwixt Britain and Holland?

22½

$$\begin{array}{r} 22\frac{6}{7} : 40 :: 240 \text{ Sterl.} \\ \hline 4) 160 \\ \hline 4 \\ \hline 12) 420 \end{array}$$

35 Answer.

*Exam. 9.* When the bank guilder is worth 22 d. Sterling, what ought to be the price of the current guilder, agio 4 per cent.?

$$\begin{array}{r} 104 : 100 :: 22 \\ \hline 104 ) 2200 \end{array}$$

21 $\frac{2}{3}$  d. Answer.

Holland exchanges with the trading nations upon the continent as follows :

With France, upon the French crown, for 54 grotes, more or less,  
 With Hamburg, upon the dollar, for 32 Flemish schillings, more or less,  
 With Spain, upon the ducat of 375 maravillas, for 97 grotes, more or less,  
 With Portugal, upon the crusado of 400 reas, for 44 grotes, more or less,  
 With Genoa, upon the piastre of 5 liv. banco, for 92 grotes, more or less,  
 With Venice, upon the ducat of 24 gros banco, for 88 grotes, more or less,  
 With Leghorn, upon the piastre of 20 sols d'or, for 86 grotes, more or less,  
 With Geneva, upon the crown of 60 sols curf, for 90 grotes, more or less,

according to the course.

And the computation is the same with the exchanges betwixt Britain and the same places.

#### Art. IV. Exchange with the Austrian Netherlands.

Antwerp was once the metropolis for trade of the whole Seventeen Provinces, although Amsterdam and Rotterdam are at this day by far its superiors. It is however the principal place of exchange in the Austrian Netherlands, and still hath a considerable trade. The par of a pound Sterling at Antwerp, according to Sir Isaac Newton, is 34 s. 11 d. Flemish; but later authors make it different; some 35 s. 6 $\frac{1}{2}$  d. others 35 s. 2 d. and others 38 $\frac{2}{3}$  s.; but the course is allowed by all to run from 30 to 40 schillings Flemish per pound Sterling.

Examples

*Examples of exchanges with Antwerp.*

*Ex. 1.* How much Flemish will answer a London draught of L. 374, 19 s. Sterling, exchange at 37 s. 6 d. per pound Sterling?

$$\begin{array}{r}
 374 \ 19 \ 0 \text{ for } 20 \\
 187 \ 9 \ 6 \text{ for } 10 \\
 93 \ 14 \ 9 \text{ for } 5 \\
 46 \ 17 \ 4\frac{1}{2} \text{ for } 2 \ 6 \\
 \hline
 & & 37 \ 6
 \end{array}$$

L. 703 0 7½ Flemish. *Answer.*

*Exam. 2.* How much Sterling will answer an Antwerp bill of L. 703 : 0 : 7½ Flemish, exchange at 37 s. 6 d. Flemish per pound Sterling?

$$\begin{array}{r}
 37 \ 6 \\
 2 \\
 \hline
 75 \text{ sixpences} : 1 :: 5)28121.25 \text{ sixpences.} \\
 \hline
 5)5624.25 \\
 \hline
 3)1124.85 \\
 \hline
 & & 374.95 = L. 374, 19 s. Ster.
 \end{array}$$

In the last example one sixpence is added to the product of 40 out of the 7½ d. and the remaining three halfpence reduced to the decimal of sixpence ; the division is made by the component parts of 75.

The different towns in Britain exchange with London for a small premium in favour of the latter, more or less, according to the demand for bills ; and, in the same manner, Antwerp and the other towns in the Netherlands exchange with Amsterdam from  $\frac{1}{2}$  to  $1\frac{1}{2}$  per cent. in favour of Amsterdam, and the calculations in both cases are made as in factorage. We shall give an example of each.

*Exam. 1.* Glasgow draws on London for L. 760 at 30 days,  $\frac{1}{2}$  exchange ; how much money is received for the draught ?

760

$$\begin{array}{r} 3 \ 16 \text{ for } \frac{1}{4} \text{ per cent.} = \frac{1}{100} \\ 19 \text{ for } \frac{1}{4} \text{ per cent.} = \frac{1}{4} \text{ of } \frac{1}{100} \end{array}$$

$$\underline{\underline{4 \ 15 \text{ exchange at } \frac{5}{6}}}$$

*L. 764-15 paid for the bill. Answer.*

*Exam. 2.* Antwerp draws on Amsterdam for *L. 760 Flemish*, exchange at  $1\frac{1}{2}$  per cent.; how much Flemish will pay the bill at Antwerp?

760

$$\begin{array}{r} 7 \ 12 \text{ at } 1 \text{ per cent.} = \frac{1}{100} \\ 3 \ 16 \text{ at } \frac{1}{2} = \frac{1}{2} \text{ of } \frac{1}{100} \end{array}$$

$$\underline{\underline{11 \ 8 \text{ exchange at } 1\frac{1}{2}}}$$

*L. 771 8 paid at Antwerp. Answer.*

## Art. V. EXCHANGE with HAMBURG.

### I N T R O D U C T I O N.

Although the funds of the bank of Hamburg are much inferior to those of the bank of Amsterdam, yet the integrity and exactness with which it is conducted, hath procured it a very great reputation all over Europe, and still more particularly in the north.

The inspection of this bank comes not under the cognisance of the senate; but as the citizens and corporations are sureties, they chuse four directors by a majority of voices from among the principal of their own number. These directors must see that the regulations of the bank be duly observed, and impress the cashiers with money without touching the treasure, when there is a demand. Though the capital of this bank is supposed to be very considerable, yet it is very difficult to conclude concerning it with any degree of certainty, as the clerks and managers are bound to secrecy by oath; which hath likewise this effect, that no seizure can be made of any deposit or bank account, as no one knows how another stands with the bank. None but citizens can have the

the benefit of a bank-account, or of lodging money there by way of deposit, without interest. The least sum which can be wrote in bank is 100 marks lubs, and every sum betwixt 100 and 300 marks lubs that is wrote in bank, is taxed with a duty of 2 schillings lubs at entry, but any sum exceeding 300 marks is entered *gratis*.

The stated hours of transacting business in the bank are from seven to ten in the morning; but the bank is kept open to five in the afternoon for such as have a mind to write in bank at the expense of 2 sols lubs for each sum.

Every account that is opened with the bank costs an expense of 50 rix-dollars, or 150 marks lubs. Loans are made out of this bank upon a deposit of silver, jewels, precious stones, &c. which are received upon inventory; and if not relieved at the end of six months, by paying the principal and a moderate interest, they are auctioned at the bar of the bank upon a day advertised for their sale and delivery.

They keep their accounts in the bank, and through the city, either in rix-dollars, sols, and deniers lubs, or in marks, sols, and deniers lubs. The rix-dollar is worth 3 marks, or 48 sols lubs, weighing 532 grains.

The livre-gros or pound Flemish is equal to  $7\frac{1}{2}$  marks lubs, or 20 sols gros, or 120 sols lubs.

The mark lubs is divided sometimes into 32 gros, but more generally into 16 schillings lubs, and each of these into 12 phennings.

Hamburg exchanges with Britain in schillings and grotes Flemish, and the par of their rix-dollar is reckoned at 4 s. 6 d. Sterling, so that the par of L. 1 Sterling is 13 marks 5 schillings lubs, 35 s.  $6\frac{3}{4}$  d. Flemish.

The value of the rix-dollar being every where known, and its standard invariable, it is applied to the valuation of all kinds of merchandises as well as coins. The principal current coins in these parts are those of Denmark and Holstein, Lubeck and Hamburg, which, taking them at the par, may be valued as follows.

A tryling $\frac{1}{4}$ of a phenning - - -	L. 0 0 0 $1\frac{3}{8}$ Sterl.
A sixling $\frac{1}{2}$ of a phenning - - -	0 0 0 $\frac{3}{4}$
A phenning $\frac{1}{16}$ of a schilling lubs - - -	0 0 0 $\frac{3}{8}$
One schilling lubs $\frac{1}{16}$ of a mark - - -	0 0 1 $\frac{1}{8}$
The dollar = 2 marks - - -	0 3 0
The rix dollar = 3 marks - - -	0 4 6
The ducat of $6\frac{1}{4}$ marks - - -	0 9 4 $\frac{1}{2}$

The current money hath been so much adulterated of late years, that the agio hath risen from 15 to 20, to 30, and 40 per cent.; but all bills of exchange are paid in bank.

*Examples of exchange with Hamburg.*

*Exam.* 1. London draws on Hamburg for L. 500 *Sterling*; how many marks must be paid at Hamburg, the exchange at 35 s. 6 d. Flem. *banco per pound Sterling*?

As 20 s. Flem. =  $7\frac{1}{2}$  marks, it will be  $20 : 7.5 :: 35.5$

The two first terms abridged by 5     $4 : 1.5 :: 35.5$

1.5

4) 53.25

13.3125 marks

= L. 1 *Sterl.*

Hence 1.5 will be a constant multiplier, and 4 a constant divisor, for finding the marks contained in the course of exchange, equal to a pound *Sterling*.

L.      marks.      L.      m.      s.

Then 1 : 13.3125 :: 500 : 6656 4 lubs.

Or in other words, since 1 is the first term, multiply the marks in one pound by the given number of pounds, and the product answers the question.

For 13.3125

500

— marks. s.

6656.25 = 6656 4 lubs. *Answer.*

Or by practice thus:

$$\begin{array}{r} 35 \ 6 \\ \quad 3 \\ \hline 8) 106 \ 6 \\ \quad 13 \ 5 \end{array}$$

Schillings Flemish are reduced into marks by multiplying by 6, and dividing by 16; hence the reason of this operation, which is only an abridgment of that.

Then 500 L.

13 marks.

6500 for 13 marks

125 for 4 s. or  $\frac{1}{4}$  of a mark.

$31\frac{1}{4}$  for 1 s. or  $\frac{1}{4}$  of  $\frac{1}{4}$ .

6656 4 marks, as before.

*Exam.*

*Exam. 2.* Hamburg draws on London for 6656 marks, 4 schillings lubs; what Sterling must be paid at London, the exchange at 35 s. 6 d. Flem. banco per pound Sterling?

$$\begin{array}{r} 35.5 \\ - 1.5 \\ \hline 4) 53.25 \end{array}$$

$$\begin{array}{r} 13.3125 ) 6656.25 \text{ (500 l. Sterling. Answer.)} \\ \quad 6656.25 \\ \hline \quad (0) \end{array}$$

Or by practice, thus.

$$\begin{array}{r} 35.6 \quad 6656.4 \\ - 6 \quad \quad 16 \\ \hline 213 \quad \quad 106500 \text{ (500 as before.)} \\ \quad 1065 \\ \hline \quad (0) \end{array}$$

*Exam. 3.* Hamburg is indebted to London in the nett proceeds of a consignment, amounting to 5600 marks current money; for how much Sterling may London draw, exchange at 36 s. Flem. banco, agio 30 per cent.? Cancel a cipher in each of the first terms, and then it will be

C. B.

$$13 : 10 :: 5600$$

$$\begin{array}{r} 10 \\ \hline 13) 56000 \\ \quad 4333\frac{1}{3} \text{ banco.} \\ \quad 3 \quad 8 \\ \hline 98 \quad 34666\frac{2}{3} \text{ (353.742 fere = L. } 353.14-10 \\ \quad 294 \\ \hline \quad 526 \\ \quad 490 \\ \hline \quad 366 \\ \quad 294 \\ \hline \quad 726 \\ \quad 686 \\ \hline \quad 406 \\ \quad 383 \\ \hline \quad (23) \end{array}$$

*Exam.*

## 318 VARIETIES in PROPORTION. Part IV.

*Exam. 4.* When the exchange with Hamburg is at 36 s. 6 d. Flem. banco per pound Sterling, what is the price of the mark lubs?

s. Fl. s. lubs,

$$20 = 120$$

$$10 = 60$$

$$5 = 30$$

$$\frac{1}{2} = 6$$

$$0.6 = 3$$

---

d.

$$219 : 240 :: 16$$

16

$$219) \overline{3840} (17\frac{1}{2})$$

219

---

1650

1533

---

117

## O B S E R V A T I O N.

When Britain exchanges upon the imaginary price of 1 l. Sterling, as with the Netherlands and Hamburg, the British merchant will find his account, in remitting when the course is high, and drawing when it is low; as it is better to get 36 s. Flemish for a pound Sterling than 35 s. If what hath been said in the last observation has been duly considered and digested, an example will be sufficient illustration.

*Examples of drawing and remitting, &c.*

*Exam. 1.* When the exchange was at 36 s. 6 d. Flemish, Britain remitted to Amsterdam L. 5000 Sterling, but upon the fall of the exchange to 35 s. Flemish, Britain drew to be reimbursed; for how much Sterling may Britain draw to bring home this remittance?

*Sixpences.*

$$73 : 5000 :: 70$$

73

$$7) \overline{36500.10}$$

L. 5214 5 8*4* Answer.

*Exam.*

Var. XI. EXCHANGE with HAMBURG. 319

*Exam. 2.* When the exchange was at 36 s. 6 d. Flemish, Britain remitted to Amsterdam 5000 guilders; but when the exchange fell to 35 s. Britain drew for the same number of guilders; what advantage to Britain attended the negotiation?

First, 36.5	2dly, 35	5000
.3	.3	—
<u>5) 10.95</u>	<u>10.5</u>	<u>3) 1000</u>
<u>5000</u>	<u>2.1</u>	<u>7) 3333.33</u>
2.19 ) 1000.00(456 12 5. at 36 s. 6 d.	Drawn 476 3 9 $\frac{3}{4}$	
876	Remitted 456 12 5	
<u>1240</u>		
1095		
<u>—</u>	Gained 19 11 4 $\frac{1}{2}$	
1450		
1314		
<u>—</u>		
136		
20		
<u>—</u>		
2720		
219		
<u>—</u>		
530		
438		
<u>—</u>		
92X12=1104		
1095		
<u>—</u>		
9		

Hamburg exchanges with other trading nations as follows.

With France, upon the crown of 60 sols, for 27 schill. lubs, more or less,  
 With Spain, upon the ducat of 375 marvades, for 93 gros, more or less,  
 With Portugal, upon the crusade of 400 reas, for 42 gros, more or less,  
 With Venice, upon the banc ducat of 24 gros, for 86 gros, more or less,  
 With Vienna, upon 100 Hamburg rix-dollars banco, for 139 rix-dollars of  
     the empire, more or less,  
 With Nuremberg, on ditto, for 135 dollars of Nur. more or less,

according to the course.

E X A M P L E S.

*Exam. 1.* Reduce 975 marks 2 $\frac{1}{2}$  schillings lubs, bank-money

ney of Hamburg, into livres, sols, and deniers tournois of France; exchange at  $27\frac{1}{4}$  sols lubs banco per crown, of 60 sols tournois?

**Exchange at  $27\frac{1}{4}$**      $975 \frac{2}{3}$

$$\begin{array}{r} 16 \\ \hline 649 \\ 15602\frac{1}{3} \\ \hline 24 \\ \hline 649) 394456 \\ \hline Cr. 576 58 5 \\ \hline 3 \end{array}$$

Livres 1730 18 5 French money. *Answer.*

**Exam. 2.** In 1730 liv. 18 sols, 5 den. how many Hamburg marks, exchange at  $27\frac{1}{4}$  sols lubs per crown, of 60 sols tournois?

$$\begin{array}{r} 3) 1730 18 5 \\ \hline \text{Crowns} 576 58 5 \\ \hline 47\frac{1}{4} \end{array}$$

$$\begin{array}{l} 4032 \\ 1152 \\ 24 = \frac{1}{4} \text{ of } 576 \\ 13\frac{1}{2} \text{ for } 30 \text{ sols} \\ 9 \text{ for } 20 \\ 1\frac{1}{4} \text{ for } 4 \\ 1\frac{1}{4} \text{ for } 4 \\ \frac{1}{6} \text{ for } - 4 \text{ deniers} \end{array}$$

$$\begin{array}{r} 4) 15602\frac{1}{3} \\ \hline 4) 3900\frac{1}{3} \end{array}$$

Marks 975 2 *Answer.*

After the same manner may the exchange betwixt Hamburg and any other place, when the exchange is made on the piece, be computed. When the exchange is made on the 100 rix-dollars, the computation is similar to that betwixt Britain and the plantations.

Art.

## Art. VI. EXCHANGE with FRANCE.

Paris and Bourdeaux are the principal places of exchange in France; and indeed, in these places, the business of exchange is particularly studied. Accounts are kept throughout the French dominions, in livres, sols, and deniers, divided as the British pound. In exchanging with France, we pay so many pence Sterling for their crown or ecu of 3 livres, or 60 sols tournois. As they have not always any piece of coin of that value, this ideal crown, or crown of exchange is distinguished from the real crown, or ecu d'argent, by the name of the *crown of 60 sols tournois*.—The exchange between France and other countries varies more than any other, occasioned by the frequent alterations of their coin, which is the effect of an arbitrary government. The king may at any time, by an arret, order the money to be recoined, at what number of crowns he pleases out of the mark, which must render their value variable in other places, whatever value royal authority may stamp upon them in France. By the last arret of the 15th of June 1726,  $8\frac{1}{2}$  crowns are appointed to be coined out of the mark of 7 ounces, 17 pennyweights, and 12 grains English, and each of these crowns to pass for 6 livres; so that, in that case, the mark contains 50 livres 5 sols, which is intrinsically worth 40 s.  $8\frac{1}{2}$  d. *Sterling*. Wherefore the present par of 3 livres, or crown of 60 sols tournois, is 29.149 pence English, and whatever is paid more or less than this, must be a gain or a loss. According to this par their coins may be estimated as follows:

A denier - - - - -	L. 0 0 0 $\frac{13}{320}$
A liard of 3 deniers - - - - -	0 0 0 $\frac{39}{320}$
A dardene of 2 liards - - - - -	0 0 0 $\frac{39}{160}$
A sol of 2 dardenes - - - - -	0 0 0 $\frac{39}{80}$
A frank of 20 sols or 1 livre - - - - -	0 0 9 $\frac{1}{4}$
A crown of exchange 60 sols - - - - -	0 2 5 $\frac{1}{4}$
A double crown of 6 livres - - - - -	0 4 10 $\frac{1}{4}$
A lewis d'or of 8 crowns - - - - -	0 19 6

They have likewise half and quarter crowns, and half and quarter franks.

To compute the exchanges with France.

*Cafe 1.* If French money is required, let the course of exchange in pence be the first term, 1 crown or 3 livres according to the requisition, be the second term, and the given Sterling reduced to pence the third, then will the fourth proportional be the answer.

*Case 2.* If Sterling be required, reverse the last rule, or make the calculation by practice.

These rules universally hold in computing the exchanges with those places where the exchange is made on any fixed piece, the price whereof is determined by the course.

When France exchanges with other countries, who have likewise a piece of exchange, 100 of the one is given for so many of the other, and the calculations made in that proportion.

*Examples of the exchanges with France.*

*Exam. 1.* London draws on Paris for L. 566 : 13 : 4; how many livres, &c. will clear the draught at Paris, the exchange being at 31 d. per ecu?

$$\begin{array}{r}
 566 \quad 13 \quad 4 \\
 \underline{-} \quad 20 \\
 11333 \\
 \underline{-} \quad 12 \\
 1133 \\
 \underline{-} \quad 12 \\
 \text{d. livres} \quad \underline{\quad} \\
 31 : \quad 3 :: \quad 136000 \\
 \underline{-} \quad 3 \\
 31) \quad 408000 \quad (13161 \quad 5 \quad 9\frac{1}{2} \\
 \underline{-} \quad 31 \\
 98 \\
 \underline{-} \quad 93 \\
 50 \\
 \underline{-} \quad 31 \\
 190 \\
 \underline{-} \quad 186 \\
 40 \\
 \underline{-} \quad 31 \\
 9 \\
 \underline{-} \quad 20 \\
 180 \\
 \underline{-} \quad 155 \\
 25 \\
 \underline{-} \quad 12 \\
 300 \\
 \underline{-} \quad 279 \\
 \text{21}
 \end{array}$$

*Exam.*

*Exam. 2.* London received at Paris 13161 livres 5 sols and 9½ deniers; for how much Sterling was the draught, the exchange being at 31 d. *Sterling per ecu?*

$$\begin{array}{r}
 L. 3 : 31 :: 13161.263 \text{ Or thus } 3) 13161 \quad 5 \quad 9\frac{1}{2} \\
 \underline{31} \\
 39483789 \\
 \underline{3) 407999.153} \\
 12) 135999.717 \\
 \underline{20) 11333 \quad 3\frac{3}{4}} \\
 566 \quad 13 \quad 3\frac{3}{4}
 \end{array}
 \qquad
 \begin{array}{r}
 8) 4387 \quad 1 \quad 11 \\
 \underline{30) 548 \quad 7 \quad 9} \text{ at } 30 \text{ d.} \\
 18 \quad 5 \quad 7 \text{ at } 1
 \end{array}
 \qquad
 L. 566 \quad 13 \quad 4 \text{ Answer.}$$

### Art. VII. EXCHANGE with SPAIN.

The monies of Spain are of two sorts, the one called *plate money*, and the other *vellon*. A rial vellon is worth in Spain 8½ quarts copper money, and the rial of old plate of exchange is worth 16 vellon, and the effective rial of new plate 17; which makes a difference between these two species of rials as 32 to 17, or 53½ per cent.

By the word *plate*, is understood silver money, wherein some of the merchants keep their accounts; and that which is used for the negotiation of foreign exchanges is distinguished by the name of *old plate*, which is ideal in the same sense with the exchange crown of France, or the pound Sterling of Britain.

The foreign bankers or remitters at Madrid, Cadiz, Seville, &c. keep their accounts in piastras, rials, and mervadies old plate, reckoning 34 mervadies to a rial, and 8 rials to a piastre, the par of which is 3 s. 7 d. *Sterling*.

The shopkeepers of Madrid, the customhouse, and other dealers within the kingdom, keep their accounts in rials and mervadies vellon. Some merchants, particularly in Valencia, keep their accounts in piastras, sols, and deniers, divided as the French livre, or British pound.

The duobloon of exchange is equal to 4 piastras, or 32 rials.

The ducat, or ducado current, is equal to 11 rials old plate,

plate, or 374 mervadies ; but the ducat of exchange is equal to 375 mervadies.

The piastre of exchange is likewise reckoned at 15 rials vellon, and 2 mervadies. — It is proper that the drawer of a bill upon Spain should expressly stipulate the payment to be made either in gold or silver, or the bearer of the bill may sustain a loss of  $1\frac{1}{2}$  per cent.

The Spanish silver and copper coins, from the above par, may be estimated as follows.

A mervadie	- - - - -	L	0 0	$0\frac{43}{272}$
A quartil = 2 mervadies	- - - - -	0 0	0	$\frac{43}{136}$
A rial plate = 17 quartiles, or 34 mervadies	- - - - -	0 0	$5\frac{3}{8}$	
A pistreine = 2 rials plate	- - - - -	0 0	$10\frac{3}{4}$	
A dollar, old plate of Seville = 10 rials	- - - - -	0 4	6	
A dollar of new plate = 8 rials plate	- - - - -	0 3	7	
Mexico ditto	- - - - -	0 4	6	
Pillar ditto	- - - - -	0 4	$6\frac{3}{4}$	
Peru ditto old plate	- - - - -	0 4	5	
A cross dollar	- - - - -	0 4	$4\frac{3}{4}$	

The gold coins are pistoles and their fractions : the pistole is worth 4 dollars, or 17 s. 11 d. and the fractions in proportion.

The course between Britain and Spain is always below par, from 35 to 40 pence per piastre.

#### Examples of exchanges with Spain.

Exam. 1. London remits to Cadiz L. 576 : 12 :  $2\frac{3}{4}$  Sterling, exchange at  $37\frac{1}{8}$  d. per piastre ; how much will be received for this remittance at Cadiz ?

d. piaſt.	L.
$37\frac{1}{8} : 1 ::$	576 12 $2\frac{3}{4}$
	<u>20</u>
303	<u>—</u>
	11532
	<u>12</u>
	138386
	<u>8</u>
	<u>107094</u>
	carried forward

piaſt. ri. mer.

$$\begin{array}{r}
 303) 1107094 (3653.775 = 3653 \ 6 \ 7 \text{ Answer.} \\
 909 \\
 \hline
 1980 \\
 1818 \\
 \hline
 1629 \\
 1515 \\
 \hline
 1144 \\
 999 \\
 \hline
 2350 \\
 2121 \\
 \hline
 2290 \\
 2121 \\
 \hline
 1690 \\
 1515 \\
 \hline
 175
 \end{array}$$

*Exam. 2.* Cadiz remits to London 3653 piaſt. 6 ri. 7 mer. at  $37\frac{7}{8}$  exchange per piaſtre; what will the remittance amount to in London?

$$\begin{array}{r}
 3653.775 \\
 \hline
 \frac{7}{8} \quad 365.3775 = 2 \text{ s.} \\
 \frac{1}{2} \quad 182.6887 = 1 \\
 \frac{1}{4} \quad 22.8361 = 1\frac{1}{2} \\
 \frac{3}{8} \quad 5.7090 = \frac{3}{8} \\
 \hline
 576.6113 = L. 576 12 2\frac{3}{8} \text{ Answer.}
 \end{array}$$

Rials vellon are reduced into rials of exchange, and the contrary; piaſtres effective into rials of exchange, and rials vellon, as in the following examples.

*Exam 1.* Reduce 5005 ri. 16 mer. vellon to rials of ex-change.

ri.

ri. mer.  
 $\underline{5005 \quad 16}$   
 $8\frac{1}{2}$

$40044 =$  the product of 8, to which 4 is added for 16 m.  
 $2502\frac{1}{2} =$  the product of  $\frac{1}{2}$

4)  $42546\frac{1}{2} =$  the product of  $8\frac{1}{2}$  quarts in a rial vellon.

4)  $10636\frac{5}{8}$

$2659\frac{5}{8}$  rials of exchange. *Answer.*

*Exam. 2.* Reduce 2659 ri. 5 mer. of exchange, to rials vellon.

$2659 \quad 5$   
 $16$  quarts in a rial of exchange.

8.5)  $42546\frac{1}{2}$

Rials 5005 16 mer. vellon. *Answer.*

*Exam. 3.* Reduce 250 piastres effective to rials vellon.

8

$\underline{\quad 2000}$   
 $\underline{\quad 8\frac{1}{2}}$   
 $\underline{\quad 16000}$   
 $\underline{\quad 1000}$   
 $\underline{\quad 16) \quad 17000}$

1062 16 mervadies vellon. *Answer.*

In most of the inland towns of Spain they keep their accounts in mervadies, hundreds and thousands of mervadies, and exchange upon the ducat of 375 mervadies.

### E X A M P L E.

Spain is indebted to Britain in 5745  $\omega$  575 mervadies, at 52 d. Sterling per ducat of 375 mervadies; how much Sterling will even the account in Britain?

5)

5) 5745 ω 575

12) 1149. 115 for 4 s. Sterl.

Qd. 1149. 115 for 4 d.

L. 1244. 874 = L. 1244. 17. 6 Answer,

### Art. VIII. EXCHANGE with PORTUGAL.

In Lisbon, and, in general, throughout the Portuguese dominions, accounts are kept in milreas and reas, reckoning 1000 of the latter to one of the former. The milrea is no real coin, but an imaginary piece of account, of which a crusado of silver contains  $\frac{480}{1000}$ , or 0 ω 480 = 480 reas. The par of a milrea is 5 s. 7½ d. according to which the gold monies of Portugal are as follows.

The piece of 25 ω 600 double Joannes	- - -	L. 7 4 0
Ditto of 24	- - -	6 15 0
Ditto of 12 ω 800 single Joannes	- - -	3 12 0
Ditto of 12	- - -	3 7 6
Ditto of 6 ω 400 half Joannes	- - -	1 16 0
Ditto of 4 ω 800 moidore stamped	- - -	1 7 0
Ditto of 3 ω 200 quarter Joannes	- - -	0 18 0
Ditto of 2 ω 400 half moidore	- - -	0 13 6
Ditto of 1 ω 600 $\frac{1}{8}$ Joannes	- - -	0 9 0
Ditto of 1 ω 200 quarter moidore	- - -	0 6 9
Ditto of 0 ω 800 $\frac{1}{16}$ Joannes or testoon piece	0	4 6

The silver monies as follow.

The crusado of 400 reas not stamped	- - -	L. 0 2 3
Ditto of 480 reas stamped in 1643	- - -	0 2 8 $\frac{1}{2}$
The 12 vintin piece of 240 reas	- - -	0 1 6
The 5 ditto of 100 reas	- - -	0 0 9
The 2 $\frac{1}{2}$ vintin ditto of 50 reas	- - -	0 0 4 $\frac{1}{2}$

The copper coins as follow.

The vintin piece of 20 reas	- - -	0 0 1 $\frac{1}{2}$
The half and quarter ditto, according to the same proportion.	- - -	
The course of exchange betwixt Britain and Lisbon is betwixt 5 and 5 s. 8 d.	- - -	

EXAMPLES.

## EXAMPLES.

*Exam. 1.* London remits to Oporto L. 578 : 19 : 6, exchange at 5 s. 3 d. per milrea; what will be received at Oporto?

$$\begin{array}{r} s. \ d. \\ 5 \ 3 : 1 \\ \hline 4 \end{array} \begin{array}{r} m. \ d. \\ 578 \ 19 \ 6 \\ \hline 20 \end{array}$$

$$21 \text{ threepen. } \begin{array}{r} 11579 \\ 4 \\ \hline 3) 46318 \\ 7) 15439 \end{array}$$

$$\begin{array}{r} 15439 \\ - 15439 \\ \hline 0 \end{array}$$

2205  $\frac{1}{2}$  619 Answer.

mil. reas.

*Exam. 2.* Oporto remits to London 2205  $\frac{1}{2}$  619, exchange at 5 s. 3 d. per milrea; how much Sterling will this be reckoned at in London?

$$4) 2205.619$$

$$\begin{array}{r} 551.40475 \text{ at } 5 \text{ s.} \\ 27.5702375 \text{ at } 3 \text{ d.} \\ \hline \end{array}$$

$$578.9749875 = L. 578-19-6 Answer.$$

## Art. IX. EXCHANGE with GENOA.

St George's bank hath a prodigious sway in the republic of Genoa, and is constituted of such branches of the public revenue as hath been mortgaged by the government for the payment of such sums as have been borrowed during the exigencies of the commonwealth. The administration is during life, by which means it hath become a kind of inferior senate, which often breaks the uniformity of their aristocratic government.

Accounts are kept in the bank, in piafres or pezzoes, which are divided into soldi and denari, as the British pound; but some of the merchants keep their accounts in lires or liras, soldi and denari, divided as before: this money is only  $\frac{1}{2}$  of the value of the other, as the Scots money is  $\frac{1}{2}$  of Sterling.

The

The par of { a pezzoe or piastre is 54 d.  
 { a lire or lira - - 10 $\frac{4}{5}$

According to which their coins may be estimated as follows:

A denari	- - - - -	L.	0	0	0 $\frac{9}{14\frac{4}{5}}$
A soldi or 12 denari	- - - - -	0	0	0	$\frac{9}{14\frac{4}{5}}$
A chevalet or 4 soldi	- - - - -	0	0	1 $\frac{1}{2}$	
A testoon or 30 soldi	- - - - -	0	1	1 $\frac{1}{2}$	
A genoini or 6 testoons	- - - - -	0	6	9	
A pistole	- - - - -	0	15	0	
A Spanish pistole	- - - - -	0	17	11	

The exchange runs between 45 and 50 d.

*Examples of exchange.*

Exam. 1. In 784 pez. 19 s. 6 d. lire money, how much money of exchange?

$$\begin{array}{r} \text{p. s. d.} \\ 5) 784 \ 19 \ 6 \text{ lire money.} \\ \hline \end{array}$$

156 19 10 $\frac{4}{5}$  exchange money. Answer.

Exam. 2. Reduce 156 pez. 19 s. 10 $\frac{4}{5}$  d. exchange-money, to lires.

$$\begin{array}{r} \text{156 19 10}\frac{4}{5} \\ \hline \text{5} \\ \hline \text{784 19 6 lire money. Answer.} \end{array}$$

Exam. 3. London is indebted to Genoa in L. 1710 16 s. 4 d.; for how many pezzoes may Genoa value on London, the exchange at 47 $\frac{1}{2}$  d.?

$$\begin{array}{r} \text{d. } \frac{1}{2} \text{ p. pez. L. s. d. } \frac{1}{2} \text{ p.} \\ 47\frac{1}{2} = 95 : 1 :: 1710 \ 16 \ 4 = 821192 \\ \hline \text{Pezzoes 8644 2 6 Answer.} \end{array}$$

Exam. 4. Genoa is indebted to London in 8644 pez. 2 s. 6 d.; for how much Sterling may London value on Genoa, the exchange at 47 $\frac{1}{2}$  per pezzoe?

$$6) 8644 \quad 2 \quad 6$$

$$\begin{array}{r} 8) 1440 \quad 13 \quad 9 \quad \text{for } 40 \\ 2) 180 \quad 1 \quad 8\frac{1}{2} \quad \text{for } 5 \\ \hline 90 \quad 0 \quad 10\frac{1}{2} \quad \text{for } 2\frac{1}{2} \end{array}$$

1710 16 4 Answer.

*Exam. 5.* London draws on Genoa for L. 1710 : 16 : 4 Sterling ; how much lire money will pay the draught, exchange at 48 d. per piastre ?

$$1710 \quad 16 \quad 4$$

5 times 4 s. in a pound Sterling.

$$\begin{array}{r} \text{Pezzoes } 8554 \quad 1 \quad 8 \text{ of exchange.} \\ \hline 5 \text{ lires in a pezzoe.} \end{array}$$

42770 8 4 lire money. Answer.

*Exam. 6.* Genoa is indebted to London in 42770 pez. 8 s. 4 d. lire money ; how much Sterling will this amount to, the exchange at 4 s. per pezzoe ?

$$5) 42770 \quad 8 \quad 4$$

5) 8554 1 8 pezzoes of exchange.

L. 1710 16 4 Sterling, at 4 s. exchange.

#### Art. X. EXCHANGE with LEGHORN.

In Leghorn accounts are kept in piastry, soldi, and denari, divided as at Genoa. Some likewise keep their accounts in liras or lires, divided as the piastry ; but this money is only  $\frac{1}{3}$  of the money of exchange.

The par with London is 4 s. 4 d. but the course runs from 45 to 50 d. only.

The coins of Leghorn may be estimated as follows :

A denari - - - - -	L. 0 0 0 $\frac{1}{3}$ $\frac{1}{3}$
A quatrini, - - - 4 denari	0 0 0 $\frac{1}{3}$ $\frac{1}{3}$
A soldi, - - - 3 quatrini	0 0 0 $\frac{1}{3}$ $\frac{1}{3}$
A craca or grain, 5 quatrini	0 0 0 $\frac{1}{3}$ $\frac{1}{3}$
A julio or paulo 8 grains	0 0 5 $\frac{1}{3}$
A piastry of exchange - - - - -	4 4

A ducat of 150 soldi - - - - L. 0 5 5  
 A pistole of 21 lires - - - - 0 15 6

*Examples of exchange.*

*Exam. 1.* London draws on Leghorn for L. 465 : 19 : 6 Sterling ; what must be paid at Leghorn, the exchange at 46 d. per piastre ?

$$\begin{array}{r} 46 : 1 :: 465 19 6 \\ \quad\quad\quad 20 \\ \hline 9319 \\ \quad\quad\quad 6 \\ \hline 23) 55917 \end{array}$$

Piastres 2431 3 5 $\frac{3}{4}$  *Answer.*

*Exam. 2.* Leghorn draws on London for 2431 piaſt. 3 s. 5 $\frac{3}{4}$  d. ; what will pay the bill at London, exchange at 46 d. per piastre ?

$$\begin{array}{r} 6) 2431 3 5\frac{3}{4} \\ \hline 8) 405 3 11 \text{ for } 40 \text{ d.} \\ 5) 50 12 11\frac{1}{2} \text{ for } 5 \\ 10 2 6\frac{1}{2} \text{ for } 1 \\ \hline 465 19 .5 \end{array}$$

465 19 .5 *Answer.*

*Exam. 3.* London is indebted to Leghorn in 7456 piaſt. 9 s. 6 d. lire money ; what Sterling stands as an equivalent in the London merchants books, the exchange being at 50 d. per piastre ?

$$\begin{array}{r} 6) 7456 9 6 \\ \hline 6) 1089 4 11 \text{ money of exchange.} \\ 4) 181 10 9\frac{5}{6} \text{ at } 40 \text{ d.} \\ 45 7 8\frac{1}{2} \text{ at } 10 \\ \hline 226 18 6\frac{1}{4} \text{ at } 50 \end{array}$$

226 18 6 $\frac{1}{4}$  at 50 *Answer.*

By estimating the fractions sometimes above their value, and at other times below the value, as seems most convenient, perplexing denominators are avoided, and the answer found with sufficient exactitude.

## Art. XI. EXCHANGE with VENICE.

The bank of Venice, or *banco del giro*, is a public depository for the reception of the merchants and traders money, established by an edict of the republic, enacting, that the payment of merchandises by wholesale, and bills of exchange, is to be made in bank; and that all debtors and creditors shall be obliged, the one to carry their money to the bank, and the other to receive their payment in bank, by transfers from the one account to the other: but to remedy the inconvenience that may arise for want of ready money to satisfy the demands of strangers, or supply the necessities of a retail-trade, a ready-money bank account hath been opened, for effective payments, which, instead of diminishing the funds of the bank, hath been found, by experience, greatly to have increased them.

— The funds of this bank are fixed to 5,000,000 of ducats, for the administration of which the republic is guarantee.

— The accounts of the bank are kept in livres, sols, and deniers gros: the livre is equal to 10 ducats bank, or 240 gros, the ducat being equal to 24 gros.

Money of exchange is always understood to be that of ducats in bank, which is imaginary, 100 whereof make 120 ducats current money; so that the difference betwixt bank and current money is an agio of 20 per cent. though the brokers have invented another agio to be added, which is more or less according to bargain.

The par of a ducat banco is 4 s. 4 d. *Sterling*, and the course between 45 and 50 d.

The Venetian coins are as follow:

A picoli - - - - -	L. 0 0 0 $\frac{3}{4}$
A soldi, or 12 picoli - - - - -	0 0 0 $\frac{50}{60}$
A jule, or 18 soldi - - - - -	0 0 5 $\frac{75}{80}$
A testoon, or 3 jules - - - - -	0 1 5 $\frac{10}{12}$
A ducat current, or 124 soldi - - - - -	0 3 4
A chequin, or 17 lires - - - - -	0 9 2

Lire money is divided as the British pound, and 1 ducat banco is worth  $7\frac{4}{5}$  lires.

*Examples of exchange.*

*Exam. 1.* London draws on Venice for L. 541, 18 s. *Sterling*; how much must be paid at Venice, exchange at 45 $\frac{5}{6}$  d. per ducat banco?

541 18

20

---

10838

12

---

*duc. s. d.*45.625) 130056.000 (2850 10 10 $\frac{4}{7}$  *Answer.*  
91250

---

388060

---

365000

---

230600

---

228125

---

24750

20

---

495000

---

45625

---

38750

12

---

465000

---

45625

---

8750

*Exam. 2.* Venice draws on London for 2850 *duc.* 10 *sol.* 10 $\frac{4}{7}$  *den.* *banco*, exchange at 45 $\frac{1}{2}$  *d.* per *ducat*; how much Sterling will pay the draught?

6) 2850 10 10 $\frac{4}{7}$ 

---

8) 475 1 9 $\frac{2}{7}$  at 40 *d.*

---

8) 59 7 9 at 5 *d.*

---

7 8 5 $\frac{1}{2}$  at  $\frac{1}{2}$ 

---

*L. 541 18 0 at 45 $\frac{1}{2}$  Answer.*

*Exam. 3.* Venice is indebted to London in 4789 *duc.* 19 *s.* 3 *d.* current money; how much Sterling may London draw for, agio at 20 *per cent.* when the exchange is at 4 *s.* 1 *d.* per *ducat banco*?

$$6 : 5 :: 4789 \ 19 \ 3 \text{ current money.}$$

5

$$\underline{6) \ 23949 \ 16 \ 3}$$

$$\underline{\underline{10) \ 3991 \ 12 \ 8\frac{1}{2} \text{ banc. money.}}}$$

$$\underline{2) \ 399 \ 3 \ 3\frac{1}{4} \text{ at } 2}$$

$$\underline{199 \ 11 \ 7\frac{5}{8} \text{ at } 1}$$

$$\underline{\underline{12) \ 199 \ 11 \ 7\frac{5}{8} \text{ at } 1}}$$

$$\underline{16 \ 12 \ 7\frac{5}{8} \text{ at } 1}$$

$$\underline{\underline{L. \ 814 \ 19 \ 2\frac{1}{8} \text{ at } 4 \ 1}}$$

The first two terms are abridged by cancelling the ciphers, and reducing the remaining figures to their lowest terms.

The foregoing are the remarkable places of exchange in Europe with which Britain hath occasion to negotiate, as all the other places with which we may have commerce receive and make payments by the medium of Amsterdam, Hamburg, or Venice; and if the examples of converting the money of one country into that of another hath been properly attended to, it may reasonably be presumed, when the course of exchange is known, that there will be very little difficulty in reducing Sterling to its equivalent value in the money of account in any other country, and the contrary.

#### O B S E R V A T I O N.

When Britain exchanges on the piece of foreign money, as the French crown, Venetian ducat, &c. Britain ought to remit when the exchange is low, and draw when it is high, to negotiate with advantage. The reason will be obvious; for L. 100 will go farther in purchasing ducats, crowns, milreas, &c. when the course of exchange is at 40 d. than when it is at 50 d. and 100 crowns will go farther in paying a debt due by France to London when the exchange is at 32 d. than when it is only at par.

#### E X A M P L E S.

*Exam. 1.* London is indebted to France in 1000 crowns, for payment of which one bill of 500 crowns is purchased at  $31\frac{1}{2}$  d. and another at 30 d. for the remaining 500; how much

much Sterling was paid for the bills, and what difference per cent. in the purchase?

*Cr.*                      *d.*    *d.*    *d.*

First, 500 exch. at 31 $\frac{1}{2}$  2dly, 30 : 1.5 :: 100  
8) 500 1.5

2|0) 62 10 at 30 d. 30) 150.0  
3 2 6 at 1 $\frac{1}{2}$  d.

5 per cent. diff.

65 12 6 at 31½ d.  
62 10 0 at 30 d.

L. 128 2 6 was paid Sterling.

*Exam. 2.* When the exchange was at 5 s. London remitted to Portugal L. 500 Sterling, but when the exchange rose to 5 s. 6 d. London drew to be reimbursed of this remittance; for how much Sterling was the bill sold?

*Sixpences.*    *L.*    *Sixpences.*

10 : 500 :: 11 Or  $500 \times 4 = 2000$  mil. at 5s.

10) 5500 40) 22000

L. 550 Answer.

L. 550 *Sterling.*

*Exam. 3.* When the exchange was at 31 d. Britain remitted to France 1000 crowns, but when the exchange rose to 32 d. Britain drew for the same number of crowns; what was paid for the remittance, and how much was received for the draught?

1000 crowns at 31 d.

125 at 30*d.*

4 3 4 at 1 d.

L. 129 3 4 paid at 31 d.

4 3 4 at 1 d.

133 6 8 rec'd at 32 d.

*Note.* In exchanging with the other places of Europe not mentioned, we use the medium of Amsterdam and Hamburg, which hath rendered Amsterdam despotic in the article of exchanges.

## Variety XII. ARBITRATION OF EXCHANGES.

## INTRODUCTION.

THE Lombards, one of the many nations that established themselves in Italy after the ruin of the Roman empire, and who have bestowed their name upon one of the finest countries in it, were the first who studied and struck out this new path of commerce to themselves, which hath since been found not only advantageous to those who thoroughly understand, and properly prosecute it, but of peculiar importance to trade in general. The way those ingenious people went to work was this: They entered into partnership, kept houses in most of the trading cities of Europe, at each of which some one of the partners constantly resided, and by keeping a regular correspondence one with another, concerning the state of trade, and the exchanges in their respective cities, they so managed this new branch of business, that their houses soon became great and reputable, their capitals large, and the cities in which they resided opulent and populous, by being made the centre of their exchanges.

To be quick and accurate in arbitrating exchanges, a perfect acquaintance with, and a thorough practice in all the computations in the preceding part of this treatise will be requisite; to which must be added, a knowledge of the intrinsic value of foreign monies, according to the most accurate assays which have been made for that purpose.

A knowledge of the natural causes of the rise and fall of the course of exchange between nation and nation, or between one trading city and another of the same nation, will contribute not a little to the purposes of arbitration.

That the course of exchange is the touchstone by which the state of trade can be infallibly discovered, hath been allowed not only by great statesmen, and speculative politicians, but by the most skilful and sagacious practical traders; and therefore it is the business of every exchange-negotiator, who would make the most of this delicate branch, to consider with attention, where the balance of trade lies among the European nations, at all points of time; for by that means only he can embrace his opportunities of profit; and these almost daily betwixt some nation or other, provided he hath a credit and correspondence extensive enough for the purpose. It very rarely happens, in a comparison of the courses of exchange, among

among several places together, that they are found to ebb and flow in an exact equality of proportion; for as the balance of trade differs between different nations, so the course of exchange will be in favour of some, and against others. This being the case, the judgment of the exchange-negotiator consists in vigilantly observing, from a proper comparison of the courses of exchange, where the greatest inequality of proportion lies, that he may thereby discover with certainty, where he will find his account in drawing and remitting to some places preferably to others; for where-ever the greatest inequality is found, it is there, that negotiations of this kind will be attended with the greatest profit; and by following this inequality, and altering the channel of his correspondence with the fluctuation of the exchanges, he will daily improve his fortune, enliven trade, by his intelligence and correspondence, open new paths of commerce, and be an instrument of making the place he lives in more conspicuous, by being the centre of his exchange-negotiations.

### SECT. I. Simple arbitration of exchanges.

*Case 1.* When a factor hath orders from his employer to remit a certain sum of money to a place, provided he can do it, at a certain price of exchange mentioned, and at the same time, to draw for his reimbursement upon some other place at a certain price of exchange; to find whether the advantage in performing the one part of the commission will be sufficient to compensate for the loss that may arise from the other; so that, in case he finds the negotiation would be to his constituent's loss, he may write him for new orders, or wait till the course of exchange be more in his favour; observe the following

### R U L E.

Let the price of exchange assigned for remitting be the first term, the price of exchange assigned for drawing be the second term, and the real course at which the remittance can be made, the third term; then will the fourth proportional give the price at which the draught ought to be made, to be at par with the price of the remittance.

By comparing which with the price of bills upon that place where the draught is to be made, the factor or correspondent will easily see, whether he ought to obey the order or not.

## E X A M P L E S.

*Exam. 1.* A factor at London receives an order to remit to Venice 1000 ducats, at 4*s.* *Sterling* per ducat, and for this purpose, to put himself in cash by drawing on Spain at 3*s.* 2*d.* per piastre; when this order came to hand, bills for Venice were at 50*d.*; at what price must London draw upon Spain to compensate the advance on the remittance to Venice by the rise of the exchange?

$$\begin{array}{r} d. \quad d. \quad d. \\ 48 : 50 :: 38 \\ \hline 12 \quad 25 \quad 19 \\ \hline 19 \\ \hline 12) 475 \\ \hline 39\frac{7}{12} \end{array}$$

39*7*/*12* per piastre, advance 1*7*/*12*

## O B S E R V A T I O N.

Since the factor, agreeable to the course of exchange, at the time he received the order, is obliged to remit at 50*d.*, which is 2*d.* advance on each ducat, he must draw 39*7*/*12* *d.* for his piastre, otherwise his employer would be a loser in whatever he drew below that price: but if bills on Spain had risen more in proportion than bills on Venice, for instance, to 4*d.* per piastre, the order could be performed with  $\frac{5}{12}$  *d.* advantage on each piastre.

For, first, 1000 ducats at 4*s.* =  $1000 \times \frac{1}{4} = L. 200$   
2dly,  $L. 200 = 4 \frac{80}{99} d.$  in a piastre = 1263.158 piastras.

That is, the factor in London must sell 1263.158 of his constituent's piastras before he is put in cash to make the remittance of 1000 crowns; but when he finds the course of exchange hath varied since his employer's intelligence, he may first find what money the purchase of 1000 ducats will require at 50*d.*; and supposing the course with Spain had risen to 40, he can save of the money arising from the sale of his constituent's piastras about  $L. 2 : 10 : 3\frac{1}{4}$ .

*Exam. 2.* An order comes to Amsterdam to remit to Genoa at 82*d.* per pezzoe, and draw upon London at 33*s.* 4*d.* per *l. Sterling*; when the order came to hand, bills for Genoa were at 85*d.*; how must the pound Sterling be valued, to compensate the loss by the remittance to Genoa?

$$\begin{array}{r}
 82 : 33\frac{1}{2} :: 85 \\
 \underline{-} \qquad \qquad \qquad \underline{33\frac{1}{2}} \\
 2805 \\
 \underline{-} \qquad \qquad \qquad \underline{28 \ 4} \\
 82) 2833 \ 4 \\
 \underline{-} \qquad \qquad \qquad \underline{34 \ 6\frac{1}{2}}
 \end{array}$$

Consequently Amsterdam must sell the London bill at  $34 s. 6\frac{1}{2}$  to be at par; whatever is below that, would be a loss; and whatever is above it, a profit.

*Exam. 3.* A merchant in Amsterdam is indebted to London in a certain sum, which he orders his friend at Paris to remit at  $30 d.$  per crown, and to value on him for his reimbursement at  $54$  grotes per ecu; upon receipt of the order, he cannot purchase a London bill under  $30\frac{1}{2} d.$  but he can sell a bill on Amsterdam at  $54\frac{3}{4}$  grotes; whether can the order be obeyed with loss or profit?

$$\begin{array}{r}
 d. \qquad \text{grotos.} \qquad d. \\
 30 : 54 :: 30\frac{1}{2} \\
 \underline{-} \qquad \underline{-} \qquad \underline{9} \\
 5 \qquad 9 \qquad \underline{\underline{}} \\
 5) 274\frac{1}{2} \\
 \underline{-} \qquad \qquad \qquad \underline{\underline{}} \\
 54\frac{9}{10} \text{ grotes. } \text{Answer.}
 \end{array}$$

Hence this order can be performed with profit; because, if the remittance to London be at  $30\frac{1}{2} d.$  the draught on Amsterdam may be in proportion to the order at  $54\frac{9}{10}$  grotes; but the crowns remitted to London may be paid at Amsterdam with less money, since the course is only at  $54\frac{3}{4}$ .

*Exam. 4.* Rome is indebted to Naples in  $3000$  stamped crowns; for recovering which debt, Naples gives an order to Lyons to draw upon Rome at  $42$  stamped crowns for  $100$  French crowns; and reserving  $\frac{2}{3}$  for commission, to remit the proceeds to Naples, at  $75\frac{1}{2}$  ducats per  $100$  crowns; when this order came to hand, bills for Naples were at  $74\frac{2}{3}$ ; how must Lyons draw upon Rome, so as to remit to Naples the number of ducats intended by the order, and save  $\frac{2}{3}$  commission?

$$\begin{array}{r}
 75\frac{1}{2} : 42 :: 74\frac{5}{6} \\
 \hline
 6 \\
 \hline
 453 \\
 \hline
 448 \\
 \hline
 42 \\
 \hline
 896 \\
 \hline
 1792 \\
 \hline
 453) \overline{18816} (41.5364 \\
 \hline
 1812 \\
 \hline
 696 \\
 \hline
 453 \\
 \hline
 2430 \\
 \hline
 2265 \\
 \hline
 1650 \\
 \hline
 1359 \\
 \hline
 2910 \\
 \hline
 2718 \\
 \hline
 1920 \\
 \hline
 1812 \\
 \hline
 \end{array}$$

Hence Lyons must draw upon Rome at  $41.5364 = 41\frac{13}{17}$  nearly; as will appear more intelligibly in the following process.

$$\begin{array}{r}
 42 : 100 :: 3000 \\
 \underline{100} \\
 \hline
 6) \underline{300000} \\
 7) \underline{50000} \\
 \hline
 7142.857 \text{ French crowns} \\
 .004
 \end{array}$$

28.571428 $\frac{2}{3}$  per cent. commission.  
7114.285572 nett proceeds to be remitted.  
75 $\frac{1}{2}$  per 100.

35571427860  
49799999004  
3557142786

5371.28560686 ducats to be remitted to Naples according to the order at 42 to Rome, and  $7\frac{1}{2}$  to Naples;

Naples; and if the amount at 41.5366 crowns to Rome, and at  $74\frac{2}{3}$  ducats to Naples, be found in the same manner, the same number of ducats will come out in the process as before.

*Exam. 5.* Amsterdam is indebted to Glasgow in 1000 guilders as a balance of nett proceeds, which Glasgow can dispose of at 22 d. per guilder; but to make a trial in the exchange way, orders M. Chappelle at Paris to value on Amsterdam at  $54\frac{3}{4}$  grotes per crown, and remit the proceeds to Mr Adair, London, reserving  $\frac{1}{2}$  commission for his trouble at the current course; when this order came to hand, the course had not altered, and the commission was executed according to order; the course of exchange betwixt London and Paris was at 32 d. per ecu, at which exchange Mr Adair receives bills of exchange upon London, which Glasgow is accordingly advised of, and draws for, with the advantage of 2 per cent. for the London bill. — What is the difference of these transactions to Glasgow of selling the guilders, or being remitted via Paris?

guild.	d.
First, 1000 at 22	12) 1000
	10) 83 6 8 for 20 d.
	8 6 8 for 2

Glasgow would draw L. 91 13 4 by selling the guilders.

grotes, adly, $1000 \times 40 = 40,000$	4 crowns
$54\frac{3}{4} = 219$	160000
<u>1533</u>	(730.5936 received by the draught on Amsterd.
<u>670</u>	3.6519 to be deducted for $\frac{1}{2}$ commiss. $= \frac{1}{8}$ .
<u>657</u>	<u>726.9407</u> to be remitted to London at 32 d. Sterl.
<u>1300</u>	90.8676 at 30 d. $= \frac{1}{8}$
<u>1095</u>	6.0578 at. 2 d. $= \frac{1}{15}$ of $\frac{1}{8}$
<u>2050</u>	96.9254 remitted to London.
<u>1971</u>	.4746 to be deducted for commiss. at London.
<u>790</u>	96.450 nett at London.
<u>657</u>	1.889 premium received for the London bill [at 2 per cent. $= \frac{1}{50}$ .]
<u>1330</u>	98.339
<u>1314</u>	20
<u>(16)</u>	6.780
	12
	9.360
	4
	3.440

Hence

Hence Glasgow receives, by negotiating the 1000 guilders at Paris, L. 98 6 9 $\frac{1}{4}$   
 And would have cleared by selling the guilders but } 91 13 4  
 So that there is a profit arising by the transaction of L. 6 13 5 $\frac{1}{4}$   
 Which will be found to be 5 $\frac{1}{4}$  per cent. difference.

*Case 2.* To find the arbitrated price of exchange.

The prices of exchange from one place are always given to other two places, in order to find the price of exchange betwixt those two places proportional to the prices given, which is called *the arbitrated price of exchange, or, the par by arbitration.*

In this case, the 1st and 3d terms must belong to the same country, and the 2d must be of that kind of which the price is required; the term which moves the question being always the 3d, the disposition of the terms will easily be discovered, and the 4th proportional is the par by arbitration.

*Examples of arbitrating the par.*

*Exam. 1.* Suppose bills at Paris on London at 32 d. per crown, and on Amsterdam at 54 grotes per crown; what must the price of exchange be betwixt London and Amsterdam, to be on a par with the exchange betwixt Paris and those places?

$$\begin{array}{rcccl} \text{d. Ster.} & & \text{grotes.} & & \text{s.} \\ 32 & : & 54 & :: & 20 \\ \hline & & & & \\ 4 & & 27 & & 5 \\ & & 5 & & \\ \hline & & & & \\ 4) 135 & & & & \\ & & s. & d. & \\ & & 33 & 9 & \\ & & \text{Flem. Answer.} & & \end{array}$$

*Exam. 2.* Suppose bills at London on Amsterdam are at 33 s. 9 d. Flemish per pound Sterling, and on Paris at 32 d. per ecu; what must the price of exchange between Amsterdam and Paris be, to make it on a par with the other two?

$$\begin{array}{rcccl}
 d. \text{ Sterl.} & d. \text{ Flem.} & d. \text{ Sterl.} \\
 240 & : & 405 & :: & 32 \\
 30 & & 4 & & 4 \\
 \hline
 3) & 162.0 & & &
 \end{array}$$

54 grotes *per ecu*, as in the first example,

*Exam. 3.* Suppose bills at Amsterdam on Paris are at 54 grotes *per ecu*, and on Britain at 33*s. 9d.* Flemish *per pound Sterling*; what must be the price of exchange betwixt Paris and London, to be on a par with the other two?

$$\begin{array}{rcccl}
 d. \text{ Flem.} & s. \text{ Sterl.} & d. \text{ Flem.} \\
 405 & : & 20 & :: & 54 \\
 81 & & 4 & & 4 \\
 \hline
 81) & 216 & & &
 \end{array}$$

2*s. 8d.* = 32*d. Sterl.* as before.

Hence, if a draught for *L. 200 Sterling* were remitted to Paris at 32*d. per crown*, it would be found to get credit there for 1500 crowns till they were remitted to Amsterdam, when Amsterdam would be debited for the same number of crowns at 54 grotes, which would find credit there for 2025 guilders; and if these guilders were remitted to London at 33*s. 9d. Flemish per pound Sterling*, Amsterdam would be credited for a remittance of *L. 200 Sterling*, which would even the account at all those places, with respect to this negotiation, without loss or profit.

#### O B S E R V A T I O N.

For the speculation of the ingenious merchant on the use to be made of the arbitrated par of exchange, it will not be improper to give some examples, from the London course, upon such places where the profit is seldom less than what follows, *viz.*

Suppose London draws on Amsterdam at 34*s. 10d. Flemish per pound Sterling*, and on Paris at 31*7/8d. per ecu*, the arbitral price between Amsterdam and Paris will be found to be 55*3/4*. But suppose Amsterdam advises, that the exchange for Paris is at 54*1/4* grotes *per ecu*, which is below the arbitral price, the question is, what profit presents?

Draw

Draw L. 100 *Sterling* on Paris at  $31\frac{1}{2}$ , it will debit you at Paris for 752 cr. 56 s. 5 d. and remit to Amsterdam L. 98 : 12 : 5 at 34 s. 10 d. Flemish, which credits you at Amsterdam with 1030 *guil.* 11 *fl.* 12 *pen.* *banco*; so that the profit to be made between those places on L. 100 is L. 1, 7 s. 7 d. The money received for your draught furnishes you with money to pay for your remittance, and your debit at Paris will be cleared by your credit at Amsterdam, exchange at  $54\frac{3}{4}$  *grotos per ecu*: for if  $54\frac{3}{4}$  *grotos* will pay a French crown, 1030 *guil.* 11 *fl.* 12 *pen.* *bank-money* will pay 752 *ecus*, 56 s. 5 d.

But, if, on the other hand, Amsterdam advises you at London that the exchange for Paris is at  $56\frac{1}{4}$ , which is above the arbitrated price of exchange; in this case you must alter your course of drawing, and instead of France as before, draw on Amsterdam for L. 100 *Sterling* at 34 s. 10 d. Flemish, which debits you at Amsterdam in 1045 *guilders banco*, and remit to Paris L. 98 : 13 : 10 at  $31\frac{1}{2}$ , which credits you at Paris in 743 *ecus*, 6 s. 8 d.; so that the profit, upon this supposition, will be L. 1 : 6 : 2 on the L. 100. And the money you receive for your draught furnishes you with the money to pay for your remittance as before; your debit at Amsterdam will be paid by your credit at Paris, exchange at  $56\frac{1}{4}$ : for if one French crown will pay  $56\frac{1}{4}$  *grotos* at Amsterdam, 743 *ecus*, 6 s. 8 d. will pay 1045 *bank guilders*.

Hence it is evident, that whether the advised price be above or below the arbitral price, there is always an advantage to be made by drawing and remitting; and as it very seldom happens that the advised and arbitral price are at par, advantageous opportunities of doing it may occur every post.

— In the foregoing example the difference is supposed to be only that betwixt  $55\frac{3}{4}$  and  $54\frac{3}{4}$  in one case; and in the other that between  $55\frac{3}{4}$  and  $56\frac{1}{4}$ , which is very small; and yet, when it is considered how many times it may be reiterated in the year, without the advance of a single shilling, it yields a profit superior to most other trades in which a man can be employed.

For another instance: Let it be supposed that London exchanges on Amsterdam at 34 s. 10 d. Flemish, and on Hamburg at 33 s. 5 d. the arbitral price will be found to be  $33\frac{1}{4}\frac{1}{2}$  betwixt Amsterdam and Hamburg.— If in this case the advised or real price of exchange be below the arbitrated par, suppose for instance at 32 s. then draw on Hamburg for L. 100 at 33 s. 5 d. which debits you there for 1253 *marks*,

2 s. lubs; and remit to Amsterdam L. 95 : 18 : 7 at 34 s. 10 d. which credits you at Amsterdam in 1002 *guil. 10 stiv.* bank-money: but you received L. 100 for your draught on Hamburg, and paid only for your bill on Amsterdam L. 95 : 18 : 7, so that you retain in your own hands a profit of L. 4 : 1 : 5 in negotiating in this manner L. 100.

But if, on the other hand, Amsterdam advises that the exchange betwixt Hamburg and there is at  $34\frac{1}{4}$ , which is above the arbitrated price, then draw on Amsterdam for L. 100 *Sterling*, at 34 s. 10 d. where you will be debited for 1045 guilders banco, and remit to Hamburg L. 97 : 7 : 10 at 33 s. 5 d. where you are credited in 1220 *mar. 7 s. 6 pen. lubs*; so that, without being one shilling in advance, you have L. 2, 12 s. 2 d. in your pocket for half an hour's trouble in negotiating L. 100: for the money you receive for your draught pays that for your remittance, and your debit at Amsterdam will be evened by your credit at Hamburg, exchange at  $34\frac{1}{4}$ ; for if one dollar of Hamburg will pay  $34\frac{1}{4}$  stivers at Amsterdam, 1220 *mar. 7 s. 6 pen. lubs* will clear 1045 guilders banco of Amsterdam.

From these examples it will be obvious, that an extensive credit, and a thorough knowledge in the arbitration of exchanges, make a sufficient capital to carry on this beneficial branch; and it is pity so few apply themselves to a study which would in so ample a manner requite their labour.

### Sect. II. Compound arbitration of exchanges.

When the price of exchange is given betwixt one country and another, betwixt that second and a third, and betwixt that third and a fourth, &c.; to find the arbitrated price betwixt the first and the last, observe the following

#### R U L E.

Place the antecedents in one column, and the consequents in another, to the right of the antecedents, so as to form a numerical equation, in the algebraic way of analysis, in which the first antecedent and the last consequent, to which an antecedent is required, must always be of the same denomination or species; the first consequent must be of the same denomination with the second antecedent; the second consequent with the third antecedent, &c. throughout. If a fraction is annexed to any of the numbers, both the antecedent and consequent must be multiplied into the denomination

Draw L. 100 Sterling on Paris at  $31\frac{7}{8}$ , it will debit you at Paris for 752 cr. 56 s. 5 d. and remit to Amsterdam L. 98 : 12 : 5 at 34 s. 10 d. Flemish, which credits you at Amsterdam with 1030 guil. 11 fl. 12 pen. banco; so that the profit to be made between those places on L. 100 is L. 1, 7 s. 7 d. The money received for your draught furnishes you with money to pay for your remittance, and your debit at Paris will be cleared by your credit at Amsterdam, exchange at  $54\frac{3}{4}$  grotes per ecu: for if  $54\frac{3}{4}$  grotes will pay a French crown, 1030 guil. 11 fl. 12 pen. bank-money will pay 752 ecus, 56 s. 5 d.

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For another instance: Let it be supposed that London exchanges on Amsterdam at 34 s. 10 d. Flemish, and on Hamburg at 33 s. 5 d. the arbitral price will be found to be  $33\frac{14}{17}$  betwixt Amsterdam and Hamburg.— If in this case the advised or real price of exchange be below the arbitrated par, suppose for instance at 32 s. then draw on Hamburg for L. 100 at 33 s. 5 d. which debits you there for 1253 marks,

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From these examples it will be obvious, that an extensive credit, and a thorough knowledge in the arbitration of exchanges, make a sufficient capital to carry on this beneficial branch; and it is pity so few apply themselves to a study which would in so ample a manner requite their labour.

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#### R U L E.

Place the antecedents in one column, and the consequents in another, to the right of the antecedents, so as to form a numerical equation, in the algebraic way of analysis, in which the first antecedent and the last consequent, to which an antecedent is required, must always be of the same denomination or species; the first consequent must be of the same denomination with the second antecedent; the second consequent with the third antecedent, &c. throughout. If a fraction is annexed to any of the numbers, both the antecedent and consequent must be multiplied into the denomination

of that fraction, and the proportion will still be the same. The terms being thus disposed, cancel the quantities that are the same on both sides of the equation, and abridge such quantities as are commensurable, then multiply all the antecedents into one another for a general divisor, and all the consequents for a general dividend, and the quotient will be the answer, or value of the antecedent required.

*Examples of compound arbitration.*

*Exam. 1.* Suppose London is to remit £. 500 to Spain by the way of Holland at 35 s. per pound, thence by the way of France at 58 grotes per crown, thence to Venice at 100 crowns per 60 ducats banco, and from Venice to Spain at 360 mervadies per ducat banco; how many piafres of 272 mervadies will it amount to in Spain exclusive of charges?

(1.)

Antecedents. Consequents.

1 pound	=	420 d. Flem.
58 grotes	=	1 crown.
100 crowns	=	60 ducats.
1 ducat	=	360 mervadies.
272 mervadies	=	1 piastre.

How many piafres for £. 500?

(2.)

1 = 210
29 = 1
1 = 30
1 = 45
17 = 1
— 5

Cancel the ciphers in 100 and 500, divide 272 and 360 by 8, and you will have for a new antecedent 34, and for a new consequent 45; divide 34 and 60 by 2, and you will have a new antecedent 17, and a new consequent 30; lastly, divide 58 and 420 by 2, and you will have the new antecedent 29, and new consequent 210; then will the whole stand abridged as in the second equation, and the operation be as follows.

$$\begin{array}{r}
 210 \\
 30 \\
 \hline
 6300 \\
 45 \\
 \hline
 315 \\
 252 \\
 \hline
 29 & 283500 \\
 17 & \hline
 493) & 1417500
 \end{array}$$

493) 1417500 (2875 $\frac{1}{4}$  piafres. Answer.

493)

493) 1417500 (2875 $\frac{1}{4}$  piafres. *Answer.*  
986

$$\begin{array}{r} 4315 \\ 3944 \\ \hline 3710 \\ 3451 \\ \hline 2590 \\ 2465 \\ \hline (125) \end{array}$$

Hence upon the supposition that  $42\frac{1}{2}$  d. had been the price of the piastre at the direct course, the L. 500 would only have been worth  $282\frac{1}{2}$  piafres; so that, by this method of negotiating, there would be a gain of 52 piafres, or 2 per cent. without reckoning charges.

*Exam. 2.* A banker in Paris remits to his factor in Amsterdam 455 crowns tournois; first to London at 30 d. per crown; from London to Rome at 65 d. per stamped crown; from Rome to Venice at 100 stamped crowns for 140 ducats banco; from Venice to Leghorn at 100 ducats banco for 100 piafres of Leghorn; and from Leghorn to Amsterdam at 86 grotes per piastre; how many guilders banco will be received at Amsterdam without reckoning charges?

Antecedents.	Consequents.
1 crown	= 30 pence.
65 pence	= 1 stamped crown.
100 stamped crowns	= 140 ducats.
100 ducats	= 100 piafres.
1 piastre	= 86 grotes.
How many grotes	= 455 crowns?

Four ciphers on each side of the equation being cancelled, and the other quantities being abridged, the operation will stand thus:

$$\frac{3 \times 14 \times 86 \times 91}{13 \times 40} = 632 \text{ guilders and } 2 \text{ stivers. } \text{ *Answer.*}$$

*Exam. 3.* Amsterdam being to remit to London L. 750 Flemish, sends it first to France at 54 grotes per crown, thence to Venice at 100 crowns for 56 ducats banco, thence to Hamburg

Hamburg at 100 grotes *per ducat*, thence to Portugal at 45 grotes *per crusado* of 400 reas, and from Portugal to London at 5*s.* 3*d.* *per milrea*; how much Sterling must be received in London for the remittance, and whether more or less than by a direct remittance, exchange at 35*s.* 6*d.* *per pound* Sterling, discounting commission at each place of  $\frac{1}{2}$  *per cent.*?

Antecedents.      Consequents.

54 grotes	=	1 crown.
100 crowns	=	56 ducats.
1 ducat	=	100 grotes.
45 grotes	=	400 reas.
1000 reas	=	63 pence.

How much Sterling = 750 pound Flemish?

The five ciphers on each side of the equation being cancelled, and the other quantities abridged, we will have the following numbers on each side.

Ant.	Con.
9	28
	4
	7
	5

Therefore  $28 \times 4 \times 7 \times 5$

$$\frac{= L. 435.555 \text{ Sterling.}}{9 .02 \text{ for comm. at 4 places at}} \\ \frac{\text{---}}{\text{---}} \frac{\frac{1}{2} \text{ per c. each} = \frac{4}{2} = 2 \text{ per c.}}{8.711 \text{ to be deducted.}}$$

If remitted directly

426.8444 nett proceeds.

$1.775 : 1 :: 750 :$

$422.5352$

$4.3092$  nett gain.

*Exam. 4.* A merchant of London hath credit for 1360 piastres of Leghorn, from which there is advice that a remittance can be made at 50*d.* *per piastre*. The London merchant finding he could make no more by drawing for them, orders them to be remitted in the following manner, *viz.* first to Venice at 94 piastres for 100 ducats *banco*, thence to Cadiz at 320 mervadies *per ducat*, thence to Lisbon at 630 reas *per piastre* of 272 mervadies, thence to Amsterdam at 50 grotes *per crusado* of 400 reas, from thence to Paris at 56 grotes *per crown*, and, lastly, he brings them home at 31*d.* *per crown*; what will be the arbitrated price *per piastre* between London and Leghorn, and how much will be received at London without reckoning charges?

Antecedents.

Antecedents.	Consequents.
94 piafres	= 100 ducats.
1 ducat	= 320 mervadies,
272 mervadies	= 630 reas.
400 reas	= 50 grotes.
3 crowns	= 94 pence <i>Sterl.</i> = $3 \times 31\frac{1}{3}$ .
How many pence	= 1 piastre?

Two ciphers on each side of the equation being cancelled, and the remaining quantities properly abbreviated, the operation will stand

$$\frac{35 \times 15 \times 5}{34} = 55\frac{5}{4} \text{ instead of } 50 \text{ d. by a direct remittance.}$$

Hence 1360 piafres at the arbitrated price = L. 312 10 0  
Ditto at the direct course, *viz.* 50 d. = 283 6 8  
Gained by the negotiation, - - L. 29 3 4

*Exam. 5.* Suppose you are presented at Amsterdam with bills of exchange upon Naples, for which an exchange of 82 sols *per ducat* of 10 carlins is required: suppose likewise, before you purchase these bills of exchange, you would know how many sols you can afford to give for every ducat, after a circulation through several places, and paying  $\frac{1}{2}$  per cent. commission to each correspondent through whose hands the money so circulates, for his trouble of negotiating: suppose likewise you find, that Naples, which is the chief object of consideration, exchanges with Rome, by giving 124 ducats *del regno* for 100 Roman crowns; that Rome retains a commission, and exchanges with Venice, by giving 62 crowns *d'estampe* for 100 ducats *banco*; that Venice exchanges with Amsterdam, by taking 90 deniers gros for the said ducat, and also reserves commission; that Amsterdam exchanges with Madrid at 96 deniers gros *per ducat* of 375 mervadies, and retains the commission; and lastly, that Madrid exchanges with Lyons at 76 sols *per piastre* of 272 mervadies of plate, and retains commission: these things previously supposed, the question is to satisfy the purchaser of the bills, whether he can dispose of them again in this manner with advantage?

Antecedents.	Consequents.
124 Naples ducats	= $99\frac{1}{2}$ Roman crowns, ded. comm.
62 crowns <i>d'estampe</i>	= $99\frac{1}{2}$ Venetian ducats, comm. ded.
1 ducat	= $89\frac{9}{20}$ gros, commission deducted.
96 deniers gros	= $373\frac{1}{8}$ mervadies, comm. deducted.
272 mervadies	= 76 sols of Lyons.
How many sols	= 1 ducat of Naples?

The antecedents and consequents being ranged, abridged, and cancelled, the consequents multiplied continually for a dividend, and the antecedents for a divisor, the value of the ducat will be found to be 83 sols, which will not answer the negotiator's end in being several months out of his money.

I shall conclude this extraordinary and useful subject, with an extract from the merchant's public counting-house, the author of which seems to have understood well the extent and delicacy of this rule. "Whoever," says he, "trades as a merchant, that is to say, as an exporter and importer in Europe, must of necessity have to do with drawing and remitting; and if so, he should by no means be unacquainted with those arts of making the best advantages by so doing; but this is not possible to be done, without being thoroughly skilled in their arbitration to a demonstrative exactitude. — The more general the trade of a merchant is, the more universal should his knowledge be in this particular; and those who may have views of dealing largely by exchange, will certainly find their account beyond expectation, in being fundamentally grounded in this extraordinary subject: for a trader of a good general foreign correspondence, may by this means gain more by dint of credit and skill, than others unacquainted herewith can do by dint of hard money."

### Variety XIII. Comparisfon of Weights and Measures,

**A**S it is of the utmost importance for the extensive trader, not only to be acquainted with the weights and measures of the different countries with which he may have occasion to deal, but with their relation one to another; I have in the first part of this work exhibited authentic tables of all the weights and measures of those countries with which we deal, and their subdivisions, as well as the weights and measures used in Britain; and I shall now insert two tables, representing the conformity which the weights and measures of the most noted trading places of Europe have with one another, discoverable by inspection, and then give a few examples of their comparison, by the rule of conjunct proportion, after the manner of compound arbitration.

[The tables follow, to be folded in.]

# T A B L E

A TABLE representing the conformity which the WEIGHTS of the principal TRADING CITIES of EUROPE have to those of Amsterdam, published in the year 1732, and quoted in 1747, as the most authentic of its kind, by the Count de Maurepas; with the difference only of transposing one of the columns, in order to place ENGLAND before Amsterdam for the United Provinces, and the Sieur Larue Paris, for the use of the French nation more particularly.

**N.B.** As the weight of Amsterdam, Paris, Bourdeaux, Besançon, and several other places, have but a very trifling difference, they are comprehended under those of Amsterdam, as those of Nuremberg are under Frankfort, and others in the same manner.

	A Of England Scotl. and Ire- land.	B Of Amster- dam, Paris, &c.	C Of Ant- werp or Brab- tant.	D Of Rouen, the vis- county	E Of Lyons, the city	F Of Ro- chelle.	G Of Tou- louse and Up- per Lan- guedoc.	H Of Mar- seilles and Pro- vence.	I Of Ge- neva.	K Of Ham- burg.
A 100 lb. of England, Scotland, and Ireld. London	100 —	91 8	96 8	88 —	106 —	90 9	107 11	113 —	81 7	93 —
B 100 lb. of Amsterdam, Paris, &c. — — —	109 8	100 —	105 8	96 4	116 —	99 —	118 —	123 8	89 —	102 —
C 100 lb. of Antwerp, or Brabant — — —	103 12	94 12	100 —	91 4	110 —	93 13	111 12	117 —	84 5	96 1
D 100 lb. of Rouen, the viscounty — — —	113 14	104 —	109 12	100 —	120 8	102 15	122 11	128 8	92 9	106 —
E 100 lb. of Lyons, the city — — —	94 3	86 —	90 12	82 12	100 —	85 2	101 8	106 4	76 8	87 1
F 100 lb. of Rochelle — — —	110 9	101 —	106 8	97 3	117 —	100 —	119 3	124 12	89 14	103 —
G 100 lb. of Toulouse, and Upper Languedoc —	92 6	84 12	89 6	81 8	98 5	83 15	100 —	105 4	75 7	86 —
H 100 lb. of Marseilles and Provence — — —	88 11	81 —	85 8	78 —	94 —	80 3	95 9	100 —	72 —	82 1
I 100 lb. of Geneva — — —	123 —	112 6	118 8	108 —	130 5	111 6	132 9	128 4	100 —	114 1
K 100 lb. of Hamburg — — —	107 5	98 —	103 6	94 4	113 10	97 —	115 10	121 —	87 4	100 —
L 100 lb. of Frankfort — — —	111 11	102 —	107 8	98 3	118 5	101 —	120 6	126 —	90 12	104 —
M 100 lb. of Leipsic — — —	104 5	95 4	100 —	91 12	110 8	94 4	112 6	117 12	84 12	92 —
N 100 lb. of Genoa — — —	73 —	66 $\frac{2}{3}$	70 5	64 —	77 5	66 —	78 10	82 5	59 5	68 —
O 100 lb. of Leghorn — — —	75 8	69 —	72 12	66 6	80 —	68 5	81 6	85 4	51 6	70 —
P 100 lb. of Milan — — —	65 3	59 8	62 12	57 4	69 —	58 14	70 3	73 8	53 —	60 —
Q 100 lb. of Venice — — —	65 11	60 —	63 6	57 12	69 10	59 6	70 13	74 2	53 6	61 —
R 100 lb. of Naples — — —	64 10	59 —	62 4	57 —	68 7	58 6	69 10	72 14	52 8	60 —
S 100 lb. of Seville, Cadiz, &c. — — —	103 7	94 8	99 12	91 —	109 10	93 9	111 8	116 11	84 2	96 —
T 100 lb. of Portugal — — —	95 4	87 8	92 —	84 4	101 8	86 10	103 4	108 —	77 14	89 —
V 100 lb. of Liege — — —	104 —	95 —	100 3	91 7	110 3	94 —	112 —	117 5	84 8	96 1

**N.B.** Such is the use of this table, that by means hereof, may be easily discerned, at one view, the conformity which the weights of one pound English make at Amsterdam, look for England in the first column, and from thence pass your eye along the line till you find 91 lb. 8 ounces, (reckoning 16 ounces to the pound) are equal to 100 lb. English; and in like manner you may find the agreement between any two cities.

## I.

*To be placed after p. 350.*

of EUROPE have with each other, taken from that of Mynheer Samuel Ricard,  
and, by the Sieur Jean Larue, merchant of Lyons, in his treatise dedicated to the  
RECE ENGLAND OR LONDON in the front, as Mynheer Ricard has done Holland and  
more particularly.

I Ge- ra.	K Of Ham- burg.	L Of Frank- fort, &c.	M Of Leipsic, &c.	N Of Genoa.	O Of Leg- horn.	P Of Milan.	Q Of Venice.	R Of Naples.	S Of Se- ville, Cadiz, &c.	T Of Por- tugal.	V Of Liege.
1 7	93 5	89 7	96 1	137 4	132 11	153 11	152 —	154 10	97 —	104 13	96 5
9 —	102 —	98 —	105 —	150 —	145 —	168 —	166 —	169 —	106 —	114 8	105 4
4 5	96 10	92 13	99 8	142 2	137 6	159 3	157 $\frac{1}{2}$	160 2	108 —	108 8	99 11
2 9	106 —	102 —	109 4	156 —	150 13	174 11	172 $\frac{2}{3}$	175 12	110 4	119 —	109 7
6 8	87 12	84 4	90 5	129 —	124 11	144 8	142 $\frac{3}{4}$	145 6	91 3	98 8	90 8
9 14	103 —	99 —	106 —	151 8	146 7	169 11	167 10	170 11	107 —	115 10	106 5
5 7	86 7	83 —	89 8	127 2	122 14	142 6	140 —	143 4	89 13	97 —	89 3
2 —	82 10	79 6	85 8	121 8	117 .7	136 1	134 8	136 14	85 13	92 12	85 4
0 —	114 10	110 2	118 —	168 9	163 —	188 13	186 8	189 14	119 2	128 8	118 4
7 4	100 —	89 11	102 15	147 —	142 2	164 10	162 11	165 10	103 13	112 4	103 2
0 12	104 —	100 —	107 1	153 —	147 14	171 6	169 5	172 6	108 2	116 13	107 6
4 12	92 2	93 5	100 —	142 13	138 1	160 —	158 2	161 —	101 —	109 —	100 4
9 5	68 —	65 5	70 —	100 —	96 11	112 —	110 11	112 11	70 11	76 5	70 6
1 6	70 6	67 10	72 8	103 8	100 —	119 —	114 8	116 9	73 —	79 —	72 10
3 —	60 1	58 5	62 8	89 4	86 4	100 —	98 12	100 8	63 —	68 2	62 10
3 6	61 3	58 13	63 —	90 —	87 —	100 13	100 —	101 6	63 9	68 11	63 2
2 8	60 2	57 13	62 —	88 8	85 8	99 2	98 —	100 —	62 8	67 9	62 —
4 2	96 6	92 10	99 4	141 12	137 —	158 12	156 14	159 12	100 —	108 3	99 14
7 14	89 4	85 12	91 13	131 14	126 13	147 —	145 4	148 —	92 12	100 —	92 —
4 8	96 14	93 —	99 12	142 8	137 12	159 9	157 11	160 10	100 9	108 12	100 —

ights of one place therein exhibited have with those of another : for example, if you would know how  
ng the line till you come to the column under the title of Amsterdam at the top, and you will find that  
it between any other weight of these places specified in the table.

# T A B L E

A TABLE representing the conformity which the LONG MEASURES of the principal TRADING CITIES of EUROPE  
 Sieur Jean Larue, merchant of Lyons, in his treatise dedicated to the Count de Maurepas; with the difference only  
 front, as the Sieur Larue has done Paris, for the use of the French nation more particularly.

The ells of Amsterdam, Haerlem, Leyden, the Hague, Rotterdam, and other cities of Holland, as well as the ell of Nuremberg, are equal among themselves. They are also comprehended under the ell of Amsterdam, as that of Osnaburg is under that of France and England, and the ell of Bern and Basil under that of Hamburg, Frankfort, and Leipsc.

	A Yards of Eng- land, Scotl. and Ire- land.	B Ells of France and Eng- land.	C Ells of Holland and Amster- dam.	D Ells of Ant- werp and Brussel.	E Ells of Hamb. Frankf. Leipsc. and Co- logn.	F Ells of Breslau in Sile- zia.	G Ells of Dant- zick.	H Ells of Bergue and Dron- theim.	I Ells of Sweden or Stock- holm.	K Ells of St Gall for li- nen.	L Ells of St Gall for cloth.	M Ells Gen-
A 100 Yards of England, Scotland, and Ireland	100 —	78 —	133 $\frac{1}{3}$	131 $\frac{2}{3}$	160 —	166 $\frac{3}{5}$	150 —	146 $\frac{2}{3}$	154 —	114 $\frac{2}{3}$	149 $\frac{1}{3}$	80
B 100 Ells of France and England	128 $\frac{1}{4}$	100 —	173 $\frac{1}{2}$	166 $\frac{2}{3}$	205 $\frac{1}{4}$	213 $\frac{3}{4}$	192 $\frac{4}{5}$	188 —	195 $\frac{3}{4}$	147 —	191 $\frac{1}{2}$	102
C 100 Ells of Holland or Amsterdam	75 —	57 $\frac{2}{3}$	100 —	98 $\frac{3}{4}$	120 —	125 —	112 $\frac{1}{2}$	110 —	114 $\frac{1}{2}$	86 —	112 —	60
D 100 Ells of Antwerp and Brussels	76 —	60 —	101 $\frac{1}{4}$	100 —	121 $\frac{1}{2}$	126 $\frac{3}{4}$	114 —	111 $\frac{4}{5}$	116 —	87 —	113 $\frac{4}{5}$	60
E 100 Ells of Hamburg, Frankfort, &c.	62 $\frac{1}{2}$	48 $\frac{7}{8}$	83 $\frac{1}{3}$	82 $\frac{2}{3}$	100 —	104 $\frac{1}{3}$	92 $\frac{3}{5}$	91 $\frac{3}{4}$	95 $\frac{1}{2}$	71 $\frac{2}{3}$	91 $\frac{1}{2}$	55
F 100 Ells of Breslau in Silesia	60 —	46 $\frac{2}{3}$	80 —	79 —	96 —	100 —	89 $\frac{4}{5}$	88 —	91 $\frac{1}{2}$	68 —	89 $\frac{3}{5}$	44
G 100 Ells of Dantzick	66 $\frac{3}{4}$	52 —	89 —	87 $\frac{3}{4}$	96 $\frac{3}{4}$	111 $\frac{1}{4}$	100 —	98 —	102 —	76 $\frac{3}{4}$	99 $\frac{3}{4}$	55
H 100 Ells of Bergue and Drontheim	67 $\frac{1}{4}$	52 —	90 —	89 —	108 —	112 $\frac{1}{2}$	101 $\frac{1}{2}$	100 —	103 —	77 $\frac{1}{3}$	100 $\frac{1}{3}$	55
I 100 Ells of Sweden or Stockholm	65 $\frac{3}{4}$	51 $\frac{1}{4}$	87 $\frac{1}{2}$	86 $\frac{1}{2}$	105 —	109 $\frac{1}{4}$	97 $\frac{3}{4}$	96 $\frac{1}{2}$	100 —	75 $\frac{1}{4}$	98 —	55
K 100 Ells of St Gall for linen	87 —	67 $\frac{3}{4}$	116 —	114 $\frac{1}{2}$	139 $\frac{1}{5}$	145 —	130 $\frac{1}{2}$	127 $\frac{1}{5}$	133 —	100 —	130 —	60
L 100 Ells of St Gall for cloth	67 —	52 $\frac{1}{2}$	89 $\frac{1}{3}$	88 $\frac{1}{3}$	107 $\frac{1}{5}$	111 $\frac{3}{4}$	100 $\frac{1}{2}$	98 $\frac{1}{4}$	102 $\frac{1}{4}$	76 $\frac{4}{5}$	100 —	55
M 100 Ells of Geneva	124 $\frac{3}{4}$	97 $\frac{1}{2}$	166 $\frac{2}{3}$	164 $\frac{1}{3}$	200 —	208 $\frac{1}{3}$	187 $\frac{1}{2}$	183 $\frac{1}{3}$	191 —	143 $\frac{1}{3}$	130 $\frac{2}{3}$	100
N 100 Canes of Marseilles and Montpelier	214 $\frac{1}{2}$	167 $\frac{1}{3}$	286 —	282 $\frac{4}{5}$	343 $\frac{1}{3}$	357 $\frac{1}{2}$	321 $\frac{3}{4}$	314 $\frac{1}{3}$	327 $\frac{1}{2}$	246 —	320 $\frac{1}{2}$	17
O 100 Canes of Toulouse and Upper Languedoc	199 $\frac{3}{4}$	156 —	266 $\frac{2}{3}$	263 $\frac{1}{3}$	320 —	333 $\frac{1}{2}$	300 —	193 $\frac{1}{3}$	304 —	229 $\frac{1}{3}$	298 $\frac{2}{3}$	16
P 100 Canes of Genoa of 9 palmos	245 $\frac{1}{4}$	191 $\frac{1}{3}$	327 —	323 —	392 $\frac{4}{5}$	408 $\frac{1}{4}$	367 $\frac{1}{2}$	359 $\frac{1}{4}$	374 $\frac{1}{2}$	281 $\frac{1}{4}$	366 $\frac{1}{4}$	19
Q 100 Canes of Rome	227 $\frac{1}{4}$	177 $\frac{1}{4}$	303 —	299 $\frac{1}{5}$	363 $\frac{3}{5}$	378 $\frac{3}{4}$	340 $\frac{1}{5}$	333 $\frac{1}{3}$	347 —	260 $\frac{2}{3}$	339 $\frac{4}{5}$	18
R 100 Vares of Castille and Biscay	93 $\frac{3}{4}$	73 $\frac{1}{3}$	125 —	123 $\frac{2}{3}$	150 —	156 $\frac{1}{4}$	140 $\frac{1}{3}$	137 $\frac{1}{2}$	143 $\frac{1}{4}$	107 $\frac{1}{2}$	140 —	7
S 100 Vares of Cadiz and Andalusia	91 $\frac{1}{4}$	71 $\frac{3}{7}$	122 $\frac{1}{4}$	119 —	146 $\frac{3}{5}$	152 $\frac{3}{5}$	138 —	134 $\frac{1}{2}$	139 $\frac{4}{5}$	105 —	137 —	7
T 100 Vares of Portugal or Lisbon	123 —	96 —	164 —	162 —	196 $\frac{4}{5}$	205 —	184 $\frac{1}{2}$	180 $\frac{4}{5}$	187 $\frac{3}{4}$	141 —	183 $\frac{3}{5}$	90
V 100 Covados of Portugal or Lisbon	74 —	58 $\frac{1}{2}$	100 —	98 $\frac{3}{4}$	120 —	125 —	112 $\frac{1}{2}$	110 —	114 $\frac{1}{2}$	86 —	112 —	60
W 100 Brasses of Venice	73 $\frac{1}{2}$	57 $\frac{1}{3}$	98 —	96 $\frac{3}{4}$	117 $\frac{3}{5}$	122 $\frac{1}{2}$	104 $\frac{1}{4}$	107 $\frac{3}{4}$	112 $\frac{1}{4}$	84 $\frac{1}{4}$	109 $\frac{3}{4}$	55
X 100 Brasses of Bergamo, &c.	72 $\frac{1}{4}$	55 $\frac{3}{5}$	95 —	93 $\frac{1}{3}$	114 —	118 $\frac{1}{3}$	106 $\frac{1}{4}$	104 $\frac{1}{2}$	108 $\frac{3}{4}$	81 $\frac{1}{4}$	106 $\frac{3}{5}$	55
Y 100 Brasses of Florence, Leghorn, &c.	65 $\frac{1}{3}$	50 —	85 $\frac{1}{2}$	84 $\frac{4}{5}$	102 $\frac{3}{5}$	106 $\frac{1}{3}$	96 $\frac{2}{3}$	94 —	98 —	73 $\frac{1}{2}$	95 $\frac{3}{4}$	55
Z 100 Brasses of Milan	58 $\frac{1}{2}$	45 $\frac{3}{5}$	78 —	77 —	93 $\frac{3}{5}$	97 $\frac{1}{2}$	87 $\frac{1}{4}$	85 $\frac{4}{5}$	89 $\frac{1}{4}$	67 —	87	4

N. B. By means of this table, the reader may please to observe, that 100 ells of Paris and of England make 173  $\frac{1}{2}$  of Holland; and in like manner you will easily make your computations for any quantity required. But there are more concise rules, which are practised by the most

## II.

To be placed after p. 350.

of EUROPE have with each other, published in 1747, as the most authentic of its kind, by the  
difference only of transposing one of the columns, in order to place ENGLAND or LONDON in the

L Ells of St Gall for cloth.	M Ells of Geneva.	N Canes of Mar seilles and Mont- pelier.	O Canes of Tou- louse, Albi, and Castres.	P Canes of Ge- noa of 9 palmos.	Q Canes of Rome.	R Vares of Ca- stille and Bif- cay.	S Vares of Cadiz and An- dalusia.	T Vares of Por- tugal or Lisbon.	V Cove- dos of Portu- gal or Lisbon.	W Brasses of Ve- nice.	X Brasses of Boulog- ne, Mod.	Y Brasses of Flo- rence, Legh. and Lucca.	Z Brasses of Mi- lan.
149 $\frac{1}{3}$	80 —	46 $\frac{2}{3}$	50 —	40 $\frac{2}{3}$	44 —	107 —	109 $\frac{1}{3}$	81 $\frac{1}{3}$	133 $\frac{1}{3}$	136 —	104 $\frac{1}{3}$	154 $\frac{3}{4}$	171 $\frac{1}{3}$
191 $\frac{1}{2}$	102 $\frac{3}{5}$	59 $\frac{4}{5}$	64 $\frac{1}{5}$	52 $\frac{1}{5}$	56 $\frac{4}{5}$	136 $\frac{7}{8}$	140 —	104 $\frac{1}{3}$	171 —	174 $\frac{4}{3}$	179 —	199 $\frac{1}{3}$	219 $\frac{1}{3}$
112 —	60 —	35 —	37 $\frac{1}{2}$	30 $\frac{1}{2}$	33 —	80 —	81 —	61 —	100 —	102 —	105 $\frac{1}{4}$	116 $\frac{1}{2}$	128 $\frac{1}{2}$
113 $\frac{4}{3}$	60 $\frac{3}{5}$	35 $\frac{4}{5}$	38 —	30 $\frac{4}{5}$	33 $\frac{3}{5}$	81 —	84 —	61 —	101 $\frac{1}{3}$	103 $\frac{3}{5}$	106 $\frac{3}{5}$	118 —	130 —
91 $\frac{1}{3}$	50 —	29 $\frac{1}{5}$	31 $\frac{1}{4}$	25 $\frac{1}{5}$	27 $\frac{1}{2}$	65 $\frac{4}{5}$	68 $\frac{1}{5}$	50 $\frac{4}{5}$	83 $\frac{1}{3}$	85 —	88 $\frac{1}{2}$	97 —	107 —
89 $\frac{3}{5}$	48 —	28 —	30 —	24 —	26 —	64 —	65 $\frac{1}{5}$	48 $\frac{4}{5}$	80 —	81 $\frac{3}{5}$	84 $\frac{4}{5}$	93 $\frac{1}{5}$	102 $\frac{4}{5}$
99 $\frac{3}{4}$	53 $\frac{3}{5}$	31 $\frac{1}{5}$	33 $\frac{4}{5}$	27 $\frac{1}{5}$	29 $\frac{4}{5}$	71 $\frac{1}{3}$	72 $\frac{1}{2}$	54 $\frac{1}{4}$	89 —	90 $\frac{3}{4}$	93 $\frac{3}{4}$	103 $\frac{3}{4}$	114 $\frac{1}{4}$
100 $\frac{4}{5}$	54 —	31 $\frac{1}{2}$	33 $\frac{3}{5}$	27 $\frac{4}{5}$	29 $\frac{3}{4}$	72 —	74 $\frac{3}{4}$	55 —	90 —	91 $\frac{3}{4}$	94 $\frac{3}{4}$	105 —	115 $\frac{3}{4}$
98 —	52 $\frac{1}{2}$	30 $\frac{1}{2}$	32 $\frac{3}{4}$	26 $\frac{3}{4}$	28 $\frac{3}{4}$	70 —	71 $\frac{1}{2}$	53 $\frac{1}{2}$	87 $\frac{1}{2}$	89 $\frac{1}{4}$	92 —	102 —	112 $\frac{1}{2}$
130 —	69 $\frac{3}{5}$	40 $\frac{2}{5}$	43 $\frac{1}{2}$	35 $\frac{4}{5}$	38 $\frac{1}{5}$	92 $\frac{4}{5}$	95 $\frac{1}{5}$	70 $\frac{3}{4}$	116 —	118 $\frac{3}{4}$	122 —	135 $\frac{1}{5}$	149 —
100 —	53 $\frac{3}{5}$	31 $\frac{1}{4}$	33 $\frac{1}{2}$	27 $\frac{3}{5}$	29 $\frac{1}{2}$	71 —	73 —	54 —	89 $\frac{1}{2}$	91 $\frac{1}{5}$	94 —	104 —	114 $\frac{3}{4}$
130 $\frac{2}{3}$	100 —	58 $\frac{1}{3}$	62 $\frac{1}{2}$	50 $\frac{4}{5}$	55 —	133 $\frac{1}{2}$	136 $\frac{2}{3}$	101 $\frac{2}{3}$	166 $\frac{2}{3}$	170 —	172 $\frac{4}{5}$	193 $\frac{1}{3}$	214 $\frac{1}{6}$
320 $\frac{2}{5}$	171 $\frac{3}{4}$	100 —	107 $\frac{1}{4}$	87 $\frac{1}{4}$	94 $\frac{4}{5}$	228 $\frac{4}{5}$	234 —	174 $\frac{3}{4}$	286 —	291 $\frac{3}{4}$	301 —	333 $\frac{1}{3}$	367 $\frac{1}{2}$
298 $\frac{2}{3}$	160 —	93 $\frac{1}{3}$	100 —	81 $\frac{1}{3}$	88 —	213 $\frac{1}{3}$	218 —	162 $\frac{2}{3}$	266 $\frac{2}{3}$	272 —	280 $\frac{3}{5}$	309 $\frac{1}{3}$	342 $\frac{2}{3}$
366 $\frac{4}{5}$	196 $\frac{1}{4}$	114 $\frac{1}{2}$	122 $\frac{3}{5}$	100 —	108 —	261 $\frac{3}{5}$	268 $\frac{1}{5}$	199 $\frac{1}{2}$	327 —	333 $\frac{1}{2}$	344 $\frac{1}{5}$	381 —	420 $\frac{1}{5}$
339 $\frac{4}{5}$	181 $\frac{4}{5}$	116 —	113 $\frac{2}{3}$	92 $\frac{4}{5}$	100 —	242 $\frac{4}{5}$	245 $\frac{3}{5}$	184 $\frac{4}{5}$	303 —	309 —	319 —	353 —	389 $\frac{4}{5}$
140 —	75 —	43 $\frac{1}{4}$	46 $\frac{4}{5}$	38 $\frac{1}{5}$	41 $\frac{1}{4}$	100 —	102 $\frac{1}{3}$	76 $\frac{1}{4}$	125 —	127 $\frac{1}{2}$	131 $\frac{1}{2}$	145 $\frac{3}{5}$	159 $\frac{2}{3}$
137 —	73 $\frac{1}{2}$	42 $\frac{2}{5}$	45 —	37 $\frac{1}{4}$	40 $\frac{2}{5}$	97 $\frac{2}{3}$	100 —	74 $\frac{1}{3}$	122 $\frac{1}{4}$	125 $\frac{1}{4}$	179 —	142 —	157 —
183 $\frac{3}{5}$	94 $\frac{4}{5}$	57 $\frac{4}{5}$	61 $\frac{1}{2}$	50 —	54 $\frac{1}{5}$	131 $\frac{1}{3}$	134 —	100 —	164 —	167 $\frac{1}{4}$	172 $\frac{3}{5}$	191 —	210 $\frac{3}{4}$
112 —	60 —	35 —	37 $\frac{1}{2}$	30 $\frac{1}{2}$	33 —	80 —	81 $\frac{4}{5}$	61 —	100 —	102 —	105 $\frac{1}{4}$	116 $\frac{1}{2}$	128 $\frac{1}{2}$
109 $\frac{3}{4}$	58 $\frac{3}{4}$	34 $\frac{1}{3}$	36 $\frac{3}{4}$	29 $\frac{3}{4}$	32 $\frac{1}{3}$	78 $\frac{4}{5}$	80 $\frac{1}{5}$	59 $\frac{3}{4}$	98 —	100 —	103 $\frac{1}{3}$	114 $\frac{1}{5}$	126 —
106 $\frac{4}{5}$	57 —	33 $\frac{1}{4}$	35 $\frac{3}{5}$	29 —	31 $\frac{1}{3}$	76 —	78 $\frac{1}{5}$	58 —	95 —	97 —	100 —	100 $\frac{3}{5}$	122 —
95 $\frac{4}{5}$	51 $\frac{3}{5}$	30 —	32 —	26 $\frac{1}{5}$	28 $\frac{1}{5}$	68 $\frac{4}{5}$	70 $\frac{1}{3}$	52 $\frac{1}{5}$	95 $\frac{1}{2}$	87 $\frac{1}{5}$	95 —	100 —	109 $\frac{4}{5}$
87 $\frac{3}{5}$	46 $\frac{4}{5}$	27 $\frac{3}{5}$	29 $\frac{1}{4}$	23 $\frac{1}{5}$	25 $\frac{3}{4}$	62 $\frac{4}{5}$	63 $\frac{1}{3}$	47 $\frac{1}{2}$	78 —	79 $\frac{1}{2}$	82 $\frac{1}{5}$	91 —	100 —

In the manner you will find how the measures of other places in the table correspond with each other. By the common rule of  
division by the most expert merchants. See the examples following.

*Examples of the comparison of weights and measures.*

*Exam. 1.* If 7 aunes of Paris make 9 yards of London, 36 yards of London 49 aunes of Holland, 7 aunes of Holland 9 braces of Milan, 3 braces of Milan 2 vares of Arragon, 5 vares of Arragon 2 canes of Montpelier, 9 canes of Montpelier 10 canes of Thoulouse, and 4 canes of Thoulouse 9 aunes of Troyes in Champaigne; how many aunes of Troyes will measure 100 aunes of Paris?

Antecedents.

7 aunes of Paris	=	9 yards of London.
36 yards of London	=	49 aunes of Holland.
7 aunes of Holland	=	9 braces of Milan.
3 braces of Milan	=	2 vares of Arragon.
5 vares of Arragon	=	2 canes of Montpelier.
9 canes of Montpelier	=	10 canes of Thoulouse.
4 canes of Thoulouse	=	9 aunes of Troyes.

How many aunes of Troyes = 100 aunes of Paris?

Both sides of the equation being abridged, the operation will stand.

$$\frac{3 \times 100}{2} = 150 \text{ aunes of Paris. Answer.}$$

*Exam. 2.* Suppose you owe 100 anees of wheat at Lyons; and would know what quantity you would purchase at Macon to replace them, and have no other mean of knowledge but the following, viz.

3 anees of Lyons	=	4 setiers of Paris.
1 setier of Paris	=	2 bushels of Bourdeaux.
36 bushels of Bourdeaux	=	27 muds of Amsterdam.
27 muds of Amsterdam	=	46 fanegas of Cadiz.
230 fanegas of Cadiz	=	57 anees of Macon.
How many of Macon	=	100 of Lyons?

Both sides of the equation being abridged, we will have

$$\frac{46 \times 2 \times 100}{115} = 80 \text{ anees of Macon to replace 100 at Lyons.}$$

*Exam. 3.* Suppose 100 lb. of Amsterdam weigh 100 lb. of Paris; 100 lb. of Paris, 150 of Genoa; 100 lb. of Genoa, 70 lb. of Leipsic; 100 lb. of Leipsic, 160 lb. in Milan; how many lb. of Milan will 548 lb. of Amsterdam weigh?

The

The terms in this example being equated and abridged, it will be

$$\frac{3 \times 2 \times 7 \times 528}{5 \times 5} = 920 \frac{16}{25} \text{ of Milan} = 548 \text{ lb. of Amsterdam.}$$

*Exam. 4.* Suppose a merchant of Hamburg, not knowing the proportion between the ell of that place and yard of London, and having orders to procure 81 yards of cloth, of which 7 ells of Hamburg must be had for 3 l. Sterling; how shall he discover how many pounds Sterling the 81 yards will amount to, only by knowing that 7 ells of France make 9 yards of London, and 7 ells of Holland make 4 ells of France, and that 1 ell of Holland makes  $1\frac{1}{2}$  of Hamburg?

Note, since  $1 = 1\frac{1}{2}$ , consequently  $5 = 6$ , which dispatches the fraction.

9 yards of London	=	7 ells of France.
4 ells of France	=	7 ells of Holland.
5 ditto of Holland	=	6 ditto of Hamburg.
7 ditto of Hamburg	=	3 l. Sterling.

How much Sterling for 81 yards?

Both sides of the equation being abridged, it will be

$$\frac{9 \times 7 \times 3 \times 3}{2 \times 5} = L. 56 14s. \text{ Answer.}$$

#### Variety XIV. INTREST.

##### INTRODUCTION.

Interest of money is an allowance made by the borrower to the lender for the use of money, at a certain rate *per cent.* *per annum*; which may be under the rate established by law, if it is so stipulated betwixt the parties, but never can be above it, without incurring a severe penalty on the part of the lender.

After the Jews were banished, there were, from time to time, many laws and ordinances made in this kingdom, as well by the church as state, against usury; but all to little purpose: for persons who wanted money, and were willing to give extraordinary interest, were never at a loss to be supplied,

plied, provided they were men of credit, or could find good security. Wherefore, as the practice could not be prevented, the parliament, anno 37 Hen. VIII. cap. 9. made an act to prohibit the taking more than 10 per cent. under very severe penalties. This act commenced the 31st of January 1545, and is the first wherein any rate of interest is mentioned. In this act usury is declared to be unlawful.

In less than seven years, anno 5 Ed. VI. c. 20. this act was repealed, and every gratuity whatever forbid for the use of money, under severe penalties.

Anno 13 Eliz. c. 8. this statute of Edward was repealed, and that of Henry VIII. with some additional clauses, revived.

Anno 21 Jac. I. c. 17. it was made penal to take above 8 per cent.

Anno 12 Car. II. c. 13. it was made penal to take above 6 per cent.; and anno 12 Annæ, c. 16. the interest of money was reduced to 5 per cent. from the 29th of September 1714, and this statute continues in force to this day.

### SECT. I. Simple Interest.

Simple interest is that which is paid for the loan of any principal or sum of money lent out for some time at any rate per cent. agreed on betwixt the borrower and the lender, which, according to 12 Annæ, sess. 2. c. 6. no person is to take for the loan of monies, &c. above 5 l. for the forbearance of 100 l. for the space of a year; and bonds, contra&ts, &c. made for money let at a greater interest, to be void and null, and the offender to forfeit triple value.

*Cafe 1.* The principal rate and time given, to find the interest.

### R U L E.

Multiply the principal by the rate, and that product by the time, the last product divided by 100 quotes the answer.

### E X A M P L E S.

*Exam. 1.* What is the interest of L. 426 : 5 : 9 for 6 $\frac{1}{2}$  years, at 4 $\frac{1}{2}$  per cent.?

$$\begin{array}{r}
 426.2875 \\
 -\frac{4\frac{1}{2}}{1705.1500} \\
 -\frac{213.14375}{1918.29375} \\
 -\frac{6\frac{3}{4}}{8) 11509.76250} \\
 -\frac{1438.7203125}{100) 129.484828125 = L. 129-9-8\frac{1}{4}} \\
 \end{array}
 \qquad
 \begin{array}{r}
 \text{Or thus,} \\
 426.2875 \\
 -\frac{.045}{21314375} \\
 -\frac{17051500}{191829375} \\
 -\frac{6\frac{3}{4}}{8) 115.0976250} \\
 -\frac{14.3872031}{129.4848281 = 129 L.} \\
 [9 s. 8\frac{1}{4} d.]
 \end{array}$$

It would have been the same thing, though all the decimal places had been saved in the operation, but three places next the point ; as any thing below the common subdivisions of a pound cannot be reckoned in interest, as there is no specie small enough to be offered in payment, and the least fraction extraordinary is usury.

The rationale of this, and all the other cases of simple interest, is deduced from compound or universal proportion : for

$$\begin{array}{lllll}
 L. pr. & year. & L. in. & L. prin. & years. \\
 100 : 1 & :: 4\frac{1}{2} & : 427.2875 & : 6\frac{3}{4} & : L. 129 : 9 : 8\frac{1}{4}.
 \end{array}$$

And the first and last method is the same ; for it makes no difference to multiply by  $4\frac{1}{2}$  and divide by 100, and to multiply by .045 without any division.

### C O N T R A C T I O N S.

The interest of any sum for one year may be found practically as commission, which may be afterwards multiplied into any number of given years, and divided for any aliquot parts of a year.

The last example resumed.

$$20) 426.2875$$

$$10) 21.3144 \text{ at } 5 \text{ per cent.}$$

$$2.1314 \text{ at } \frac{1}{2} \text{ being } \frac{1}{5} \text{ of } 5 \text{ per cent.}$$

[carried forward] 19.1830 at  $4\frac{1}{2}$  per cent. for 1 year.

19.1830 at  $4\frac{1}{4}$  per cent. for 1 year.  
 $\underline{6\frac{3}{4}}$

8) 115.098 for 6 years  
 $\underline{14.387}$

129.485 as before.

Or thus.

5) 426.2875

2) 85.2575 at 20 per cent.  
 $\underline{42.6287}$  at 10 per cent.

80) 127.8862 at 30 per cent.  
 $\underline{1.5973}$  at  $\frac{3}{8}$  per cent.

129.4835 at  $30\frac{3}{8} = 4\frac{1}{2} \times 6\frac{3}{4}$ .

*Exam. 2.* What is the interest of L. 575, 15s. for  $5\frac{3}{4}$  years, at 5 per cent.?

4) 575 15

143 18 9 at 25 per cent. = 5 per cent.  $\times$  5 years.

14 7 10 $\frac{1}{2}$  for  $\frac{1}{4}$  year by dividing the principal by 40.

7 3 11 $\frac{1}{4}$  for  $\frac{1}{4}$  or  $\frac{1}{2}$  of  $\frac{1}{2}$  year.

L. 165 10 6 $\frac{3}{4}$  = 5 $\frac{1}{4}$  at 5 per cent.

*Exam. 3.* What is the interest of L. 478, 10s. for  $3\frac{1}{5}$  years, at 6 per cent. per annum?

6 10) 478.5

$\underline{3\frac{1}{5}}$

2) 47.85 for 10 per cent.

19 $\frac{1}{5}$  23.925 for 5 per cent.

9.57 for 2 per cent.  $\frac{1}{5}$  of  $\frac{1}{5}$ .

9.57 for 2 per cent. ditto.

.957 for  $\frac{1}{5} = \frac{1}{25}$  of 2 per cent.

91.872 for  $19\frac{1}{5} = 6$  per cent. of  $3\frac{1}{5}$  years.

It is not accurate in the calculation of interest to take aliquot parts for months, because the year is not divided into months, consisting each of the same number of days; and it happens more frequently in business, that one hath occasion to calculate interest for days than for any fixed space of time: wherefore, to find the interest of any sum for any number of

Y y 2

days

days less than a year, multiply the given sum by the number of days, and divide the product by 7300, the quotient will give the interest at 5 per cent. which is the legal rate; but if the interest at a higher or lower rate is required, take parts of the quotient for the difference, and add or subtract accordingly.

*Exam. 4.* What is the interest of £ 547, 15s. at 5 per cent. for 320 days?

$$\begin{array}{r}
 547.75 \\
 - 320 \\
 \hline
 109550 \\
 - 164325 \\
 \hline
 7300) 1752.8000 (24.0109 = L.24-0-25 \\
 \underline{- 140} \\
 \hline
 292 \\
 - 292 \\
 \hline
 80 \\
 - 73 \\
 \hline
 700 \\
 - 657 \\
 \hline
 (43)
 \end{array}$$

The reason of this operation is deduced from the same principles with the foregoing:

$$\text{For } 100 : 365 :: 5 : 547.75 : 320 : L. 24.0109$$

Hence it is plain, that as the second and third terms remain always the same, they will always admit of the same abbreviation; and therefore when the principal is multiplied into the days, and that product divided by  $100 \times 73$ , it will have the same effect as if the work was carried on at length. As cases of this kind are more frequent than any other, I would advise the young arithmetician to get the product of 73 into all the nine digits by heart, that he may as readily divide any number by it, as by a single digit.

*Exam. 5.* What is the interest of £. 248, 19 s. for 175 days at 4 per cent.?

248.95

$$\begin{array}{r}
 248.95 \\
 175 \\
 \hline
 124475 \\
 423215 \\
 \hline
 7300) 435.6625 \\
 \hline
 5) 5.9679 \text{ at } 5 \text{ per cent.} \\
 1.1935 \text{ at } 1 \text{ per cent.} \\
 \hline
 4.7744 \text{ at } 4 \text{ per cent.} = L. 4-15-5\frac{3}{4}.
 \end{array}$$

*Exam. 6.* What is the interest of L. 150 from the 18th day of January to the 11th of November, at  $4\frac{3}{4}$  per cent.?

Days

January	13	
February	28	150
March	31	297
April	30	
May	31	73) 445.50
June	30	
July	31	20) 6.102 at 5 per cent.
August	31	305 at $\frac{1}{4}$ per cent.
September	30	
October	31	5.797 = L. 5-15-11, at $4\frac{3}{4}$ per cent.
November	11	
		297

*Exam. 7.* What is the interest of L. 384, 16 s. from the 7th of May to the 11th of December, at  $5\frac{1}{2}$  per cent.?

$$\begin{array}{r}
 384.8 \\
 219 \\
 \hline
 731.12 \\
 7696 \\
 \hline
 73) 842.712 \\
 \hline
 11.544 = L. 11-10-10\frac{1}{2}.
 \end{array}$$

When interest is to be calculated on cash-accounts, accounts-current, or any other accounts where partial payments are made, or partial debts contracted, multiply the several balances into the days they are at interest, and the sum of these products, divided by 7300, will quote the interest at 5 per cent.: for any other rate make the proper addition or deduction as before. The whole operation ought to stand as in the following examples.

*Exam.*

*Exam.* 8. On the 1st of May, I lent Ralph Newlands *per bill*, at one day's date, £. 500; which I received back in the following partial payments, *viz.* on the 13th May, £. 50; on the 4th June, £. 56; on the 14th July, £. 44; on the 23d ditto, £. 50; on the 18th August, £. 87; on the 30th ditto, £. 13; on the 21st September, £. 30; on the 18th October, £. 30; on the 29th ditto, £. 40; on the 11th November, £. 50; and on the 28th December, £. 50: what interest is due at 5 per cent.?

*Dates.* *Cash-transactions with Ralph Newlands.* exten. days pro.

May	1	Lent <i>per bill</i> at one day's date	- -	500	13	6500
	13	Received in part	- -	50		
				Bal.	450	22
June	4	Received in part	- -	56		9900
				Bal.	394	40
July	14	Received in part	- -	44		15760
				Bal.	350	9
	23	Received in part	- -	50		3150
				Bal.	300	26
Aug.	18	Received in part	- -	87		7800
				Bal.	213	12
	30	Received in part	- -	13		2556
				Bal.	200	22
Sept.	21	Received in part	- -	30		4400
				Bal.	170	27
Oct.	18	Received in part	- -	30		4590
				Bal.	140	11
	29	Received in part	- -	40		1540
				Bal.	100	13
Nov.	11	Received in part	- -	50		1300
				Bal.	50	47
Dec.	28	Received in full of principal	- -	0		2350
	73)	598.46				59846

8 3 11 $\frac{1}{2}$  interest on the above account.

*Exam.* 9. Lent John Jameson *per bill* dated 18th January, payable one day after date, £. 878 : 19 : 10; which I received back in the following partial payments, *viz.* on the 27th February, £. 57 : 15 : 7; on the 18th March, £. 37, 14 $\frac{1}{2}$ ; on the 29th April, £. 34, 11 s.; on the 12th May, £. 136,

15 s.

Var.

15 s.

L. 15

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balance

at 5 p

Dates.

Jan.

Feb.

March

April

May

June

July

Aug.

Ditto

Sept.

Oct.

Nov.

Ditto

Dec.

73)

I ha

conseq

L. 500

he gave

he paid

15 s. 7 d.; on the 19th June, L. 67 : 13 : 4; on the 15th July, L. 15 : 15 : 6; on the 25th ditto, L. 111 : 11 : 11; on the 3d October, L. 78 : 7 : 4; on the 19th November, L. 100; on the 23 ditto, L. 100; and on the 30th December received the balance of the principal: how much interest ought I to claim at 5 per cent.?

*Cash transactions with John Jameson.*

Dates.		Extended.	Days.	Products.
Jan.	18	L. p. bill at one day's date	878 19 10 40	35159 13 4
Feb.	27	Received in part	57 15 7	
March	18	Received in part	Bal. 821 4 3 19 15603 0 9	
			37 14	
April	29	Received in part	Bal. 783 10 3 42 32907 10 6	
			34 11	
May	12	Received in part	Bal. 748 19 3 13 9736 10 3	
			136 15 7	
June	19	Received in part	Bal. 612 3 8 38 23262 19 4	
			67 13 4	
July	15	Received in part	Bal. 544 10 4 26 14157 8 8	
			15 15 6	
Ditto	25	Received in part	Bal. 528 14 10 10 5287 8 4	
			111 11 11	
Oct.	3	Received in part	Bal. 417 2 11 70 29200 4 2	
			78 7 4	
Nov.	19	Received in part	Bal. 338 15 7 47 15922 12 5	
			100	
Ditto	23	Received in part	Bal. 238 15 7 4 955 2 4	
			100	
Dec.	30	Received in full cf prin <sup>t</sup>	Bal. 138 15 7 37 5134 16 7	
			138 15 7	
				187317 6 8

73) 1873.17 6 8

25 13 2 interest on the above account.

I have given Thomas Truman a cash-credit for L. 2000; in consequence of which, on the 8th January, I paid his bill for L. 500; ditto 29th, paid his draught for L. 560; February 27th he gave me a bill on the new bank at eight for L. 580; March 14th he paid me per receipt L. 140; April 29th he drew for L. 1500 at

at sight; May 18th he paid me *per my receipt* £. 1000; June 29th he drew at sight for £. 1270, and 18th July for 350; September 19th he paid *in per receipt* £. 500, and October 18th £. 250, November 11th he drew at sight for £. 750, and ditto 13th for £. 250. December 12th he paid *in per receipt* £. 580, and 30th ditto £. 420. On the 8th January thereafter, he demands a settlement: what principal and interest is he due to me for the year's transactions, interest at 5 per cent?

Dates.	Cash-transactions with Thomas Truman.	Ex.	Days.	Pro.
Jan. 8	Paid his bill	-	-	500 50 25000
Ditto 29	Paid his draught	-	-	560 29 16240
		Bal.	1060	
Feb. 27	Received in part	-	-	580 15 7200
		Bal.	480 15	
March 14	Received in part	-	-	140 0 0
		Bal.	340 46	15640
April 29	Paid	-	-	1500 0 0
		Bal.	1840 19	34960
May 18	Received in part	-	-	1000 0 0
		Bal.	840 42	35280
June 29	Paid	-	-	270 0 0
		Bal.	1110 19	21090
July 18	Paid	-	-	350 0 0
		Bal.	1460 63	91980
Sept. 19	Received in part	-	-	500 0 0
		Bal.	960 29	27840
Oct. 18	Received in part	-	-	250 0 0
		Bal.	710 24	17040
Nov. 11	Paid	-	-	750 0 0
		Bal.	1460 2	2920
Ditto 13	Paid	-	-	250 0 0
		Bal.	1710 29	49590
Dec. 12	Received in part	-	-	580 0 0
		Bal.	1130 18	20340
Ditto 30	Received in part	-	-	420 0 0
Jan. 8		Bal.	710 9	6390
				371510 Then

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Then 73) 3715.10

50 17 10	interest on the above account.
710	balance of the account.

---

760 17 10 total balance.

When cash-credits are given in the banks, accounts are generally kept as in the preceding examples, and a balance made upon every transaction, which is multiplied into the days the first convenient minute; in which case, when the period of settling their accounts comes, they have only to add up the products, divide by 7300, and the account is finished.

When partial payments are made upon bonds, bills, &c. at any interval greater than twelve months, the interest is calculated in a progressive manner, by adding the interest to the principal at the time of the first payment, and from that sum deducting the payment, &c.; and this method hath the authority of the court of session for its sanction, in cash-accounts of various kinds; only the banks are not allowed, at least do not take this privilege, because their cash-transactions are very frequent; and it amounts to the same thing, when they give no cash-accounts but what must be settled once in twelve months.

*Specimen of a progressive account.*

Borrowed on bond, of date 21st May 1754, L. 1000. Paid back on the 18th July 1755, L. 350; on the 29th September 1756, L. 150; on the 19th December 1757, L. 180; on the 18th July 1758, L. 120; on the 18th September 1759, L. 50; and on the 21st May 1761, I paid up the balance of the principal: how much more had I to pay to retire my bond, interest at 5 per cent.?

1754.	<i>May 21</i>	Borrowed on bond - - -	1000			
		Interest for 1 year and 58 days - -	57	18	10	$\frac{1}{4}$
		Amount	1057	18	10	$\frac{1}{4}$
1755.	<i>July 18</i>	Paid in to account - - -	350			
		Balance	707	18	10	$\frac{1}{4}$
		Interest for 1 year and 73 days - -	42	9	6	
		Amount	750	8	4	$\frac{3}{4}$
1756.	<i>Sept. 29</i>	Paid in to account - - -	150			
		Balance	600	8	4	$\frac{3}{4}$
		Interest for 1 year and 81 days - -	36	13	7	$\frac{1}{4}$
		Amount	637	2	0	$\frac{1}{2}$
1757.	<i>Dec. 19</i>	Paid in to account - - -	180			
		Balance	457	2	0	$\frac{1}{2}$
		Interest for 1 year and 211 days - -	36	1	4	
		Amount	493	3	4	$\frac{1}{2}$
	<i>July 18</i>	Paid in to account - - -	120			
		Balance	373	3	4	$\frac{1}{2}$
		Interest for 1 year and 62 days - -	21	16	6	$\frac{1}{2}$
		Amount	394	19	11	
1759.	<i>Sept. 18</i>	Paid in to account - - -	50			
		Balance	344	19	11	
		Interest for 2 years and 245 days - -	46	1	6	$\frac{3}{4}$
		Amount	391	1	5	$\frac{3}{4}$
1761.	<i>May 21</i>	Paid the balance of the principal	150			
		Balance due	241	1	5	$\frac{3}{4}$

*Cafe 2.* Amount, rate, and time given, to find the principal, or present worth.

### R U L E.

As the amount of £. 100 at the rate and time given is to £. 100, so is the given amount to the principal or present worth required.

*Exam. 1.* What ready money will pay a debt, due 3 years and

and 145 days hence, of L. 3998 : 12 : 10 $\frac{1}{4}$ , discounting interest at 3 per cent. per annum?

The time = 3.4

3 the interest of 100 l. for 1 year.

Add  $\frac{10.2}{100}$  } the interest } of 100 for the  
 \_\_\_\_\_ }                  } given time.  
 110.2 } the amount }

Then L. 110.2 : 100 :: 3998.6427

$$\begin{array}{r}
 & 100 \\
 110.2) & 399864.270000 \quad (3998.6427 \text{ amount.} \\
 & 3306 \quad \quad \quad 3628.53239 \text{ prin.} \\
 & \underline{6926} \quad \quad \quad \underline{370.11031} \text{ disct} \\
 & 6612 \\
 & \underline{\underline{3144}} \\
 & 2204 \\
 & \underline{\underline{9402}} \\
 & 8816 \\
 & \underline{\underline{5867}} \\
 & 5510 \\
 & \underline{\underline{3570}} \\
 & 3306 \\
 & \underline{\underline{2640}} \\
 & 2204 \\
 & \underline{\underline{4360}} \\
 & 3306 \\
 & \underline{\underline{10540}} \\
 & 9918 \\
 & \underline{\underline{622}}
 \end{array}$$

Or it will be the same thing to work as follows:

3.4

$\cdot 03$  interest of L. 1 for 1 year.

$1.102$  amount of ditto for 1 year.

Z z 2

Then

Then  $1.102 : 1 :: 3998.6427 : 3628.53239$  as before.

*Exam. 2.* What ready money will discount a bill payable 6 months hence of L. 500, interest at 5 per cent.?

$$\begin{array}{r} .05 \\ 5 \\ \hline 1.025) 500.000 \\ .205) 100.000 \\ .041) 20.000 \\ \hline \end{array}$$

L. 487 16 1 $\frac{1}{2}$  *Answer.*

Bankers and others who discount bills never use the above method, because in that case they would have no more than the interest of the money they pay down, which they reckon too small a consideration for keeping clerks and ready money by them for the purposes of discounting; and therefore, they not only deduce interest from the contents of the bill for the time it hath to run, with three days of grace, but likewise very often charge commission. It is a common practice among petty bankers, to lend a man money by discounting his own bill, and taking commission at  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and sometimes more for 3 months; when that time is elapsed, and the acceptor of the bill cannot conveniently pay, out of mere lenity, the bill is renewed for three months longer, and discounted upon an extraordinary commission, and the practice is continued for a mighty favour till the man's eyes are opened, with paying at the rate of 10 per cent. for the use of his money. There are others who go still more neatly to work in taking advantage of the necessitous. — The man who wants money offers his own acceptance at a short date to sale. — If the bill is for L. 100, the moneyed man perhaps offers him L. 80, more or less, as he can make his bargain, or as he understands the acceptor's credit depends on having the money. — This method needs little calculation. For the former take the following examples.

*Exam. 1.* A bill of L. 500 is to be discounted 120 days, to run interest at 5 per cent.; how much money will the holder receive?

$$\begin{array}{r} 500 \\ 120 \\ \hline 73) 600.00 \\ \hline 8 \ 4 \ 4\frac{1}{2} \text{ deducted from L. 500.} \\ 491 \ 15 \ 7\frac{1}{2} \text{ Answer.} \end{array}$$

*Exam.*

*Exam. 2.* A bill of L. 449, 18 s. is to be discounted, 180 days to run, interest at 5 per cent. upon allowing commission at  $\frac{1}{2}$  per cent.; what money does the holder receive?

$$\begin{array}{r}
 449.9 \\
 180 \\
 \hline
 73) 809.820 \\
 \hline
 11.093 \text{ interest.} \\
 2.2495 \\
 \hline
 13.3425 \\
 \hline
 436.5575 = L. 436 11 2
 \end{array}$$

*Exam. 3.* A bill dated May 8. payable September 15. presented June 10. for discount, was discounted by deducting interest at 5 per cent. commission  $\frac{1}{4}$ ; what will the holder receive, the contents being L. 378, 16 s.?

$$\begin{array}{r}
 378.8 \\
 97 \text{ days to run.} \\
 \hline
 26516 \\
 34092 \\
 \hline
 73) 367.436 \\
 \hline
 5.0333 \\
 .947 \\
 \hline
 5.9803 = L. 5 19 7
 \end{array}$$

*Cafe 3.* The amount, principal, and time given, to find the rate of interest,

### R U L E.

As the principal multiplied by the time is to the whole interest, so is L. 100 to the rate per cent.

At what rate of interest will L. 500 become L. 537, 10 s. in 1 year and 6 months?

$$\begin{array}{r}
 500 \\
 1.5 \\
 \hline
 5) 750.0 : 37.5 :: 100 \\
 \hline
 5) 150 \quad 7.5 \quad 50 \\
 2) 30 \quad 1.5 \quad 1.5 \\
 \hline
 15 \quad \quad \quad \\
 \hline
 15) 750 \\
 \hline
 \end{array}$$

5 per cent.

*Case 4.* The principal, amount, and rate of the interest being given, to find the time.

As the interest of the principal of 1 year, at the given rate, is to 1 year, so is the whole interest to the time required.

*Exam.* In what time will L. 500 become L. 537, 10*s.* at 5 per cent. per annum?

$$\begin{array}{r}
 5) 25 : 1 :: 37.5 \\
 \hline
 5) 37.5 \\
 \hline
 5) 7.5 \\
 \hline
 1.5 = 1 year and 6 months.
 \end{array}$$

#### Practical questions.

*Quest. 1.* I have a bond from the bank for L. 500 at 4 per cent.—Ditto with the tannery-company for L. 500 at 5 per cent. 300  $3\frac{1}{2}$  per cent. annuities, and 500 3 per cent. annuities; what may I afford to live at per annum, without breaking my stock?

L. 500 at 4 per cent.	- - -	L. 20
500 at 5 per cent.	- - -	25
300 annuities at $3\frac{1}{2}$ per cent.	-	10 10
500 annuities at 3 per cent.	-	15
		<hr/>
1800		70 10

*Quest. 2.* Being straitened, I discounted my own bill with J. D. for 500, dated the 1st January, payable in 2 months thereafter, allowing commission at  $\frac{1}{2}$  per cent. and deduction of interest.

interest at 5 *per cent.* — I was obliged to repeat the transaction six different times for that year, and paid the last bill when it fell due : at what rate of interest had I my money ?

500 for which the bill was granted.

4 3 4 interest for 2 months.  
2 10 commission.

6 13 4 charges in all.

493 6 8 received.

Then  $500 : 6 \frac{13}{4} \times 6 :: 100$

5) 40

[support.]  
8 *per cent.* which is too much for any trade to

But still more if we consider that the discounting of these bills will cost L. 6 : 13 : 4 ready money every time, excepting the first, which will raise the interest considerably higher.

Quest. 3. Sold my own acceptance of L. 100 payable 3 months after date, for L. 80 ready money ; at what rate of interest did I procure the money ?

80

.25

20.00 : 20 :: 100 : 100 per cent.

For  $100 \times 2$

= 100

2

This is truly barbarous, and yet it is practised. Such methods of eluding the law cannot be too severely punished, or too cautiously guarded against. — It would be much better for a man, who finds he can pay his creditors if a little time is granted him, to throw himself upon their mercy, than give away his subject voluntarily to such merciless usurers ; nor is it to be imagined, that his creditors would use him so unaccountably ill.

*Compound*

*Compound Interest.*

Compound interest is an allowance or gratuity made by the borrower to the lender, not only for the use of the principal, but likewise for the use of the interest, after it falls due; so that the interest being continually added to the principal, at the end of a year the sum or amount becomes a new principal for the succeeding year, and in this case the principal and interest are always increasing. It is true, the law doth not allow money to be put out at compound interest, but the interest may be exacted precisely when it is due, and lent out again, by which means all the purposes of compound interest may be answered to the lender; besides, in purchasing annuities, pensions, or leases in reversion, it is ordinary to allow the purchaser compound interest for his ready money, and therefore every dealer in these articles ought to understand it.

*Case 1.* Principal, rate, and time given, to find the interest.

To the principal add the interest for the first year, which becomes a new principal; to which add the interest thereof for another year, which becomes a principal at the end of the second year; proceed thus progressively for any number of years, and if there are days, add the interest of the last amount for that time, the difference betwixt the principal and this last amount gives the interest.

Or multiply the principal by the amount of L. 1 for a year continually for all the years proposed.

What is the compound interest of L. 500 for 3 years, at 5 per cent.?

20) 500	500
25	1.05 amount of 1 l.
	_____
20) 525     1st year.	525     1st year.
26.25	1.05
	_____
20) 551.25     2d year.	2625
27.5625	525
	_____
Amount 578.8125     3d year.	551.25     2d year.
	1.05
Interest 78.8125     3d year.	Amount 578.8125     3d year.
	_____
Interest 78.8125     3d year.	78.8125     3d year.

Ques.

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*Quest.* 2. What will L. 524, 12 s. amount to in  $3\frac{1}{2}$  years, compound interest being allowed at 4 per cent.?

$$\begin{array}{r}
 524.6 \\
 \cdot 04 \\
 \hline
 20.984 \text{ interest.} \\
 \hline
 545.584 \text{ amount.} \\
 \cdot 04 \\
 \hline
 21.82336 \text{ interest.} \\
 \hline
 567.40736 \text{ amount.} \\
 \cdot 04 \\
 \hline
 22.6963 \text{ interest.} \\
 \hline
 590.10366 \text{ amount at three years end.} \\
 11.80207 \text{ interest for 6 months.} \\
 \hline
 601.90573 \text{ amount at the given period.}
 \end{array}$$

*Cafe.* 2. The amount, rate, and time given, to find the principal or present worth.

### R U L E.

As the amount of L. 1 at compound interest, at the rate and for the time given, is to L. 1, so is the amount given to the present worth required.

*Exam.* What ready money will clear a debt due three years hence of L. 578 : 16 : 3, compound interest being allowed at 5 per cent.?

$$\begin{array}{r}
 1.05 \\
 1.05 \\
 \hline
 1.1025 \\
 1.05 \\
 \hline
 1.157625 : 1 :: 578.812500 (500 l. Answer. \\
 578.8125 \\
 \hline
 \dots \dots \dots
 \end{array}$$

*Case 3.* The principal, amount, and rate being given, to find the time.

R U L E.

Divide the amount by the principal, and that quotient by the amount of £. 1 for a year, continually till unity is quoted; the number of divisions gives the number of years.

*Exam.* In what time will £. 500 amount to £. 578 : 16 : 3, compound interest being allowed at 5 per cent.?

$$\begin{array}{r}
 500) 578.8125 \\
 \hline
 1.05) 1.157625 \quad 1 \\
 \hline
 1.05) 1.1025 \quad 2 \\
 \hline
 1.05) 1.05
 \end{array}$$

Variety XV. Of purchasing freehold or real estates.

ALL freehold or real estates, are such as are bought to continue for ever; and questions relating to such purchases, except in reversion, may be done in the most simple manner, without having recourse to the usual theorems for performing questions of that kind.

*Case 1.* When the yearly income is required, it will be as 100 : the proposed rate per cent. :: the sum to be laid out in the purchase : the yearly income.

*Exam. 1.* A gentleman would lay out £. 1000 on the purchase of an estate, but in such a manner, as he might get  $7\frac{1}{2}$  per cent. for his money, compound interest; what yearly income would he have from such an estate?

$$100 : 7.5 :: 1000 : ?$$

For 1000	Or	2 0) 100 0
7.5		<hr/>
100)	75.000	2) 50
		<hr/>
		25
		<hr/>
		75 as before.

*Exam. 2.* A gentleman hath £. 3500 to lay out on the purchase of an estate, so as to have  $4\frac{1}{2}$  per cent. per annum for his money; what would be his income?

3500

$$\begin{array}{r}
 3500 \\
 - 4 \\
 \hline
 14000 = 4 \text{ per cent.} \\
 - 1750 = \frac{1}{2} \\
 \hline
 157.50 = L. 157 10 s. per annum.
 \end{array}$$

*Cafe 2.* If the value of an estate is required, by having the yearly rent, and a certain allowance *per cent.* given; it will be as the rate *per cent.* : 100 :: the yearly rent : the value required.

*Exam. 1.* There is an estate which yields yearly L. 75; what is the value in ready money, allowing interest at  $7\frac{1}{2}$  *per cent.*?

$$\begin{array}{c}
 7.5 : 100 :: 75 : 1000 \\
 \text{For } 100 \times 75 \\
 \hline
 = 1000.
 \end{array}$$

7.5

*Exam. 2.* An estate brings of yearly rent, L. 157, 10s.; what would it sell for, allowing the purchaser  $4\frac{1}{2}$  *per cent.* for his money?

$$\begin{array}{c}
 4.5 : 100 :: 157.5 : 3500 \\
 \text{For } 100 \times 157.5 \\
 \hline
 = 3500 \text{ Answer.}
 \end{array}$$

4.5

*Cafe 3.* To find the rate *per cent.* on money laid out on the purchase of freehold estates. It will be, as the money laid out on the purchase is to the yearly rent, so is 100 to the rate *per cent.*

*Exam. 1.* Suppose L. 1000 is paid for a freehold estate, which yields of yearly rent L. 75; what rate of interest doth the purchaser get for his money?

$$\begin{array}{c}
 1000 : 75 :: 100 : 7.5 \\
 \text{For } 75 \times 100 \\
 \hline
 = 7.5 \text{ Answer.}
 \end{array}$$

1000

*Exam. 2.* Suppose an estate of L. 150 *per annum* is sold at 25 years purchase, how much *per cent.* hath the purchaser for his money?

$$\begin{array}{c}
 25 \times 150 = 3750 : 150 :: 100 : 4 \\
 \text{Or } 25 : 1 :: 100 : 4 \\
 3 A 2 \qquad \qquad \qquad \text{Variety}
 \end{array}$$

### Variety XVI. Of purchasing freehold estates in reversion.

*Case 1.* THE yearly rent of a freehold estate being known, to find the present worth of the reversion of the said estate, after the expiration of a certain number of years, at any given rate *per cent.*; find the full value of the estate *per last variety*, and then find the principal that will raise that sum, at the rate and time given, compound interest being allowed.

Suppose the reversion of a freehold estate, of L. 50 yearly rent, to commence three years hence, is to be sold immediately; what ready money ought to be paid for it, allowing the purchaser 5 *per cent.* for his money?

$$\text{First } 5 : 100 :: 50 : 1000$$

$$\text{Then } 1.05 \times 1.05 \times 1.05 : 1 :: 1000 : L. 863\text{-}19s.$$

*Case 2.* The sum given for the reversion of a freehold estate, to commence after a certain number of years, being known, to find the yearly rent, allowing the purchaser an assigned rate of interest for his money.

Find the amount of the sum given, to the time from which the reversion is to commence; and the yearly income which that amount will purchase, is the answer.

Suppose the reversion of a freehold estate to commence three years hence, is sold for L. 863, 19 s. allowing the purchaser at the rate of 5 *per cent.* for his money; what ought the yearly rent to be in this case?

$$100 : 5 :: 1.05 \times 1.05 \times 1.05 \times 863.95 : L. 50. \text{ Answer.}$$

#### *Construction of decimal tables for computations in compound interest.*

Operations by numbers in compound interest are generally attended with such heavy multiplications and divisions, that they become very troublesome, and liable to error; for which reason I have thought proper to insert the following tables; by the help of which all questions relative to compound interest may be quickly resolved, that fall within the compass of the tables.

*Construction*

*Construction of the first table.*

The first table shows the amount of 1*l.* from one year to forty, at 3,  $3\frac{1}{2}$ , 4,  $4\frac{1}{2}$ , and 5 per cent. and is effected by the continual multiplication of the amount of 1*l.* for one year at these rates, per case 1*l.* of compound interest. Thus the amount of 1*l.* the third year at 5 per cent. will be

$$1.05 \times 1.05 \times 1.05 = 1.157625, \text{ and so of any other.}$$

*Construction of the second table.*

The second table shows the present worth or value of 1*l.* payable at any period from one to forty years inclusive, and is effected by dividing 1*l.* by the amount found in the first table for the time and rate assigned. Thus 1 being divided by 1.157625 will quote .8638376 for the present worth of 1*l.* due three years hence, compound interest at 5 per cent.

*Construction of the third table.*

The third table shews the amount of 1*l.* per annum, and is easily made from the first table thus: To 1*l.* being the first year of the third table, add the first year of the first table, and the amount will be the second year of the third table; to which add the second year of the first table, and the amount will be the third year of the third table, &c. For instance,

To 1.000000  
add 1.050000 the amount of 1*l.* at 5 per cent. for 1 year.

---

2.050000 the amount for 2 years in the third table.  
1.102500 second year of the first table.

---

3.152500 third year of the third table, &c.

For the amount at the expiration of one year is the annuity itself, namely 1*l.* and 1*l.* being put out to interest at 5 per cent. will at the end of the year become 1.05, to which the annuity being added, the sum 2.05 will be the amount of the annuity at the expiration of two years, as above.

Again, 2.05*l.* the last amount, being let out to interest at the same rate for another year, it becomes at the expiration of that period 2.1525, to which adding 1*l.* the annuity, we have 3.1525, which gives the third year of the third table, as before, &c.

*Construction*

*Construction of the fourth table.*

The fourth table shews the present value of 1*l. per annum, &c.* and is easily constructed by help of the second. Thus,

The present value of the first year in the second table is the same in the fourth table; the first and second years in the second table, added together, make the second year in the fourth table, and the third year in the second table, added to the second year in the fourth table, make the third year in the fourth. For instance, at 5*per cent.*

The 1st year in 2d and 4th tables	=	.9523809
The 2d year in the 2d table	=	.9070294
The 2d year in the 4th table	=	1.8594104
The 3d year in the 2d table	=	.8638376
The 3d year in the 3d table	=	2.7232480 <i>&amp;c.</i>

For it is plain, that every number in the fourth table is the present worth or principal of that sum or number in the third table which stands against the same years, and therefore the whole construction depends upon the second case of compound interest. Suppose the present worth of 2.05*l.* the second number in the third table, were required at 5*per cent.* it would be

$$\frac{2.05}{1.1025} = 1.8594104. \text{ For as } 1.1025 : 1 :: 2.05 : 1.8594104 \\ \text{ &c.}$$

*Construction of the fifth table.*

The fifth table shews the annuity which 1*l.* will purchase, *&c.* and is made in this manner. Find the present value of 1*l. per annum* in the fourth table at the assigned rate and time; then divide unity by the present value so found, and the quotient will be the annuity which 1*l.* will purchase at the same rate for the same time. For instance, suppose the annuity were required which 1*l.* will purchase, to continue for three years, interest 5*per cent.*

In the fourth table under 5*per cent.* and opposite to three years, we have 2.7232480} 1.00000000

The 3d year in the 5th = .3672085 = the annuity for three years.

By the foregoing rules may the tables be continued to any number of years whatever.

T A B L E

## T A B L E I.

Amount of 1*l.* for years.

Years.	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
1	1.0300000	1.0350000	1.0400000	1.0450000	1.0500000
2	1.0609000	1.0712250	1.0816000	1.0920250	1.1025000
3	1.0927270	1.1087178	1.1248640	1.1411661	1.1576250
4	1.1255088	1.1475230	1.1698586	1.1925186	1.2155063
5	1.1592740	1.1876863	1.216529	1.2461819	1.2762816
6	1.1940523	1.2202553	1.2653190	1.3022601	1.3400956
7	1.2298738	1.2722792	1.3159318	1.3608618	1.4071004
8	1.2667700	1.3168090	1.3685691	1.4221006	1.4774554
9	1.3047731	1.3628973	1.4233118	1.4860951	1.5513282
10	1.3439163	1.4105987	1.4802443	1.5529694	1.6288946
11	1.3842338	1.4599697	1.5394541	1.6228530	1.7103393
12	1.4257608	1.5110686	1.6010322	1.6958814	1.7958563
13	1.4685337	1.569560	1.6650735	1.7721961	1.8856491
14	1.5125897	1.6186945	1.7316764	1.8519449	1.9799316
15	1.5579674	1.6753488	1.8009435	1.9352824	2.0789282
16	1.6047064	1.7339860	1.8739812	2.0223701	2.1828746
17	1.6528476	1.7946755	1.9479005	2.1133768	2.2920183
18	1.7024330	1.8574892	2.0258163	2.2084787	2.4066192
19	1.7535060	1.9225013	2.1068494	2.3078603	2.5269502
20	1.8061112	1.9897888	2.1911231	2.411740	2.6532977
21	1.8602945	2.0594314	2.278768	2.5202411	2.7859626
22	1.9161034	2.1315115	2.3699188	2.6336520	2.9252607
23	1.9735865	2.2061144	2.4647155	2.7521663	3.0715238
24	2.0327941	2.2833284	2.5637042	2.8760138	3.2251000
25	2.0937779	2.3632449	2.6658363	3.0054344	3.3863549
26	2.1565912	2.4459585	2.7724697	3.1406790	3.5556727
27	2.2212890	2.5315671	2.8823685	3.2820095	3.7324563
28	2.2879276	2.6201719	2.9987033	3.4296999	3.9201291
29	2.3565655	2.7118779	3.1186514	3.5840364	4.1161356
30	2.4272624	2.8067937	3.2433975	3.7453181	4.3219424
31	2.5000803	2.9050314	3.3731334	3.9178574	4.5380395
32	2.5750827	3.0067075	3.5085587	4.0899810	4.7649415
33	2.6523352	3.1119423	3.6483811	4.2740301	5.0031885
34	2.7319053	3.2208603	3.7943167	4.4663615	5.2533480
35	2.8138624	3.3335904	3.9460889	4.6673478	5.5160154
36	2.8982783	3.4502661	4.1039325	4.8777784	5.7918161
37	2.9852266	3.5710254	4.2683898	5.0968604	6.0814069
38	3.0747834	3.6960113	4.4388134	5.3262192	6.3854773
39	3.1670269	3.8253717	4.616659	5.5658990	6.7047511
40	3.2620377	3.9592597	4.8010206	5.8163645	7.0399887

T A B L E

## T A B L E II.

'The present worth of £1. for years.

Years.	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
1	.9708738	.9661836	.9615385	.9569378	.9523809
2	.9425959	.9335107	.9245562	.9157299	.9070295
3	.9151417	.9019427	.88389964	.8762966	.8638376
4	.8884870	.8714422	.8548042	.8385613	.8227025
5	.8626088	.8419732	.8219271	.8024511	.7835262
6	.8374843	.8135006	.7903145	.7678957	.7462154
7	.8130915	.7859910	.7599178	.7348285	.7106813
8	.7804092	.7594116	.7306902	.7031851	.6768394
9	.7664167	.7337310	.7025867	.6729044	.6446089
10	.7440939	.7089188	.6755642	.6439277	.6139133
11	.7224213	.6849457	.6495809	.6161987	.5846793
12	.7013799	.6617833	.6245971	.5896639	.5568374
13	.6809513	.6394041	.6005741	.5642716	.5303214
14	.6611178	.6177818	.5774751	.5399729	.5050679
15	.6418619	.5968906	.5552645	.5167204	.4810171
16	.6211669	.5767059	.5339082	.4944693	.4581115
17	.6050164	.5572038	.5137733	.4731764	.4362967
18	.5872946	.5383611	.4936281	.4528004	.4155207
19	.5702860	.5201557	.4746424	.4333018	.3957340
20	.5567758	.5025659	.4563870	.4146429	.3768895
21	.5375493	.4855709	.4388336	.3967874	.3589424
22	.5218925	.4691506	.4219554	.3797009	.3418499
23	.5066917	.4532856	.4057263	.3633501	.3255713
24	.4919337	.4379571	.3901215	.3477035	.3100679
25	.4770056	.4231470	.3751163	.3327306	.2953028
26	.4636947	.4088378	.3606892	.3184025	.2812407
27	.4501891	.3950123	.3468166	.3046914	.2678483
28	.4370768	.3816543	.3334775	.2915707	.2550936
29	.4242464	.3687482	.3206514	.2790150	.2429463
30	.4119868	.3562784	.3083187	.2670000	.2313775
31	.3999871	.3442304	.2964603	.2555024	.2203595
32	.3883370	.3325897	.2850579	.2444999	.2098662
33	.3770263	.3213427	.2740942	.2339712	.1998726
34	.3660449	.3104761	.2635521	.2238959	.1903548
35	.3553834	.2999769	.2534155	.2142544	.1812903
36	.3450324	.2898327	.2436687	.2050282	.1726574
37	.3349829	.2800316	.2342969	.1961992	.1644356
38	.3252262	.2705619	.2252854	.1877504	.1566054
39	.3157536	.2614125	.2166206	.1796655	.1491479
40	.3065568	.2525725	.2082890	.1719287	.1420457

Years
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## T A B L E

## T A B L E III.

The amount of 1*l. per annum*, or annuity for years.

Years	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
1	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
2	2.0300000	2.0350000	2.0400000	2.0450000	2.0500000
3	3.0900000	3.1062250	3.1216000	3.1370250	3.1525000
4	4.1336270	4.2149429	4.2464640	4.2781911	4.3101250
5	5.3091358	5.3624659	5.4163226	5.4707097	5.5256312
6	6.4684099	6.5501522	6.6329755	6.7168917	6.8019128
7	7.6624622	7.7794075	7.8982945	8.0191518	8.1420084
8	8.8323360	9.0516866	9.2142263	9.3800136	9.5491089
9	10.1591061	10.3684958	10.5827953	10.8021142	11.0265643
10	11.4638793	11.7313931	12.0061071	12.2882094	12.5778925
11	12.8077957	13.1419919	13.4863514	13.8411788	14.2067871
12	14.1920296	14.6019816	15.0258055	15.4640318	15.9171265
13	15.6177904	16.1130303	16.6268377	17.1599133	17.7129828
14	17.0863242	17.6762864	18.2919112	18.9321094	19.5986320
15	18.5989139	19.2956809	20.0235876	20.7840543	21.5785636
16	20.1568813	20.9710297	21.8245311	22.7193367	23.6574918
17	21.7615877	22.7050158	23.6975124	24.7417069	25.8403664
18	23.414454	24.4996913	25.6454129	26.8550837	28.1323847
19	25.1163684	26.3571805	27.6712294	29.0635625	30.5390039
20	26.8703745	28.2796318	29.7780786	31.3714228	33.0659541
21	28.6764857	30.2694707	31.9692017	33.7831368	35.7192518
22	30.5367803	32.3289022	34.2479698	36.3033779	38.5052144
23	32.4528837	34.4604137	36.6178886	38.9370299	41.4304751
24	34.4264702	36.6665282	39.0826041	41.6891963	44.5019989
25	36.4592643	38.9498567	41.6459083	44.5652101	47.7270988
26	38.5530422	41.3131017	44.3117446	47.5706446	51.1134538
27	40.7096335	43.7590602	47.0842144	50.7113236	54.6691265
28	42.9309225	46.2906273	49.9675830	53.9933332	58.4025828
29	45.2188502	48.0107903	52.9662863	57.4230332	62.3227119
30	47.5754157	51.6226773	56.0849377	61.0070697	66.4388475
31	50.0026782	54.4294710	59.3283352	64.7523878	70.7607899
32	52.5027585	57.3345025	62.7014687	68.6662452	75.2988294
33	55.0770413	60.3412101	66.2095274	72.7562263	80.0637708
34	57.7301765	63.4531524	69.8579085	77.0302565	85.0669594
35	60.4620818	66.6740127	73.6522248	81.4966180	90.3203073
36	63.2759443	70.0076032	77.5983138	86.1639658	95.8363227
37	66.1742226	73.4578693	81.7022464	91.0413443	101.6281388
38	69.1594493	77.0288947	85.9703362	96.1382048	107.7095458
39	72.2342327	80.7249060	90.4091497	101.4644240	114.0950231
40	75.4012597	84.5302778	95.0255157	107.0303231	120.7997742

## T A B L E IV.

The present worth of 1*l.* per annum, or annuity for years.

Years	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
1	0.9708758	0.9661836	0.9615385	0.9569378	0.9523809
2	1.9134697	1.8996943	1.8860947	1.8726678	1.8594104
3	2.8186114	2.8016370	2.7750910	2.7489644	2.7232480
4	3.7170984	3.6730792	3.6298952	3.5875257	3.5459505
5	4.5797072	4.5150524	4.4518223	4.3899767	4.3294767
6	5.4171914	5.3285530	5.2421369	5.1578725	5.0756921
7	6.2322329	6.1145439	6.0020547	5.8927009	5.7863734
8	7.0196022	6.8739555	6.7327448	6.5958861	6.4632128
9	7.7861089	7.6076865	7.4353314	7.2687905	7.1078217
10	8.5302028	8.3166053	8.1108955	7.9127182	7.7217349
11	9.2526241	9.0015510	8.7604763	8.5289169	8.3064142
12	9.9540040	9.6633543	9.3850733	9.1185808	8.8632516
13	10.6349553	10.3027385	9.9856473	9.6828524	9.3935730
14	11.2960731	10.9205203	10.5631223	10.2228253	9.8986409
15	11.9379351	11.5174109	11.1183868	10.7395457	10.3796580
16	12.5611020	12.0941168	11.6522949	11.2140151	10.837695
17	13.1661185	12.6513206	12.1656680	11.7071914	11.2746622
18	13.7535131	13.1896817	12.6592961	12.1599918	11.6895869
19	14.3237991	13.7098374	13.1339385	12.5952936	12.0853208
20	14.8774748	14.2124033	13.5903253	13.0079365	12.4622103
21	15.4150241	14.6979742	14.0291589	13.4947239	12.8211527
22	15.9369166	15.1671243	14.4511142	13.7844248	13.1630026
23	16.4436084	15.6204105	14.8568405	14.1477749	13.4886739
24	16.9355421	16.0583676	15.2469619	14.4954784	13.7986418
25	17.4131477	16.4815146	15.6220787	14.8282089	14.0939445
26	17.8768424	16.8903523	15.9827678	15.1466115	14.3751853
27	18.3270315	17.2853645	16.3295844	15.4513028	14.6430336
28	18.7641082	17.6670188	16.6630618	15.7428735	14.8981272
29	19.1884146	18.0357670	16.9837132	16.0218885	15.1410735
30	19.6004413	18.3920454	17.2920318	16.2883885	15.3724510
31	20.0004285	18.7362758	17.584921	16.5443909	15.5928104
32	20.3887655	19.0688656	17.873550	16.7888909	15.8026766
33	20.7657918	19.3902082	18.1476441	17.0228621	16.0025491
34	21.1318367	19.7006842	18.4111962	17.2467580	16.1929039
35	21.4872200	20.0006612	18.6646116	17.4610124	16.3741942
36	21.8322525	20.2904938	18.9082803	17.6660406	16.5468516
37	22.1672354	20.5705254	19.1425771	17.8622398	16.7112872
38	22.4924616	20.8410874	19.3678625	18.0499902	16.8678926
39	22.8082151	21.1024999	19.5844831	18.2296557	17.0170406
40	23.1147719	21.3550723	19.7927721	18.4015844	17.1590862

T A B L E

## T A B L E V.

The annuity which  $i$  l. will purchase for a number of years.

Years.	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.
1	1.0300000	1.0350000	1.0400000	1.0450000	1.0500000
2	.5226108	.5264005	.5301961	.5339976	.5378049
3	.3535304	.3569342	.3603485	.3637734	.3672086
4	.2690271	.2722511	.2754901	.2787437	.2820118
5	.2183546	.2214814	.2246271	.2277916	.2309748
6	.1845975	.1876682	.1907619	.1938784	.1970175
7	.1605064	.1635445	.1666096	.1697015	.1728198
8	.1424564	.1454767	.1485279	.1516097	.1547218
9	.1284339	.1314460	.1344930	.1375745	.1406901
10	.1172305	.1202414	.1232909	.1263788	.1295046
11	.1080775	.1110920	.1141490	.1172482	.1203889
12	.1004621	.1034840	.1065522	.1096662	.1128254
13	.0940295	.0970616	.1001437	.1032754	.1064558
14	.0885263	.0915707	.0946690	.0978203	.1010240
15	.0837666	.0868251	.0899411	.0931138	.0963423
16	.0796109	.0826848	.0858200	.0890154	.0922699
17	.0759525	.0790431	.0821985	.0854176	.0886991
18	.0727087	.0758168	.0789933	.0822369	.0855462
19	.0698139	.0729403	.0761386	.0794073	.0827450
20	.0672157	.0703611	.0735818	.0768761	.0802426
21	.0648718	.0680366	.0712801	.0746006	.0779961
22	.0627474	.0659321	.0691988	.0725457	.0759705
23	.0608139	.0640188	.0673091	.0706825	.0741368
24	.0590474	.0622728	.0655868	.0689870	.0724709
25	.0574279	.0606740	.0640120	.0674390	.0709525
26	.0559383	.0592054	.0623674	.0660214	.0695643
27	.0545642	.0578524	.0612385	.0647195	.0682919
28	.0532932	.0566027	.0600130	.0635208	.0671225
29	.0521147	.0554454	.0588799	.0624146	.0660455
30	.0510193	.0543713	.0579301	.0613915	.0650514
31	.0499989	.0533724	.0568554	.0604435	.0641321
32	.0490466	.0524415	.0559486	.0595632	.0632804
33	.0481561	.0515724	.0551036	.0587445	.0624900
34	.0473220	.0507597	.0543148	.0579819	.0617554
35	.0465393	.0499984	.0535773	.0572705	.0610717
36	.0458038	.0492842	.0528869	.0566058	.0604345
37	.0451116	.0486133	.0522396	.0559840	.0598398
38	.0444593	.0479821	.0516319	.0554017	.0592842
39	.0438439	.0473878	.0510668	.0548557	.0587646
40	.0432624	.0468273	.0505235	.0543431	.0582782

*The application of the foregoing tables.*

*Prob. 1.* Principal, rate, and time being given, to find the amount.

Multiply the amount of 1*l.* found in the first table at the rate and for the time given, by the proposed principal, and the product gives the answer.

*Exam. 1.* What will 1000*l.* amount to in 20 years, compound interest being allowed at 5 per cent. per annum?

In the first table, under 5 per cent. and opposite to 20 years we have

$$\begin{array}{r} 2.6532977 \\ \hline 1000 \end{array}$$

L. 2653 5 11 $\frac{1}{4}$ . Answer.

*Exam. 2.* What will 1000*l.* amount to, forborn 14 years and 3 months, at 5 per cent. compound interest?

$$\begin{array}{r} 1.9799316 \\ \hline 1000 \\ \hline 80) 1979.9316 \\ \quad\quad\quad 247491 \\ \hline 2004.6807 \end{array}$$

Hence if any sum is let out at 5 per cent. compound interest; it will double itself in something less than 14 years and  $\frac{1}{2}$ .

*Prob. 2.* Any principal, rate, and amount being given, to find the time.

Divide the amount by the principal, and the quotient will be the amount of 1*l.* at the given rate, which will be found in the first table under that rate, even with the time required.

*Example.* In what time will 1000*l.* become 2653*l.* 5*s.* 11 $\frac{1}{4}$  d. compound interest being allowed at 5 per cent. per annum?

$$\begin{array}{r} 1000) 2653.2977 \\ \hline \end{array}$$

2.6532977 which is found opposite to 20 years.

*Prob. 3.* Principal, time, and amount given, to find the rate of interest:

Divide the amount by the principal, and the quotient will be

be the amount of 1*l.* which being found in the first table even with the given time, and under the rate required.

*Example.* At what rate per cent. per annum will 1000*l.* become 2653*l.* 5*s.* 11*1*/<sub>4</sub>*d.* in 20 years?

$$1000) \underline{2653.2977}$$

$$\underline{2.6532977} \text{ under } 5 \text{ per cent.}$$

*Prob. 4.* Amount, rate, and time being given, to find the principal.

Divide the amount given by the amount of 1*l.* found in the first table, and the quotient will give the answer.

*Example.* What principal sum will amount to 2653*l.* 5*s.* 11*1*/<sub>4</sub>*d.* forborn 20 years, interest upon interest at 5 per cent.?

$$2.6532977) \underline{2653.2977000}$$

$$\underline{1000} \text{ Answer.}$$

*Prob. 5.* Principal, rate, and time being given, to find the annuity.

Multiply the annuity which 1*l.* will purchase at the rate and for the time given, found in the fifth table, by the principal, and the product will be the annuity required.

*Exam. 1.* A gentleman hath 1000*l.* which he would sell for an annuity to continue 20 years; what would be his yearly income for that time, supposing interest upon interest allowed him at 5 per cent.?

$$.0802425$$

$$\underline{1000}$$

$$\underline{80.2425 = L. 80-4-10\frac{1}{4}} \text{ Answer.}$$

*Exam. 2.* A farmer renewes a 19 years lease for 200*l.* gravity, to be paid at signing the lease; but for the farmer's convenience it was changed to an yearly additional rent; how much must be paid annually, compound interest at 5 per cent. to compensate the loss the landlord sustains of 500*l.* ready money?

$$.082745$$

$$\underline{200}$$

$$\underline{16.549000 = L. 16-10-11\frac{1}{4}} \text{ Answer.}$$

*Prob.*

*Prob. 6.* Principal, annuity, and rate being given, to find the time.

Divide the annuity by the principal, and the quotient will be the annuity which 1*l.* will purchase, at the given rate, which will be found in the fifth table under that rate, and even with the time required.

*Example.* An annuity of 80*l.* 4*s.* 10*d.* is purchased for 1000*l.* at the rate of 5 per cent. compound interest; what time ought that annuity to continue?

1000) 80.2425

.082425 even with 20 years. *Answer.*

*Prob. 7.* Principal, annuity, and time given, to find the rate.

Divide the annuity by the principal, and the quotient will be the annuity which 1*l.* will purchase for the given time, which will stand even with the time, and under the required rate in the fifth table,

*Example.* If an annuity of 80*l.* 4*s.* 10*d.* to continue 20 years, be purchased for 1000*l.* what rate of interest hath the purchaser for his money?

1000) 80.2425

.082425 under 5 per cent. *Answer.*

*Prob. 8.* Annuity, rate, and time given, to find the amount.

Find the amount of 1*l. per annum*, at the rate and for the time given in table 3. by which multiply the annuity, and the product will be the amount required.

*Example.* What will an annuity of 80*l.* 4*s.* 10*d.* amount to in 20 years, compound interest being allowed at 5 per cent. per annum?

The amount of 1*l.* for 20 years *per table 3.*

33.0659541
80.2425
1053247705
661319082
1322638164
661319082
2645276328
2653.29482186925

2653.29482186925 *Answer.*

Which will be found equal to the amount of 1000*l.* in 20 years.

*Prob.*

*Prob. 9.* Annuity, rate, and amount given, to find the time.

Divide the amount by the annuity, and the quotient will be the amount of 1*l. per annum* at the given rate, which will be found in the third table under that rate, and even with the time required.

*Example.* In what time will an annuity of 80*l. 4s. 10 $\frac{1}{4}$ d.* amount to 2653*l. 5s. 10 $\frac{3}{4}$ d.* compound interest at 5*per cent.*?

80.2425) 2653.29482186925

33.659541 even with 20 years.

*Prob. 10.* Annuity, time, and amount given, to find the rate.

Divide the amount by the annuity, and the quotient will be the amount of 1*l. per annum*, for the given time; which will be found in the third table even with the given time, and below the required rate.

*Example.* At what rate *per cent. per annum* will an annuity of 80*l. 4s. 10 $\frac{1}{4}$ d.* amount to 2653*l. 5s. 10 $\frac{3}{4}$ d.* in 20 years?

80.2425) 2653.29482186925

33.0659541 in the 3d tab. below 5*per cent.*

*Prob. 11.* Amount, rate, and time given, to find the annuity.

Divide the amount given by the amount of 1*l.* found in the third table, at the rate and time given, and the quotient will be the annuity required.

*Exam.* What annuity will amount to L. 2653.29482186925 in 20 years, compound interest at 5*per cent.*?

33.0659541) 2653.29482186925

80.2425 annuity. *Answer.*

*Prob. 12.* Principal sum in reversion, rate, and time given, to find the present value.

Multiply the present value of 1*l.* for the rate and time given, as in table 2. by the principal, and the product will be the answer.

*Example.* What is the present value of 2653*l. 5s. 10 $\frac{3}{4}$ d.* payable at the expiration of 20 years, at the rate of 5*per cent. per annum*?

2653. *&c.*

$$\begin{array}{r} 2653.29482186925 \\ \hline 3768895 \end{array}$$

1000. &c. Answer.

*Prob. 13.* Annuity, time in reversion, and rate being given, to find the present value.

In the fourth table find the present value of 1*l. per annum*, at the given rate, both for the time in being and the time in reversion, added together, and subtract the time in being from the other, and multiply the remainder by the annuity, the product answers the question.

*Example.* What is it worth in present money to add 14 years to a term of 7 years in being, and thereby make up the term of 21 years, the rent or annuity being 50*l.* computing at the rate of 5 per cent. *per annum*?

The present value of 1*l. per annum* in the 4th table for 21 years is - - - 12.8211527

For 7 years 5.7863734

$$\begin{array}{r} \text{Difference} \quad 7.0347793 \\ \text{Multiplied by} \quad 50 \\ \hline \end{array}$$

351.7389650 Answer.

*Prob. 14.* An annuity several times in reversion, and rate being given, to find the several present values.

Find the present value of *L. 1 per annum*, per table fourth, at the given rate, and for the several given times, which being severally multiplied by the annuity, the products will be the several present values of that annuity, for the several times given; subtract the several present values, the one from the other, and the several remainders answer the question.

*Example.* *A* has a term of 7 years in an estate of *L. 50 per annum*; *B* has a term of 14 years in the same estate in reversion, after the seven years are expired; and *C* hath a farther term of 20 years, after the expiration of 21 years; it is required to know, what is the present value of the several terms, computing at the rate of 5 per cent.

The present value of 1*l.* for 41 years  $\times 50 = 864\ 14\ \frac{1}{4}$

Ditto ditto for 21 years  $\times 50 = 641\ 1\ \frac{1}{4}$

Ditto ditto for 7 years  $\times 50 = 289\ 6\ \frac{1}{4}$

Therefore

Therefore  $L.$  864  $\frac{1}{4}$   $4\frac{1}{4}$   
 $641 \quad 1 \quad 1\frac{1}{4}$

$\underline{223 \quad 13 \quad 2\frac{1}{2}}$  *C's term.*

$641 \quad 1 \quad 1\frac{1}{4}$   
 $289 \quad 6 \quad 4\frac{1}{4}$

$\underline{351 \quad 14 \quad 9\frac{1}{4}}$  *B's term.*

$\underline{575 \quad 7 \quad 11\frac{1}{4}}$  the sum of *B's* and *C's*.

$\underline{289 \quad 6 \quad 4\frac{1}{4}}$  *A's term.*

*Prob. 15.* Any annuity in fee simple, and rate being given, to find the present value.

Multiply the present value of  $L. 1$  per annum, per table fourth, at the given rate by the annuity, and the product gives the answer.

*Example.* What is the value of an estate of  $L. 100$  per annum in fee simple, computing at the rate of 5 per cent. per annum?

$\begin{array}{r} 100 \\ \times 20 \\ \hline \end{array}$

$L. 2000$  *Answer.*

*Prob. 16.* The principal and rate given, to find the annuity in fee simple.

This is the reverse of the last problem, therefore divide the principal by the present value of 1 l. per annum in fee simple, and the quotient will be the answer.

*Example.* What annuity in fee simple will 2000 l. purchase, at the rate of 5 per cent. per annum?

$\begin{array}{r} 2000 \\ \hline \end{array} = 100$  *Answer.*  
 $20$

#### Promiscuous examples.

*Exam. 1.* A person having 12 years to run in a lease of an estate of 60 l. per annum, of 60 l. for 40 years, would know what present money he must pay, in order to complete the lease by adding 28 years thereto, computing at 5 per cent. compound interest?

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3 C

By

By table 4. the present value of 1*l. per annum*, at 5*per cent.* for 40 years, is } 17.1590862  
 By the same table, the value of 1*l. per annum* at that rate, for 12 years to run, is } 8.8632516

Difference	8.2958346
Multiply by	60

*Answer.* 497.7500760

*Exam. 2.* For a lease of certain profits for 7 years, *A* offers to pay 150*l.* gratuity, and 300*l. per annum*; *B* offers 400*l.* gratuity, and 250*l. per annum*; *C* bids 650*l.* gratuity, and 200*l. per annum*; and *D* offers 1800*l.* for the whole purchase, without any yearly rent: query, which is the best offer, and what the difference, computing at 5*per cent.*?

By table 4. the present worth of 300*l. per annum*, for 7 years, at 4*per cent.* is } L. 1800.6164  
 To which add 150.

Value of <i>A</i> 's offer	1950.6164
----------------------------	-----------

The present worth of 250*l. per annum* for 7 years } 1500.5136  
 Add 400

Value of <i>B</i> 's offer	1900.5136
----------------------------	-----------

The present worth of 200*l. per annum* for 7 years } 1200.4109  
 Add 650

Value of <i>C</i> 's offer	1850.4109
----------------------------	-----------

*D*'s offer 1800.

Hence it appears that *A*'s is the best offer, and that rejecting the decimals, he bids 50*l.* more than *B*, 100*l.* more than *C*, and 150*l.* more than *D*.

*Exam. 3.* Which is most advantageous, a term of 19 years of an estate of 100*l. per annum*, or the reversion of such an estate for ever, at the expiration of the said 19 years, computing at the rate of 4*per cent.* compound interest?

A freehold estate of 100*l.* per annum, at 4 per cent. is worth *L.* 2500  
And by table 4. the present worth of 100*l.* annuity for 19 years at 4 per cent. is = 1313.39385

Consequently the reversion is worth 1186.60615

Hence it appears, that the first term of 19 years, is better than the reversion ever after, by 126.7877 = *L.* 126 : 15 : 9.

*Exam.* 4. What annuity would be sufficient to pay off a debt of 100 millions at 4 per cent. compound interest, in 30 years?

By table 5. even with 30 years and under 4 per cent. .0578301  
1000000000  
—

5.783.010.

So that supposing the national debt to be 100 millions, the interest at 4 per cent. would be 4.000.000  
—

It would require a sinking fund of 17.83.010 to clear the national debt in 30 years.

### Variety XVII. ANNUITIES on LIVES.

AS annuities upon lives are daily bought and sold, and at this time become a very considerable branch of business, I shall here offer the public, in miniature, the substance of what the best writers upon this subject have communicated at greater length.

The value of an annuity for life, depends as well upon the interest which money bears, as the probability of the life continuing a longer or shorter time; the former of which is generally settled by law, but the latter can only be determined by observation.

## T A B L E I.

Shewing the probabilities of life from the latest observations  
made at LONDON.

No of persons, curr.	Ages	No of persons, curr.	Ages	No of persons, curr.	Ages	No of persons, curr.	Ages
1280	born.	462	20	294	40	130	60
410 *		7 *		10 *		7 *	
870	1	455	21	284	41	123	61
170 *		7 *		10 *		6 *	
700	2	448	22	274	42	117	62
65 *		7 *		10 *		6 *	
635	3	441	23	264	43	111	63
35 *		7 *		9 *		6 *	
600	4	434	24	255	44	105	64
20 *		8 *		9 *		6 *	
580	5	426	25	246	45	99	65
16 *		8 *		9 *		6 *	
564	6	418	26	237	46	93	66
13 *		8 *		9 *		6 *	
551	7	410	27	228	47	87	67
10 *		8 *		8 *		6 *	
541	8	402	28	220	48	81	68
9 *		8 *		8 *		6 *	
532	9	394	29	212	49	75	69
8 *		9 *		8 *		6 *	
524	10	385	30	204	50	69	70
7 *		9 *		8 *		5 *	
517	11	376	31	196	51	64	71
7 *		9 *		8 *		5 *	
510	12	367	32	188	52	59	72
6 *		9 *		8 *		5 *	
504	13	358	33	180	53	54	73
6 *		9 *		8 *		5 *	
498	14	349	34	172	54	49	74
6 *		9 *		7 *		4 *	
492	15	340	35	165	55	45	75
6 *		9 *		7 *		4 *	
486	16	331	36	158	56	41	76
6 *		9 *		7 *		3 *	
480	17	322	37	151	57	38	77
6 *		9 *		7 *		3 *	
474	18	313	38	144	58	35	78
6 *		9 *		7 *		3 *	
468	19	304	39	137	59	32	79
6 *		10 *		7 *		3 *	
462	20	294	40	130	60	29	80

Note, The numbers marked thus \* are supposed to die off yearly.

## T A B L E

## T A B L E II.

For the valuation of annuities upon one life.

Age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.	Age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.
6	14.1	16.2	18.8	41	10.2	11.4	13.0
7	14.2	16.3	18.9	42	10.1	11.2	12.8
8	14.3	16.4	19.0	43	10.0	11.1	12.6
9	14.3	16.4	19.0	44	9.9	11.0	12.5
10	14.3	16.4	19.0	45	9.8	10.8	12.3
11	14.3	16.4	19.0	46	9.7	10.7	12.1
12	14.2	16.3	18.9	47	9.5	10.5	11.9
13	14.1	16.2	18.7	48	9.4	10.4	11.8
14	14.0	16.0	18.5	49	9.3	10.2	11.6
15	13.9	15.8	18.3	50	9.2	10.1	11.4
16	13.7	15.6	18.1	51	9.0	9.9	11.2
17	13.5	15.4	17.9	52	8.9	9.8	11.0
18	13.4	15.2	17.6	53	8.8	9.6	10.7
19	13.2	15.0	17.4	54	8.6	9.4	10.5
20	13.0	14.8	17.2	55	8.5	9.3	10.3
21	12.9	14.7	17.0	56	8.4	9.1	10.1
22	12.7	14.5	16.8	57	8.2	8.9	9.9
23	12.6	14.3	16.5	58	8.1	8.7	9.6
24	12.4	14.1	16.3	59	8.0	8.6	9.4
25	12.3	14.0	16.1	60	7.9	8.4	9.2
26	12.1	13.8	15.9	61	7.7	8.2	8.9
27	12.0	13.6	15.6	62	7.6	8.1	8.7
28	11.8	13.4	15.4	63	7.4	7.9	8.5
29	11.7	13.2	15.2	64	7.3	7.7	8.3
30	11.6	13.1	15.0	65	7.1	7.5	8.0
31	11.4	12.9	14.8	66	6.9	7.3	7.8
32	11.3	12.7	14.6	67	6.7	7.1	7.6
33	11.2	12.6	14.4	68	6.6	6.9	7.4
34	11.0	12.4	14.2	69	6.4	6.7	7.1
35	10.9	12.3	14.1	70	6.2	6.5	6.9
36	10.8	12.1	13.9	71	6.0	6.3	6.7
37	10.6	11.9	13.7	72	5.8	6.1	6.5
38	10.5	11.8	13.5	73	5.6	5.9	6.2
39	10.4	11.6	13.3	74	5.4	5.6	5.9
40	10.3	11.5	13.2	75	5.2	5.4	5.6

## T A B L E

## TABLE III.

For the valuation of annuities upon two joint lives.

For

Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.	Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.
6	11.3	12.7	14.4	41	7.2	8.0	8.9
7	11.5	12.9	14.6	42	7.1	7.8	8.7
8	11.6	13.0	14.7	43	7.0	7.7	8.6
9	11.6	13.0	14.7	44	6.9	7.6	8.5
10	11.6	13.0	14.7	45	6.7	7.4	8.3
11	11.5	12.9	14.6	46	6.6	7.3	8.2
12	11.4	12.8	14.5	47	6.5	7.2	8.1
13	11.3	12.7	14.3	48	6.4	7.1	7.9
14	11.2	12.5	14.1	49	6.3	7.0	7.8
15	11.0	12.3	13.9	50	6.2	6.8	7.6
16	10.8	12.1	13.7	51	6.1	6.7	7.4
17	10.7	11.9	13.5	52	6.0	6.6	7.3
18	10.5	11.7	13.2	53	5.9	6.5	7.2
19	10.3	11.5	13.0	54	5.8	6.3	7.0
20	10.1	11.3	12.8	55	5.7	6.2	6.9
21	10.0	11.2	12.6	56	5.6	6.1	6.7
22	9.8	11.0	12.4	57	5.5	6.0	6.6
23	9.7	10.8	12.2	58	5.4	5.8	6.4
24	9.5	10.6	12.0	59	5.3	5.7	6.3
25	9.4	10.5	11.8	60	5.2	5.6	6.1
26	9.2	10.3	11.6	61	5.1	5.5	6.0
27	9.1	10.1	11.4	62	5.0	5.4	5.9
28	8.9	9.9	11.2	63	4.9	5.3	5.7
29	8.8	9.8	11.0	64	4.8	5.1	5.5
30	8.6	9.6	10.8	65	4.7	5.0	5.4
31	8.5	9.4	10.6	66	4.6	4.9	5.3
32	8.3	9.2	10.4	67	4.5	4.8	5.1
33	8.2	9.1	10.2	68	4.4	4.6	4.9
34	8.1	8.9	10.0	69	4.3	4.5	4.8
35	8.0	8.8	9.9	70	4.2	4.4	4.6
36	7.8	8.6	9.7	71	4.1	4.3	4.5
37	7.6	8.4	9.5	72	3.9	4.1	4.3
38	7.5	8.3	9.3	73	3.8	4.0	4.2
39	7.4	8.2	9.2	74	3.7	3.8	4.0
40	7.3	8.1	9.1	75	3.6	3.7	3.8

## TABLE

## T A B L E IV.

For the valuation of annuities upon the longest of two lives.

Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.	Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.
6	16.9	19.7	23.3	41	13.2	14.9	17.0
7	17.0	19.8	23.4	42	13.1	14.7	16.8
8	17.1	19.9	23.5	43	13.0	14.5	16.5
9	17.1	19.9	23.5	44	12.9	14.3	16.3
10	17.1	19.9	23.5	45	12.8	14.2	16.1
11	17.1	19.9	23.5	46	12.6	14.0	15.8
12	17.0	19.8	23.4	47	12.5	13.8	15.6
13	16.9	19.7	23.3	48	12.4	13.6	15.3
14	16.7	19.5	23.1	49	12.2	13.4	15.1
15	16.6	19.3	22.9	50	12.1	13.3	14.9
16	16.4	19.1	22.6	51	11.9	13.1	14.6
17	16.2	18.9	22.4	52	11.8	12.9	14.4
18	16.1	18.7	22.1	53	11.6	12.7	14.1
19	15.9	18.5	21.9	54	11.5	12.5	13.9
20	15.7	18.3	21.6	55	11.3	12.3	13.6
21	15.6	18.2	21.3	56	11.2	12.1	13.4
22	15.4	18.0	21.1	57	11.0	11.9	13.1
23	15.3	17.8	20.8	58	10.9	11.7	12.8
24	15.1	17.6	20.6	59	10.7	11.5	12.5
25	15.0	17.4	20.3	60	10.5	11.2	12.2
26	14.9	17.3	20.1	61	10.3	11.0	12.0
27	14.7	17.1	19.9	62	10.1	10.8	11.7
28	14.6	16.9	19.7	63	9.9	10.5	11.4
29	14.5	16.8	19.5	64	9.7	10.3	11.1
30	14.4	16.6	19.3	65	9.4	10.0	10.8
31	14.2	16.4	19.1	66	9.2	9.7	10.5
32	14.1	16.2	18.9	67	8.9	9.4	10.2
33	14.0	16.1	18.7	68	8.7	9.2	9.9
34	13.9	15.9	18.5	69	8.5	8.9	9.5
35	13.8	15.8	18.3	70	8.2	8.6	9.2
36	13.7	15.6	18.1	71	8.0	8.4	8.9
37	13.6	15.5	17.9	72	7.7	8.1	8.6
38	13.5	15.3	17.7	73	7.5	7.8	8.2
39	13.4	15.2	17.5	74	7.2	7.5	7.9
40	13.3	15.0	17.3	75	6.9	7.2	7.6

## T A B L E

## TABLE V.

For the valuation of annuities upon three joint lives.

Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.	Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.
6	9.7	10.6	11.7	41	5.5	6.1	6.8
7	9.9	10.8	11.9	42	5.4	6.0	6.7
8	10.0	10.9	12.0	43	5.4	5.9	6.5
9	10.0	10.9	12.0	44	5.3	5.8	6.4
10	10.0	10.9	12.0	45	5.2	5.7	6.3
11	9.9	10.8	11.9	46	5.1	5.6	6.2
12	9.8	10.7	11.8	47	5.0	5.5	6.1
13	9.6	10.5	11.6	48	5.0	5.4	5.9
14	9.5	10.4	11.4	49	4.9	5.3	5.8
15	9.3	10.2	11.2	50	4.8	5.2	5.7
16	9.2	10.0	11.0	51	4.7	5.1	5.6
17	9.0	9.8	10.8	52	4.7	5.1	5.5
18	8.8	9.6	10.6	53	4.6	5.0	5.4
19	8.6	9.4	10.4	54	4.5	4.9	5.3
20	8.4	9.2	10.2	55	4.4	4.8	5.2
21	8.2	9.0	10.0	56	4.4	4.7	5.1
22	8.1	8.9	9.8	57	4.3	4.6	5.0
23	7.9	8.7	9.6	58	4.2	4.5	4.9
24	7.7	8.5	9.4	59	4.1	4.4	4.8
25	7.6	8.3	9.2	60	4.0	4.3	4.6
26	7.4	8.1	9.0	61	3.9	4.2	4.5
27	7.3	8.0	8.8	62	3.8	4.1	4.4
28	7.1	7.8	8.6	63	3.7	4.0	4.3
29	7.0	7.7	8.5	64	3.7	3.9	4.2
30	6.8	7.5	8.3	65	3.6	3.8	4.1
31	6.7	7.4	8.2	66	3.5	3.7	3.9
32	6.5	7.2	8.0	67	3.4	3.6	3.8
33	6.4	7.1	7.9	68	3.3	3.5	3.7
34	6.2	6.9	7.7	69	3.2	3.4	3.6
35	6.1	6.8	7.6	70	3.1	3.2	3.4
36	6.0	6.7	7.4	71	3.0	3.1	3.3
37	5.9	6.5	7.2	72	2.9	3.0	3.1
38	5.8	6.4	7.1	73	2.8	2.9	3.0
39	5.7	6.3	7.0	74	2.6	2.7	2.8
40	5.6	6.2	6.9	75	2.5	2.6	2.7

## TABLE

## T A B L E VI.

For the valuation of annuities upon the longest of three lives.

Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.	Mean age.	Years purch. at 5 per cent.	Years purch. at 4 per cent.	Years purch. at 3 per cent.
6	18.0	21.0	25.0	41	14.6	16.4	18.9
7	18.1	21.1	25.1	42	14.5	16.3	18.7
8	18.2	21.2	25.2	43	14.4	16.2	18.5
9	18.2	21.2	25.2	44	14.3	16.0	18.2
10	18.2	21.2	25.2	45	14.2	15.9	18.0
11	18.2	21.2	25.2	46	14.1	15.7	17.7
12	18.1	21.1	25.1	47	13.9	15.5	17.5
13	18.0	21.0	25.0	48	13.8	15.2	17.2
14	17.9	20.9	24.8	49	13.7	15.1	17.0
15	17.8	20.7	24.6	50	13.5	14.9	16.7
16	17.6	20.5	24.3	51	13.4	14.7	16.5
17	17.5	20.3	24.1	52	13.2	14.5	16.2
18	17.3	20.1	23.8	53	13.1	14.3	15.9
19	17.2	19.9	23.5	54	12.9	14.1	15.7
20	17.0	19.7	23.3	55	12.8	13.9	15.4
21	16.9	19.5	23.1	56	12.6	13.7	15.1
22	16.8	19.4	22.8	57	12.5	13.5	14.8
23	16.6	19.2	22.6	58	12.3	13.2	14.5
24	16.5	19.0	22.3	59	12.1	12.9	14.1
25	16.4	18.8	22.1	60	11.9	12.7	13.8
26	16.3	18.7	21.9	61	11.7	12.5	13.5
27	16.1	18.5	21.6	62	11.5	12.2	13.1
28	16.0	18.3	21.4	63	11.3	11.9	12.8
29	15.9	18.2	21.2	64	11.0	11.6	12.5
30	15.8	18.0	21.0	65	10.8	11.4	12.2
31	15.6	17.8	20.8	66	10.5	11.1	11.8
32	15.5	17.7	20.6	67	10.2	10.8	11.5
33	15.4	17.6	20.4	68	9.9	10.5	11.2
34	15.3	17.4	20.2	69	9.6	10.2	10.9
35	15.2	17.3	20.0	70	9.3	9.9	10.5
36	15.1	17.2	19.9	71	9.0	9.6	10.2
37	15.0	17.0	19.7	72	8.7	9.2	9.8
38	14.9	16.9	19.5	73	8.4	8.9	9.5
39	14.8	16.7	19.3	74	8.1	8.6	9.1
40	14.7	16.6	19.1	75	7.8	8.2	8.7

*Use and explanation of the first table.*

Suppose it was required to find the probability which a person of 40 years of age hath to live 30 years longer, that is, to the age of 70; look in the table against 40 years and 70 years, fronting which will be found 294 and 69 respectively; shewing that out of 294 persons living of 40 years of age, no more than 69 arrive at the age of 70: therefore, since the whole number of persons living at the beginning of this term, is to the number remaining alive at the end of it, in the ratio of 294 to 69; the number of chances that a person of 40 years hath to live 30 years longer, will be to the number of all the chances, that he hath both to live beyond, and die within 30 years, in the same ratio of 294 to 69; consequently  $\frac{69}{294}$  is the measure of the probability required; the probability that any event will happen, being always to be considered as the ratio of the chances which that event hath to happen, to all the chances which that event hath to happen and fail. Hence in ordinary cases may the premiums for insurance on lives be estimated, and annuities bought and sold, in proportion to the risk; which is obvious from this table. For instance, suppose it were required to find the value of an annuity of 100*l.* for a life of 20 years of age, interest 4 per cent. — Because the present value of 100*l.* due at the expiration of one year, is *L.* 96.15; it is plain that *L.* 96.15 would be the value of the first year's annuity paid down, was the purchaser sure of recovering it; but the probability of his living one year appearing from the table to be only  $\frac{455}{462}$ , the sum of *L.* 96.15 must be diminished in the ratio of 462 to 455, in order to make a just deduction out of it, for the contingency of his dying before the end of one year, which will reduce it to 94.7, equal to the true value of the first year's annuity. After the same manner may the value of the second year's annuity be calculated; for since the probability of receiving the second year's annuity is in the ratio of 462 to 448, the present value of 100*l.* to be received at the end of two years, when the proportional discount is allowed for  $\frac{448}{462}$ , will be found to be *L.* 89.65. In like manner may the 3d, 4th, 5th, 6th, &c. years annuity be computed to the utmost extent of life, the sum of all which will be the value of the annuity required.

*Problems applying the use of the foregoing tables.*

*Prob. 1.* To find the value of an annuity for an assigned life.

### R U L E.

In table 2. against the given age, and under the assigned rate of interest, will stand the number of years purchase, which an annuity upon that life is worth.

### E X A M P L E S.

*Exam. 1.* Suppose one of 20 years of age would sell an annuity of 100*l.* during his life, what ready money would the annuity be worth, interest being allowed at 4 *per cent.*?

### S O L U T I O N.

First, opposite to 20 years and under 4 *per cent.* we have in table 2. 14.8, which ascertains the value to be so many years purchase, and by multiplying 14.8 into the annuity 100*l.* we have 1480*l.* for the answer or present worth.

*Exam. 2.* A widow lady, with 200*l.* a-year jointure, aged 30 years, marries a young merchant, who, to enlarge his capital, proposes to sell the jointure; what ready money should he receive, discounting interest at 5 *per cent.*?

### S O L U T I O N.

Opposite to 30 and under 5 *per cent.* we have 11.6 for the year's purchase, which multiplied into 200 = 2320*l.* Answer.

*Prob. 2.* To find the value of an annuity upon two assigned joint lives.

*Case 1.* If the two lives be equal, the answer is found at once *per table 3.*

### E X A M P L E.

Suppose it were required to find the value of an annuity, 50*l.* for instance, upon two joint lives, each 20 years, interest 5 *per cent.*

### S O L U T I O N.

Opposite to 20 and under 5 *per cent.* in table 3. we have  
3 D. 2

10.1 the number of years purchase, which multiplied by 50 l. the annuity, produceth 505 l. for the answer.

*Case 2.* If the given ages be unequal, but neither less than 25, nor exceeding 50, take half the sum of the two for a mean age, and then proceed as in *case 1.*

*Example.* There are two joint lives upon an annuity of 500 l. the age of the one is 30, and that of the other 40; what is the present worth of that annuity, interest 4 per cent.?

### S O L U T I O N.

$$\begin{array}{r} 30 \\ 40 \\ \hline 2) \underline{70} \end{array}$$

35 mean age per table 3. = 8.8 years purchase.

500 annuity.

L. 4400 Answer.

*Case 3.* If one or both ages be without the limits 25 and 50, multiply the difference by one half the lesser of the two values, and divide the product by the greater, and the quotient added to the lesser value will give the answer very near.

### E X A M P L E S.

*Exam. 1.* Let the one age be 12 and the other 60 upon an annuity of 100 l. interest 5 per cent.

### S O L U T I O N.

Per table 3. 12 = 11.4

60 = 5.2

6.2 difference.

2.6 the lesser value.

$$\begin{array}{r} 372 \\ 124 \\ \hline 11.4) \underline{16.12} \end{array}$$

1.414 quotient.

5.2 lesser value.

6.614 sum.

100

L. 661.4 Answer.

*Exam.*

*Exam. 2.* Let the age of the one be 8, and the other 30; annuity and interest as above.

## S O L U T I O N.

$$\begin{array}{r}
 8 = 11.6 \\
 30 = 7.8 \\
 \hline
 & 3.8 \\
 & 3.9 \\
 \hline
 11.6) 14.82 \\
 \hline
 & 1.279 \\
 & 7.8 \\
 \hline
 & 9.079 \\
 & 100 \\
 \hline
 & 907.9 \text{ Answer.}
 \end{array}$$

*Prob. 3.* To find the value of an annuity upon two lives to continue as long as either of them is in being.

*Case 1.* If the lives are equal, find the given age in tab. 4, and against it, under the proposed rate of interest, will be the number of years purchase required.

## E X A M P L E.

Suppose each of the ages 30, the annuity 100*l.* interest 3 per cent.

## S O L U T I O N.

Opposite to 30, table 4. under 3 per cent. 19.3

100

*L.* 1930 *Answer.*

*Case 2.* If both ages be between 25 and 50, take half their sum for a mean age, and then proceed as before.

## E X A M P L E.

Suppose one of the ages 28, and the other 48, annuity 100*l.* interest 5 per cent.

S O-

## S O L U T I O N.

$$\begin{array}{r}
 28 \\
 48 \\
 \hline
 2) 76 \\
 \hline
 38 = 13.5 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{l}
 \text{Or} \quad 28 = 14.6 \\
 48 = 12.4 \\
 \hline
 2) 27.0 \\
 \hline
 13.5
 \end{array}$$

1350 Answer.

*Cafe 3.* If the given ages do not correspond to the limitations 25 and 50, find the value of the two joint lives by the last case of the preceding problem, which subtract from the sum of the values of the two single lives, and there will remain the required value of an annuity upon the longest life.

## E X A M P L E.

Let the one age be 12, and the other 60 upon an annuity of 100*l.* interest 5 per cent.

## S O L U T I O N.

$$\begin{array}{l}
 \text{Per table 4. } 12 = 17 \\
 \text{and } 60 = 9.2 \\
 \hline
 \end{array}$$

26.2 sum.

$$\begin{array}{l}
 \text{Per prob. 2. case 3. value } \} \\
 \text{of 2 such joint lives. } \} \\
 \hline
 \end{array}
 6.614$$

19.586 = *L.* 1958.6 Answer.

*Prob. 4.* To find the value of an annuity upon three joint lives.

*Cafe 1.* If all the lives are equal, the number of years purchase is found at once *per table 5.*

## E X A M P L E.

Suppose there are three joint lives upon an annuity of 100*l.* interest 5 per cent. each 25 years of age — required the value.

S O-

## S O L U T I O N.

$$\text{Per table 5. } 25 = \frac{7.6}{100}$$

L. 760 Answer.

*Cafe 2.* If all the three ages be between 15 and 55, and the difference betwixt the greatest and least not more than 15,  $\frac{1}{2}$  of the sum of their ages will be a mean age, with which proceed as in case 1.

## E X A M P L E.

Suppose there are three joint lives upon an annuity of 100*l.* 5 per cent interest, viz. *A*, *B*, and *C*; *A* is 20 years of age, *B* 26, and *C* 30; required the value.

$$\begin{array}{r} 20 \\ 26 \\ 30 \\ \hline 2) 76 \\ \hline 38 = 5.8 \\ 100 \\ \hline 580 \end{array}$$

*Cafe 3.* If one or all of the ages be without the limitations prescribed in the last case, multiply the sum of the three corresponding values by the square of the least of them, reserving the product; then multiply the two greater values into each other, and to the double of the product add the square of the lesser value, divide the reserved product by this sum, and subtract the quotient from twice the lesser value, and the remainder will give the years purchase.

## E X A M P L E.

Let the three ages be 13,  $3\frac{1}{2}$ , and 53, interest 4 per cent.

The value corresponding to 13 = 10.5

to  $3\frac{1}{2}$  = 7.3

and to 53 = 5

$$\begin{array}{r} \text{Sum} = 22.8 \\ 5 \times 5 = 25 \\ \hline \end{array}$$

Product to be reserved 570 Then

Then  $10.5 \times 7.3 = 76.65$

2

$$\begin{array}{r} 153.30 \\ - 25 \\ \hline \end{array}$$

$178.3 (570.0)$

L.

$5 \times 2 = 10 - 3.2 = 6.8 \times 100 = 680$  Answer.

**Prob. 5.** To find the value of an annuity upon the longest of three lives.

*Cafe 1.* If the ages be all equal, the answer will be found at once by table 6.

### E X A M P L E.

Suppose the three ages be each 30 years for 100 l. annuity, interest 5 per cent.

### S O L U T I O N.

Per table 6.  $30 = 15.8$  years purchase.  
100 annuity.

L. 1580 value.

*Cafe 2.* If none of the ages be below 10 or above 60, and the difference betwixt the greatest and least of them not exceeding 15; to double the sum of the two least add the greatest, and take  $\frac{1}{2}$  of the sum for a mean age, with which proceed as in the last cafe.

### E X A M P L E.

Let the proposed ages be 16, 24, and 30; annuity 100 l. and interest 5 per cent.

### S O L U T I O N.

$16 \times 2 = 32$   
 $24 \times 2 = 48$  } twice the sum of the two least.  
30 the greatest.

5) 110

Mean age  $\frac{22}{100} = 19.4$  per table 6.

L. 1940 value.

*Cafe*

Var

S  
U  
V

*Case 3.* If the given ages do not correspond to the limitations in the last case; find the value answering to the greatest of the given ages *per table 4.* and the values corresponding to all the three several ages by *table 6.* and let the difference of the two values answering to the greatest age be taken and reserved; let the square of the greater of these two be divided by the product of the two other remaining values; multiply the square of the quotient by the reserved difference, then this last product, added to the value of an annuity for the two youngest lives, will be the value required.

## E X A M P L E.

Let the three ages be respectively 20, 36, and 60; annuity 100*l.* and interest 4 *per cent.*

## S O L U T I O N.

$$\begin{array}{l} 60 = 12.7 \text{ } per \text{ } table \text{ } 6. \\ 60 = 11.2 \text{ } per \text{ } table \text{ } 4. \end{array}$$


---

1.5 difference.

$$\begin{array}{l} \text{Then } 20 = 19.7 \\ \quad\quad\quad 36 = 17.2 \end{array} \left. \begin{array}{l} \text{per table 6.} \\ \text{per table 6.} \end{array} \right\}$$


---

$$338.84) \overline{161.29} = 12.7 \times 12.7$$

$$\begin{array}{r} .5 \times .5 = .25 \\ \hline 1.5 \end{array}$$


---

$$\begin{array}{l} \text{Value of the two youngest lives } \\ \text{by case 2. prob. 3. } \end{array} \left. \begin{array}{l} \cdot 375 \\ = 17 \end{array} \right\}$$

$$\begin{array}{r} 17.375 \\ \hline 100 \end{array}$$


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1737.5 Answer.

## Variety XVIII. Mensuration of Surfaces and Solids.

## SECT. I. Of Surfaces.

SURFACES are measured by the superficial inch, foot, or yard, according to the stated measures of different countries,

and the measures peculiar to different artists, as carpenters, plasterers, surveyors of land. The superficial inch is any small surface of one inch length and breadth; and because 12 inches make 1 foot of long measure,  $12 \times 12 = 144$  inches make 1 superficial foot, 9 feet make 1 yard,  $30\frac{1}{4}$  yards make 1 perch, 40 perches make 1 rood, and 4 roods make 1 acre of land-measure. The superficial content of every surface is found by the proper rule of its figure, whether square, triangle, polygon, or circle.

The superficial content or area of rectangular square figures, whether equal or oblong, is obtained by multiplying the length by the breadth; the product gives the content of the same name with the dimensions taken, either inches, yards, or feet, accordingly.

*Exam. 1.* What is the superficial content of a board 15.5 feet long, and 3.5 feet broad? Here the dimensions being taken in feet, the product will be superficial feet,  $15.5 \times 3.5 = 54.25$ , the answer required. To perform this and the like by the sliding rule: Set 1 on the line *A* to 15.5 on the line *B*; look for 3.5 on the line *A*, and opposite to this you have 54.25 on the line *B*; or on the gunter two foot scale, extend the compass from 1 to 15.5; that distance laid the same way from 3.5 will reach to 54.25, the answer required.

*Exam. 2.* What is the content of a deal 14.5 feet long and 15 inches broad? Here the length being given in feet, and the breadth in inches, say, If 12 inches give 14.5 feet, what will 15 inches give?  $14.5 \times 15 \div 12 = 18.125$ , which is the answer in feet. By the sliding rule, set 12 on *A* to 14.5 on *B*; and opposite to 15 on *A* you have 18.125 on *B*: or on the gunter, the extent from 12 to 14.5 will reach the same way from 15 to 18.125. If there were a number of such deals to be measured, suppose 50, you need only multiply the content of one 18.125 by 50, the product 906.25 is the content of the whole.

The superficial content of a triangle is found by multiplying  $\frac{1}{2}$  of the base by the perpendicular falling on that base from the opposite angle. For example, let it be required to find in acres the area of a triangular piece of ground, 46 chains by the base, and 18 by the perpendicular.  $23 \times 18 = 414$  square chains, which divided by 10, the number of square chains in an acre, quotes 41.4 acres. By the sliding rule, set 10 on *A* to 23 on *B*; and opposite to 18 on *A* you have

41.4 acres on *B*; on the gunter, the extent from 10 to 23 reaches from 18 to 41.4.

The superficial content of a trapezoid, or the segment of <sup>a</sup> triangle cut by a line parallel to the base, is found by adding the parallel sides together, and multiplying  $\frac{1}{2}$  of that sum by the perpendicular breadth, the product is the content.

An example of this in slaters work: Suppose a pavilion roof the opposite sides equal trapezoids; let one of the sides be 81 feet long below, and 62 above, the breadth 30; let the other side be 54 feet below, and 41 above, the breadth 30; required the number of superficial yards of slating in the whole?

$$81+62=143 \div 2=71.5 \text{ Then } 54+41=95 \div 2=47.5 \\ \text{Add } 47.5$$

$$119 \times 30 = 3570 \div 9 = 396\frac{2}{3} \text{ yards, which doubled}$$

for the opposite sides give  $793\frac{1}{3}$  yards in all. By the sliding rule, set 9 on *A* to 119 on *B*, because the dimensions are taken in feet, and the content required in yards; then look for 30 on *A*, against which you find 396.6 on *B*, which double for the answer.

Trapeziums, or irregular quadrilateral figures, are divided by a diagonal into two triangles, and the content of each found, by the rule proper to a triangle mentioned above. An example in land-measure. Suppose a quadrilateral piece of ground, the diagonal 22 chains 40 links, the perpendicular falling on this diagonal from one of the opposite angles 12 chains 30 links, and from the other opposite angle 9 chains 25 links; required the content in acres? Add the 2 perpendiculars together, 12 chains 30 links + 9 chains 25 links = 21 chains 55 links, by which multiply half the diagonal 11:20; the product is 241.36 square chains, this divided by 10=24 acres, and .136 remaining; bring the remainder to rods by 4=.544, and then to perches by 40, the whole content is 24 acres 0 rods 21 perches. By the sliding rule, set 10 on *A* to 21.5 on *B*, and opposite to 11.2 on *A*, you have 24.1 acres on *B*. Here observe, that all irregular polygons are in the same manner divided into triangles; and the content of each found, and added into one sum.

Regular polygons are distinguished by the number of equal  
3 E 2 sides;

sides ; thus a figure having 5 equal sides and 5 equal angles, is termed a *pentagon* ; a figure having six equal sides is called a *hexagon*. The general rule whereby the superficial content of any polygon is found, is this : Take  $\frac{1}{2}$  the circumference, or the length of one side multiplied by  $\frac{1}{2}$  the number of sides, multiply this by the perpendicular falling from the centre of the figure on any of the sides ; the product gives the superficial content. An example of paviours work.

Suppose a piece of pavement in the form of a regular hexagon, the length of the side 35 feet, the perpendicular 30.3 feet ; required the area in superficial yards ?  $35 \times 3 = 105$  the half circumference which  $\times 30.3$  gives 3181.5, this divided by 9 gives 353.5 yards, the answer. By the rule set 9 on A to 105 on B ; and against 30.3 you have 353.5, the same answer.

The area of a circle is found by squaring the diameter, and multiplying the product by this constant factor .7854, which expresseth the area of a circle whose diameter is unity. Suppose a table round and circular 5 feet diameter, required its area in square feet ?  $5 \times 5 = 25 \times .7854 = 19.635$  feet, the answer ; were it an oval table 6 feet long and 4 broad, the content would be  $6 \times 4 = 24 \times .7854 = 18.849$ .

### SECT. II. Of Solids.

SOLIDS are measured by the solid inch, foot, or yard. A solid or cubical inch is a body of a die-figure, its length, breadth, and depth being 1 inch,  $\frac{1}{12}$  of these inches, that is  $12 \times 12 \times 12$ , make one cubical foot, and 27 cubical feet make one cubical yard. Also 282 solid inches make 1 gallon English ale-measure ; 231 inches make 1 gallon wine-measure ; 2150 inches are 1 malt bushel ; 40 solid feet make 1 tun of wood. Now, the solid content of every body is found by rules adapted to its particular figure.

Cubes and parallelopipedons are so called, because they are contained under six surfaces, the opposite ones whereof are parallel and equal ; and their solid content is found by multiplying the length by the breadth, and that product by the depth.

*Exam. 1.* Required in solid feet the content of a log equal at both ends, the length 16.5 feet, and 14 inches every way by the side ; or, as carpenters term it, 14 inches square. First, square 14, that is,  $14 \times 14 = 196$ , then say, as 144 inches to the length 16.5 feet, so is 196 inches to the answer in feet ;

16.5

$16.5 \times 196 = 3234$ , which divided by 144 gives 22.46 feet. This and the like is readily performed by the sliding rule; set 12 on the girt line marked *D*, to the length on *C*, then look for 14 on *D*, and opposite to it you have the answer on *C* 22.46: or by the gunter, extend your compass from 12 to 16.5, that distance twice turned over will reach from 14 to 22.4, as before.

*Exam. 2.* Required the content in malt bushels of a chest 40 inches long, 20 inches broad, and 15 deep;  $40 \times 20 = 800 \times 15 = 12000$ , which divided by 2150 inches in a malt bushel, gives 5.581 the answer. By the sliding rule first find a mean proportional between the length 40, and the breadth 20, which is 28.28. Then set 46.36 the gauge point for malt marked on Everard's sliding rule *M S*, on the line *D*, to 15 on *C*, and opposite to 28.28 on *D*, you have the answer on *C*.

Prisms of all kinds, whether square, triangular, or polygonal, are measured by one general rule. Find the superficial content or area of the base by the proper rule of sect. I.; this multiplied by the length or height of the prism, gives the solid content. What is the content in ale-gallons of a triangular prism, the triangular base whereof is 36 inches in the side, and 24 inches by the perpendicular falling on that side? The altitude being 40 inches,  $\frac{1}{2}$  of  $36 = 18 \times 24 = 432 \times 40 = 17280$ , this divided by 282 inches in one ale-gallon gives 61.27 gallons. Pyramids may be reduced to the same rule, being  $\frac{1}{3}$  of their circumscribing prism.

The segments of pyramids, as are all square logs greater at one end than at the other, are generally measured by carpenters in the following manner: Take the girt of the log in the middle,  $\frac{1}{4}$  part of which note down; multiply it by itself taken in inches, then say, as 144 is to the length of the log in feet, so is that product to the content in feet. *Example.* Suppose a square log 60 inches round in the middle, and 24 feet long; required its content in solid feet; and how many tuns of wood are contained in 20 of these logs? First,  $\frac{1}{4}$  of 60 = 15, which  $\times 15$  gives 225; then  $144 : 24 :: 225 : 37.5$ , the content of one log, which multiplied by 20 the number of logs, and divided by 40 solid feet in a tun of wood, gives the number of tuns required, 18 tuns 30 solid feet. By the sliding rule, set 12 on the line *D*, to 24 the length in feet on the line *C*; and against 15 on *D*, you have the answer on *C* 37.5; or on the gunter extend from 12 to 15, that distance doubled

doubled or twice turned over the same way reaches from 24 to 37.5, the content of 1 log in feet. Then to find the content of 20 logs in tuns, set 40 on *A* to 20 on *B*, and against 37.5 on *A* you have 18.75 on *B*.

A cylinder is measured by multiplying the square of the diameter by .7854, and multiplying that product by the length. *Example.* Suppose an equal round piece of timber 14 inches diameter, and 5 feet long; what is its solid content?  $14 \times 14 = 196$ , multiply by the length, taken in feet  $196 \times 5 = 980$ , and divide by this constant divisor 183.34, which is a gauge point for solid feet in round timber arising from the division of 144 by .7854, the quotient 5.345 gives the solid content of the piece in feet before the slobes are taken off.

By this rule all round timber, whether equal at both ends or not, is readily measured both by the pen and sliding rule, the diameter in the middle being known.

Suppose a round log 15 feet long, and the circumference in the middle 66 inches, required its solid content in feet, before the slobes are taken off? First, find the diameter by this proportion, as  $22 : 7 :: 66 : 21$ . Square 21, that is  $21 \times 21 = 441$ , multiply by the length in feet,  $441 \times 15 = 6615$ , this divided by the above-mentioned point 183.34 gives the answer 36.08 feet.

By the sliding rule, set 13.5 (the square root of 183.34) found on the line *D*, to 15 feet the length on *C*, and opposite to 21 the diameter on *D*, you have the same answer on *C* 36.08; or because it may seem tedious to find the diameter by the circumference, to shorten the work, say, as 42.54 a point is to 66 the circumference, so is the length 15 feet to 23.27 a fourth number; again say, as 42.54 is to 66 :: so is 23.27 to 36.05 the answer. Set 42.54 on *D* to 15 the length on *C*, and opposite to 66 the circumference on *D*, you have the answer 36.05 on *C*: or on the gunter, extend your compass from 42.54 to the circumference 66, that distance twice turned over the same way, from 15 the length, reaches to the same answer as before.

#### Cask-gauging.

There are many different canons drawn from stereometry for the gauging of casks; the following is both exact, and the most expeditious, by pen and sliding rule: Take the dimensions of the cask in inches; namely the diameter at the bung, the diameter

diameter at the top, and the length of the cask; subtract the top diameter from the bung diameter, and note down the difference; if the staves of the cask are much curved and bulging between the bung and the top, multiply the difference by .7; if they do not bulge so much, multiply by .65; if they bulge yet less, multiply by .6; if they are straight, or almost straight, multiply by .55, and add the product to the top diameter; the sum will be a mean diameter by which the cask is reduced to a cylinder. Square the mean diameter thus found; then multiply by the length of the cask. Divide the product by 359 for beer-gallons, or 294 for wine-gallons.

*Exam. 1.* Required the content in ale gallons of a round hogshead, the bung diameter 48 inches, the top diameter 36 inches, the length 60 inches?

Bung diam.	48
Top diam.	36
Difference	12
×	.7
	8.4
Add the top diam.	36.0
The mean diam.	44.4
	44.4
	1776
	1776
	1776
Squared	1971.36
The length	60
For ale gallons	359) 118281.60 (329.47 ale gallons,

1077
1058
718
3401
3231
1706
1436
2700
2513

By the sliding rule.

On the line *D* you have 18.9 the gauge point for ale gallons marked *AG*. Set this to the length of the cask 60 on *C*, and against 44.4 the mean diameter on *D*, you have the answer on *G* 329.5 as above.

*Exam. 2.* Required the content in wine gallons of a pipe 36 inches by the bung, 26 by the top, and length 54 inches?

Bung diam.	36
Top diam.	26
Difference	10
x .6	.6
	6
Add the top diam.	26
The mean diam.	32
	32
	64
Squared	1024
Length	54
	4096
	5120

Divisor for wine 294) 55296 (188.08 wine gallons.

294	
	2589
	2352
	2376
	2352
	2400
	2352
	48

By the sliding rule.

On the line *D* you have 17 the gauge point for wine-gallons marked *WG*. Set this to the length 54 on *C*; and against 32 the mean diameter on *D*, you have the answer on *C* 188.08 as before

*The End of the FIRST VOLUME.*



